value of x. Determine the average rate of change for these values of x and y. When the average rate of change has stabilized to a constant value, this is the instantaneous rate of change.

13. **a)** and **b)** Only a and k affect the instantaneous rate of change. Increases in the absolute value of either parameter tend to increase the instantaneous rate of change.

Chapter Review, pp. 510–511

1. **a)** $y = \log_4 x$ **c)** $y = \log_{\frac{3}{4}} x$
b) $y = \log_a x$ **d)** $m = \log_p q$

2. **a)** vertical stretch by a factor of 3, reflection in the x-axis, horizontal compression by a factor of $\frac{1}{2}$
b) horizontal translation 5 units to the right, vertical translation 2 units up
c) vertical compression by a factor of $\frac{1}{2}$, horizontal compression by a factor of $\frac{1}{5}$
d) horizontal stretch by a factor of 3, reflection in the y-axis, vertical shift 3 units down

3. **a)** $y = \frac{2}{5}\log x - 3$
b) $y = -\log\left[\frac{1}{2}(x - 3)\right]$
c) $y = 5\log(-2x)$
d) $y = \log(-x - 4) - 2$

4. Compared to $y = \log x$, $y = 3\log(x - 1) + 2$ is vertically stretched by a factor of 3, horizontally translated 1 unit to the right, and vertically translated 2 units up.

5. **a)** 3 **c)** 0
b) -2 **d)** -4

6. **a)** 3.615 **c)** 2.829
b) -1.661 **d)** 2.690

7. **a)** $\log 55$ **c)** $\log_5 4$
b) $\log 5$ **d)** $\log 128$

8. **a)** 1 **c)** $\frac{2}{3}$
b) 2 **d)** 3

9. It is shifted 4 units up.

10. **a)** 5 **c)** -2
b) 3.75 **d)** -0.2

11. **a)** 2.432 **c)** 2.553
b) 3.237 **d)** 4.799

12. **a)** 0.79; 0.5
b) -0.43

13. 5.45 days

14. **a)** 63 **c)** 9
b) $\frac{10\,000}{3}$ **d)** 1.5

15. **a)** 1 **c)** 3
b) 5 **d)** $\pm\sqrt{10\,001}$

16. 10^{-2} W/m^2

17. $10^{-3.8}$ W/m^2

18. 5 times

19. 3.9 times

20. $\frac{10^{4.7}}{10^{2.3}} = 251.2$
$\frac{10^{12.5}}{10^{10.1}} = 251.2$
The relative change in each case is the same. Each change produces a solution with concentration 251.2 times the orignial solution.

21. Yes; $y = 3(2.25^x)$

22. 17.8 years

23. **a)** 8671 people per year
b) 7114; The rate of growth for the first 30 years is slower than the rate of growth for the entire period.
c) $y = 134\,322(1.03^x)$, where x is the number of years after 1950
d) **i)** 7171 people per year
ii) 12 950 people per year

24. **a)** exponential; $y = 23(1.17^x)$, where x is the number of years since 1998
b) 331 808
c) Answers may vary. For example, I assumed that the rate of growth would be the same through 2015. This is not reasonable. As more people buy the players, there will be fewer people remaining to buy them, or newer technology may replace them.
d) about 5300 DVD players per year
e) about 4950 DVD players per year
f) Answers may vary. For example, the prediction in part e) makes sense because the prediction is for a year covered by the data given. The prediction made in part b) does not make sense because the prediction is for a year that is beyond the data given, and conditions may change, making the model invalid.

Chapter Self-Test, p. 512

1. **a)** $x = 4^y$; $\log_4 x = y$
b) $y - 6^x$; $\log_6 y - x$

2. **a)** horizontal compression by a factor of $\frac{1}{2}$, horizontal translation 4 units to the right, vertical translation 3 units up
b) vertical compression by a factor of $\frac{1}{2}$, reflection in the x-axis, horizontal translation 5 units to the left, vertical translation 1 unit down

3. **a)** -2 **b)** 5

4. **a)** 2 **b)** 7

5. $\log_4 xy$

6. 7.85

7. **a)** 2 **b)** $1\frac{3}{4}$

8. **a)** 50 g
b) $A(t) = 100(0.5)^{\frac{t}{5730}}$
c) 1844 years
d) -0.015 g/year

9. **a)** 6 min
b) 97°

Chapter 9

Getting Started, p. 516

1. **a)** $f(-1) = 30$, $f(4) = 0$
b) $f(-1) = -2$, $f(4) = -5\frac{1}{3}$
c) $f(-1)$ is undefined, $f(4) \doteq 1.81$
d) $f(-1) = -20$, $f(4) - -0.625$

2. $D = \{x \in \mathbf{R} \mid x \neq 1\}$
$R = \{y \in \mathbf{R} \mid y \neq 2\}$
There is no minimum or maximum value; the function is never increasing; the function is decreasing from $(-\infty, 1)$ and $(1, \infty)$; the function approaches $-\infty$ as x approaches 1 from the left and ∞ as x approaches 1 from the right; vertical asymptote is $x = 1$; horizontal asymptote is $y = 2$

3. **a)** $y = 2|x - 3|$
b) $y - -\cos(2x)$
c) $y = \log_3(-x - 4) - 1$
d) $y = -\frac{4}{x} - 5$

4. **a)** $x = -1, \frac{1}{2}$, and 4
b) $x = -\frac{5}{3}$ or $x = 3$
c) $x - 5$ or $x - -2$
Cannot take the log of a negative number, so $x = 5$.
d) $x = -\frac{3}{4}$
e) $x = -3$
f) $\sin x = \frac{3}{2}$ or $\sin x = -1$. Since $\sin x$ cannot be greater than 1, the first equation does not give a solution; $x = 270°$

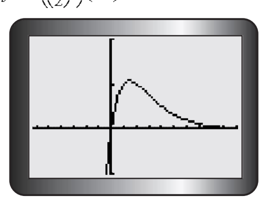
5. **a)** $(-\infty, -4) \cup (2, 3)$
b) $\left(-2, \frac{3}{2}\right) \cup [4, \infty)$

6. **a)** odd **c)** even
b) neither **d)** neither

7. Polynomial, logarithmic, and exponential functions are continuous. Rational and trigonometric functions are sometimes continuous and sometimes not.

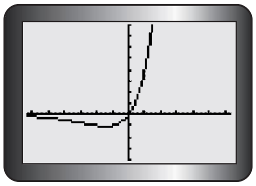
Lesson 9.1, p. 520

1. Answers may vary. For example, the graph of $y = \left(\left(\frac{1}{2}\right)^x\right)(2x)$ is

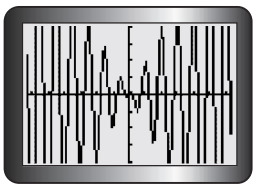


Answers

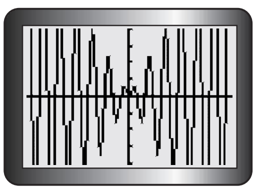
2. **a)** Answers may vary. For example, $y = (2^x)(2x)$;



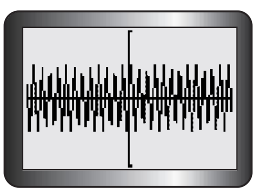
b) Answers may vary. For example, $y = (2x)(\cos(2\pi x))$;



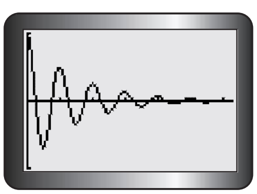
c) Answers may vary. For example, $y = (2x)(\sin(2\pi x))$;



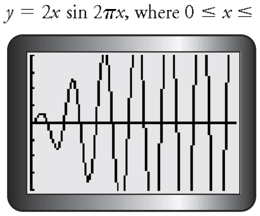
d) Answers may vary. For example, $y = (\sin 2\pi x)(\cos 2\pi x)$;



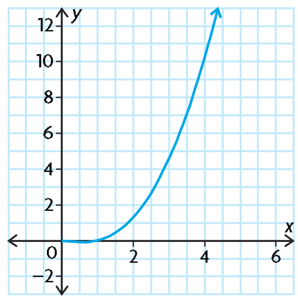
e) Answers may vary. For example, $y = \left(\frac{1}{2}\right)^x(\cos 2\pi x)$, where $0 \le x \le 2\pi$;



f) Answers may vary. For example, $y = 2x \sin 2\pi x$, where $0 \le x \le 2\pi$;



3. Answers will vary. For example,
$y = x^2$
$y = \log x$
The product will be $y = x^2 \log x$.



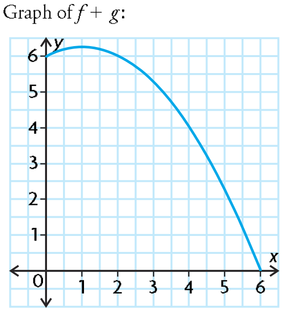
Lesson 9.2, pp. 528–530

1. **a)** $\{(-4, 6), (-2, 5), (1, 5), (4, 10)\}$
b) $\{(-4, 6), (-2, 5), (1, 5), (4, 10)\}$
c) $\{(-4, 2), (-2, 3), (1, 1), (4, 2)\}$
d) $\{(-4, -2), (-2, -3), (1, -1), (4, -2)\}$
e) $\{(-4, 8), (-2, 8), (1, 6), (3, 10), (4, 12)\}$
f) $\{(-4, 0), (-2, 0), (0, 0), (1, 0), (2, 0), (4, 0)\}$

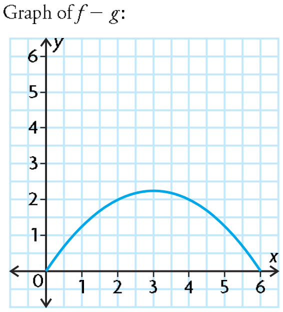
2. **a)** 10
b) 2; $(f + g)(x)$ is undefined at $x = 2$ because $g(x)$ is undefined at $x - 2$.
c) $\{x \in \mathbf{R} \mid x \ne 2\}$

3. $\{x \in \mathbf{R} \mid -1 \le x < 1\}$

4. Graph of $f + g$:



Graph of $f - g$:

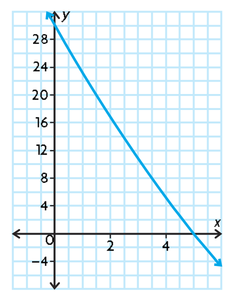


5. **a)** $f + g = |x| + x$
b) The function is neither even nor odd.

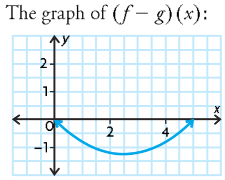
6. **a)** $\{(-6, 7), (-3, 10)\}$
b) $\{(-6, 7), (-3, 10)\}$
c) $\{(-6, -5), (-3, 4)\}$
d) $\{(-6, 5), (-3, -4)\}$
e) $\{(-9, 0), (-8, 0), (-6, 0), (-3, 0), (-1, 0), (0, 0)\}$
f) $\{(-7, 14), (-6, 12), (-5, 10), (-4, 8), (-3, 6)\}$

7. **a)** $\dfrac{2(2x + 1)}{3x^2 - 2x - 8}$
b) $\left\{x \in \mathbf{R} \mid x \ne -\dfrac{4}{3} \text{ or } 2\right\}$
c) $\dfrac{17}{84}$
d) $-\dfrac{11}{84}$

8. The graph of $(f + g)(x)$:



The graph of $(f - g)(x)$:



9. **a)** $f(x) + g(x) = 2^x + x^3$
The function is not symmetric.
The function is always increasing.
zero at $x = -0.8262$
no maximum or minimum
period: N/A
The domain is all real numbers. The range is all real numbers.
$f(x) - g(x) = 2^x - x^3$
The function is not symmetric.
The function is always decreasing.
zero at $x - 1.3735$
no maximum or minimum
period: N/A
The domain is all real numbers. The range is all real numbers.
b) $f(x) + g(x) = \cos(2\pi x) + x^4$
The function is symmetric across the line $x = 0$.
The function is decreasing from $-\infty$ to -0.4882 and 0 to 0.4882 and increasing from -0.4882 to 0 and 0.4882 to ∞.
zeros at $x = -0.7092, -0.2506, 0.2506, 0.7092$

relative maximum at $x = 0$ and relative minimums at $x = -0.4882$ and $x = 0.4882$
period: N/A
The domain is all real numbers. The range is all real numbers greater than -0.1308.
$f(x) - g(x) = \cos(2\pi x) - x^4$
The function is symmetric across the line $x = 0$.
The function is increasing from $-\infty$ to -0.9180 and -0.5138 to 0 and 0.5138 to 0.9180; decreasing from -0.9180 to -0.5138 and 0 to 0.5138 and 0.9180 to ∞.
zeros at $x = -1, -0.8278, -0.2494, 0.2494, 0.8278, 1$
relative maxima at -0.9180, 0, and 0.9180; relative minima at -0.5138 and 0.5138
period: N/A
The domain is all real numbers. The range is all real numbers less than 1.

c) $f(x) + g(x) = \log(x) + 2x$
The function is not symmetric.
The function is increasing from 0 to ∞.
no zeros
no maximum or minimum
period: N/A
The domain is all real numbers greater than 0. The range is all real numbers.
$f(x) - g(x) - \log(x) - 2x$
The function is not symmetric.
The function is increasing from 0 to approximately 0.2 and decreasing from approximately 0.2 to ∞.
no zeros
maximum at $x \doteq 0.2$
period: N/A
The domain is all real numbers greater than 0. The range is all real numbers less than or equal to approximately -1.1.

d) $f(x) + g(x) = \sin(2\pi x) + 2\sin(\pi x)$
The function is symmetric about the origin.
The function is increasing from $-0.33 + 2k$ to $0.33 + 2k$ and decreasing from $0.33 + 2k$ to $1.67 + 2k$.
zero at k
minimum at $x = -0.33 + 2k$
maximum at $x = 0.33 + 2k$
period: 2
The domain is all real numbers. The range is all real numbers between -2.598 and 2.598.
$f(x) - g(x) - \sin(2\pi x) - 2\sin(\pi x)$
The function is symmetric about the origin, increasing from $0.67 + 2k$ to $1.33 + 2k$ and decreasing from $-0.67 + 2k$ to $0.67 + 2k$
zero at k
minimum at $0.67 + 2k$ and maximum at $1.33 + 2k$
period: 2
The domain is all real numbers.
The range is all real numbers between -2.598 to 2.598.

e) $f(x) + g(x) = \sin(2\pi x) + \frac{1}{x}$
The function is not symmetric.
The function is increasing and decreasing at irregular intervals.
The zeros are changing at irregular intervals.
The maximums and minimums are changing at irregular intervals.
period: N/A
The domain is all real numbers except 0.
The range is all real numbers.
$f(x) - g(x) = \sin(2\pi x) - \frac{1}{x}$
The function is not symmetric.
The function is increasing and decreasing at irregular intervals.
The zeros are changing at irregular intervals.
The maximums and minimums are changing at irregular intervals.
period: N/A
The domain is all real numbers except 0.
The range is all real numbers.

f) $f(x) + g(x) = \sqrt{x - 2} + \frac{1}{x - 2}$
The function is not symmetric.
The function is increasing from 3.5874 to ∞ and decreasing from 2 to 3.5874.
zeros: none
minimum at $x = 3.5874$
period: N/A
The domain is all real numbers greater than 2. The range is all real numbers greater than 1.8899.
$f(x) - g(x) - \sqrt{x - 2} - \frac{1}{x - 2}$
The function is not symmetric.
The function is increasing from 2 to ∞.
zero at $x = 3$
no maximum or minimum
period: N/A
The domain is all real numbers greater than 2. The range is all real numbers.

10. a) The sum of two even functions will be even because replacing x with $-x$ will still result in the original function.
b) The sum of two odd functions will be odd because replacing x with $-x$ will still result in the opposite of the original function.
c) The sum of an even and an odd function will result in neither an even nor an odd function because replacing x with $-x$ will not result in the same function or in the opposite of the function.

11. a) $R(t) = 5000 - 25t - 1000\cos\left(\frac{\pi}{6}t\right)$; it is neither odd nor even; it is increasing during the first 6 months of each year and decreasing during the last 6 months of each year; it has one zero, which is the point at which the deer population has become extinct; it has a maximum value of 3850 and a minimum value of 0, so its range is $\{R(t) \in \mathbf{R} \mid 0 \le R(t) \le 3850\}$.
b) after about 167 months, or 13 years and 11 months

12. The stopping distance can be defined by the function $s(x) = 0.006x^2 + 0.21x$. If the vehicle is travelling at 90 km/h, the stopping distance is 67.5 m.

13. $f(x) = \sin(\pi x)$; $g(x) = \cos(\pi x)$

14. The function is neither even nor odd; it is not symmetrical with respect to the y-axis or with respect to the origin; it extends from the third quadrant to the first quadrant; it has a turning point between $-n$ and 0 and another turning point at 0; it has zeros at $-n$ and 0; it has no maximum or minimum values; it is increasing when $x \in (-\infty, -n)$ and when $x \in (0, \infty)$; when $x \in (-n, 0)$, it increases, has a turning point, and then decreases; its domain is $\{x \in \mathbf{R}\}$, and its range is $\{y \in \mathbf{R}\}$.

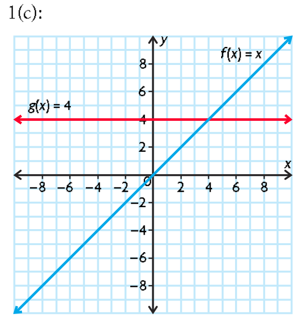
15. a) $f(x) = 0$; $g(x) = 0$
b) $f(x) = x^2$; $g(x) = x^2$
c) $f(x) = \frac{1}{x - 2}$; $g(x) = \frac{1}{x - 2} + 2$.

16. $m = 2, n = 3$

Lesson 9.3, pp. 537–539

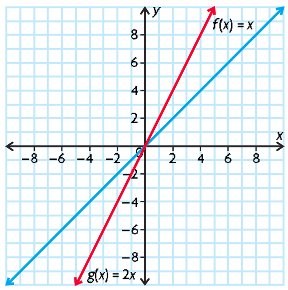
1. a) $\{(0, -2), (1, -10), (2, 21), (3, 60)\}$
b) $\{(0, 12), (2, -20)\}$
c) $4x$
d) $2x^2$
e) $x^3 - 3x + 2$
f) $2^x\sqrt{x - 2}$

2. a) 1(c):

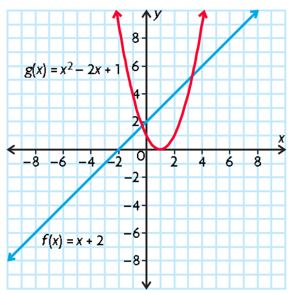


Answers

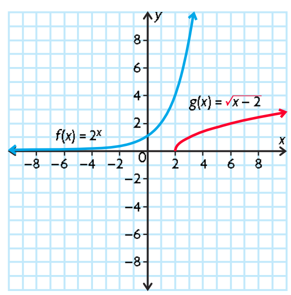
1(d):



1(e):

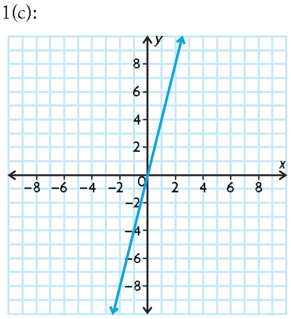


1(f):

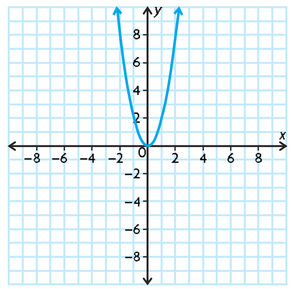


b) 1(c): f: $\{x \in \mathbf{R}\}$; g: $\{x \in \mathbf{R}\}$
1(d): f: $\{x \in \mathbf{R}\}$; g: $\{x \in \mathbf{R}\}$
1(e): f: $\{x \in \mathbf{R}\}$; g: $\{x \in \mathbf{R}\}$
1(f): f: $\{x \in \mathbf{R}\}$; g: $\{x \in \mathbf{R} \mid x \geq 2\}$

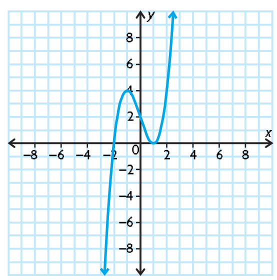
c) 1(c):



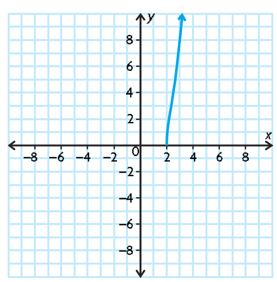
1(d):



1(e):



1(f):



d) 1(c): $\{x \in \mathbf{R}\}$
1(d): $\{x \in \mathbf{R}\}$
1(e): $\{x \in \mathbf{R}\}$
1(f): $\{x \in \mathbf{R} \mid x \geq 2\}$

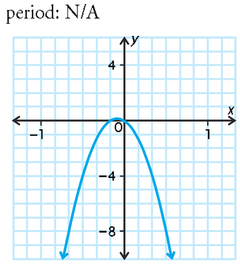
3. $\{x \in \mathbf{R} \mid -1 \leq x \leq 1\}$

4. **a)** $x^2 - 49$
b) $x + 10$
c) $7x^3 - 63x^2$
d) $-16x^2 - 56x - 49$
e) $\dfrac{2 \sin x}{x - 1}$
f) $2^x \log(x + 4)$

5. 4(a): D = $\{x \in \mathbf{R}\}$; R = $\{y \in \mathbf{R} \mid y \geq -49\}$
4(b): D = $\{x \in \mathbf{R} \mid x \geq -10\}$;
R = $\{y \in \mathbf{R} \mid y \geq 0\}$
4(c): D = $\{x \in \mathbf{R}\}$; R = $\{y \in \mathbf{R}\}$
4(d): D = $\{x \in \mathbf{R}\}$; R = $\{y \in \mathbf{R} \mid y \leq 0\}$
4(e): D = $\{x \in \mathbf{R} \mid x \neq -1\}$; R = $\{y \in \mathbf{R}\}$
4(f): D = $\{x \in \mathbf{R} \mid x > -4\}$;
R = $\{y \in \mathbf{R} \mid y \geq 0\}$

6. 4(a): The function is symmetric about the line $x = 0$.
The function is increasing from 0 to ∞.
The function is decreasing from $-\infty$ to 0.
zeros at $x = -7, 7$
The minimum is at $x - 0$.
period: N/A
4(b): The function is not symmetric.
The function is increasing from -10 to ∞.
zero at $x = -10$
The minimum is at $x = -10$.
period: N/A
4(c): The function is not symmetric.
The function is increasing from $-\infty$ to 0 and from 6 to ∞.
zeros at $x = 0, 9$
The relative minimum is at $x = -6$. The relative maximum is at $x = 0$.
period: N/A
4(d): The function is symmetric about the line $x - -1.75$.
The function is increasing from $-\infty$ to -1.75 and is decreasing from -1.75 to ∞.
zero at $x = -1.75$
The maximum is at $x = -1.75$.
period: N/A
4(e): The function is not symmetric.
The function is increasing from $-\infty$ to 0 and from 6 to ∞.
zeros at $x = 0, 9$
The relative minima are at $x = -4.5336$ and 4.4286. The relative maximum is at $x = -1.1323$.
period: N/A
4(f): The function is not symmetric.
The function is increasing from -4 to ∞.
zeros: none
maximum/minimum: none
period: N/A

7.



8. **a)** $\left\{x \in \mathbf{R} \,\middle|\, x \neq -2, 7, \frac{\pi}{2}, \text{ or } \frac{3\pi}{2}\right\}$
b) $\{x \in \mathbf{R} \mid x > 8\}$
c) $\{x \in \mathbf{R} \mid x \geq -81 \text{ and } x \neq 0, \pi, \text{ or } 2\pi\}$
d) $\{x \in \mathbf{R} \mid x \leq -1 \text{ or } x \geq 1,$ and $x \neq -3\}$

9. $(f \times p)(t)$ represents the total energy consumption in a particular country at time t

10. **a)** $R(x) = (20\,000 - 750x)(25 + x)$ or $R(x) = 500\,000 + 1250x - 750x^2$, where x is the increase in the admission fee in dollars

b) Yes, it's the product of the function $P(x) = 20\ 000 - 750x$, which represents the number of daily visitors, and $F(x) - 25 + x$, which represents the admission fee.

c) \$25.83

11. $m(t) = ((0.9)^t)(650 + 300t)$
The amount of contaminated material is at its greatest after about 7.3 s.

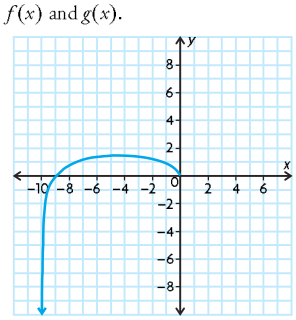
12. The statement is false. If $f(x)$ and $g(x)$ are odd functions, then their product will always be an even function. When you multiply a function that has an odd degree with another function that has an odd degree, you add the exponents, and when you add two odd numbers together, you get an even number.

13. $f(x) = 3x^2 + 2x + 5$ and
$g(x) = 2x^2 - 4x - 2$

14. **a)** $(f \times g)(x) = \sqrt{-x}\log(x + 10)$
The domain is $\{x \in \mathbf{R} \mid -10 < x \leq 0\}$.

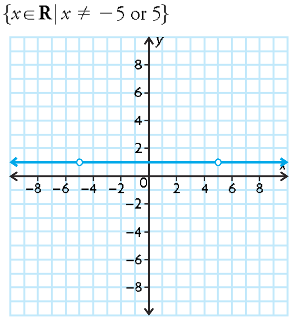
b) One strategy is to create a table of values for $f(x)$ and $g(x)$ and to multiply the corresponding y-values together. The resulting values could then be graphed. Another strategy is to graph $f(x)$ and $g(x)$ and to then create a graph for $(f \times g)(x)$ based on these two graphs. The first strategy is probably better than the second strategy, since the y-values for $f(x)$ and $g(x)$ will not be round numbers and will not be easily discernable from the graphs of $f(x)$ and $g(x)$.

c)



15. **a)** $f(x) \times \dfrac{1}{f(x)} = 1$

b) $\{x \in \mathbf{R} \mid x \neq -5 \text{ or } 5\}$



c) The range will always be 1. If f is of odd degree, there will always be at least one value that makes the product undefined and which is excluded from the domain. If f is of even degree, there may be no values that are excluded from the domain.

16. **a)** $f(x) = 2^x$
$g(x) = x^2 + 1$
$(f \times g)(x) = 2^x(x^2 + 1)$

b) $f(x) = x$
$g(x) = \sin(2\pi x)$
$(f \times g)(x) = x\sin(2\pi x)$

17. **a)** $f(x) = (2x + 9)$
$g(x) = (2x - 9)$

b) $f(x) = (2\sin x + 3)$
$g(x) = (4\sin^2 x - 6\sin x + 9)$

c) $f(x) = x^{\frac{1}{2}}$
$g(x) = (4x^5 - 3x^3 + 1)$

d) $f(x) = \dfrac{1}{2x + 1}$
$g(x) = 6x - 5$

Lesson 9.4, p. 542

1. **a)** $(f \div g)(x) = \dfrac{5}{x}, x \neq 0$

b) $(f \div g)(x) = \dfrac{4x}{2x - 1}, x \neq \dfrac{1}{2}$

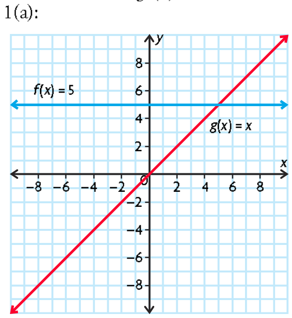
c) $(f \div g)(x) = \dfrac{4x}{x^2 + 4}$

d) $(f \div g)(x) = \dfrac{(x + 2)(\sqrt{x - 2})}{x - 2}, x > 2$

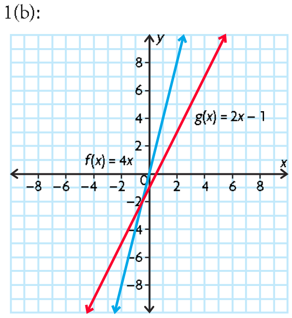
e) $(f \div g)(x) = \dfrac{8}{1 + (\frac{1}{2})^x}$

f) $(f \div g)(x) = \dfrac{x^2}{\log(x)}, x > 0$

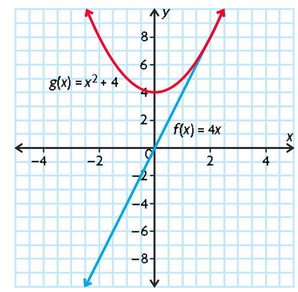
2. **a)** 1(a):



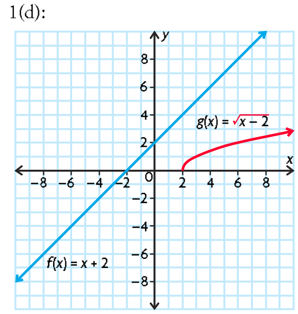
1(b):



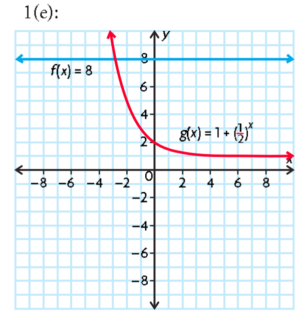
1(c):



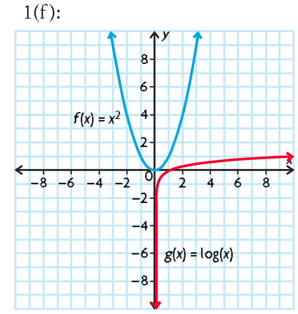
1(d):



1(e):

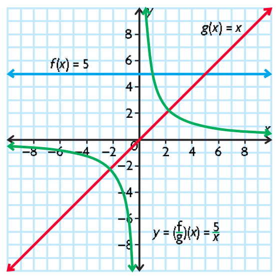


1(f):

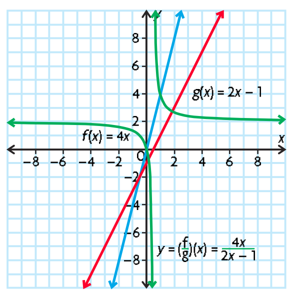


b) 1(a): domain of f: $\{x \in \mathbf{R}\}$;
domain of g: $\{x \in \mathbf{R}\}$
1(b): domain of f: $\{x \in \mathbf{R}\}$;
domain of g: $\{x \in \mathbf{R}\}$
1(c): domain of f: $\{x \in \mathbf{R}\}$;
domain of g: $\{x \in \mathbf{R}\}$
1(d): domain of f: $\{x \in \mathbf{R}\}$;
domain of g: $\{x \in \mathbf{R} \mid x \geq 2\}$
1(e): domain of f: $\{x \in \mathbf{R}\}$;
domain of g: $\{x \in \mathbf{R}\}$
1(f): domain of f: $\{x \in \mathbf{R}\}$;
domain of g: $\{x \in \mathbf{R} \mid x > 0\}$

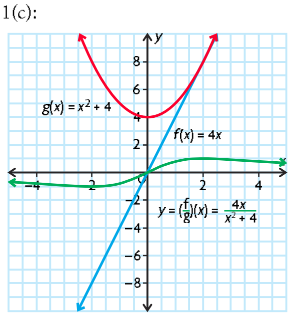
c) 1(a):



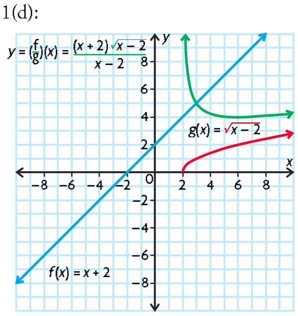
1(b):



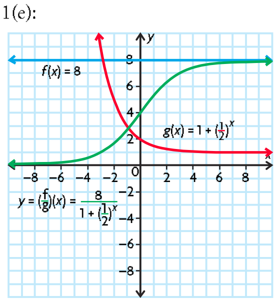
1(c):



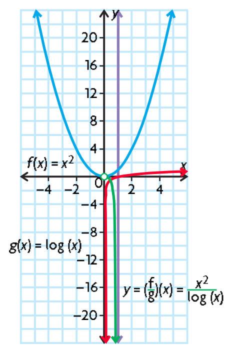
1(d):



1(e):



1(f):



d) 1(a): domain of $(f \div g)$: $\{x \in \mathbf{R} \mid x \neq 0\}$

1(b): domain of $(f \div g)$: $\left\{x \in \mathbf{R} \mid x \neq \frac{1}{2}\right\}$

1(c): domain of $(f \div g)$: $\{x \in \mathbf{R}\}$

1(d): domain of $(f \div g)$: $\{x \in \mathbf{R} \mid x > 2\}$

1(e): domain of $(f \div g)$: $\{x \in \mathbf{R}\}$

1(f): domain of $(f \div g)$: $\{x \in \mathbf{R} \mid x > 0\}$

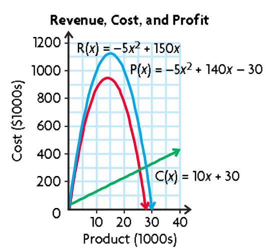
3. **a)** 2.798 cm/day
b) about 30 days
c) 6.848 cm/day
d) It slows down and eventually comes to zero. This is seen on the graph as it becomes horizontal at the top.

Mid-Chapter Review, p. 544

1. multiplication

2. **a)** $\{(-9, 2), (-6, -9), (0, 14)\}$
b) $\{(-9, 2), (-6, -9), (0, 14)\}$
c) $\{(-9, -6), (-6, 3), (0, -10)\}$
d) $\{(-9, 6), (-6, -3), (0, 10)\}$

3. **a)** $P(x) = -5x^2 + 140x - 30$
b)



c) \$738 750

4. **a)** $R(h) = 24.39h$
b) $N(h) = 24.97h$
c) $W(h) = 24.78h$
d) $S(h) = 25.36h$
e) \$317

5. **a)** $(f \times g)(x) = x^2 + x + \frac{1}{4}$

$D = \{x \in \mathbf{R}\}$

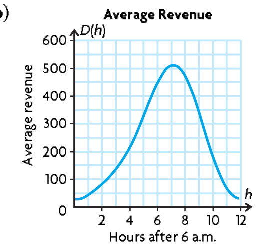
b) $(f \times g)(x) = \sin(3x)(\sqrt{x - 10})$
$D = \{x \in \mathbf{R} \mid x \geq 10\}$

c) $(f \times g)(x) = \frac{22x^3}{x + 5}$
$D = \{x \in \mathbf{R} \mid x \neq -5\}$

d) $(f \times g)(x) = 8100x^2 - 1$
$D = \{x \in \mathbf{R}\}$

6. **a)** $R(h) = 90 \cos\left(\frac{\pi}{6}h\right) \sin\left(\frac{\pi}{6}h\right) - 102 \sin\left(\frac{\pi}{6}h\right) - 210 \cos\left(\frac{\pi}{6}h\right) + 238$

b)



c) about \$470.30

7. **a)** $(f \div g)(x) = \frac{80}{x}$
$D = \{x \in \mathbf{R} \mid x \neq 0\}$

b) $(f \div g)(x) = \frac{10x^2}{x^2 - 3}$
$D = \{x \in \mathbf{R} \mid x \neq \pm\sqrt{3}\}$

c) $(f \div g)(x) = \frac{x + 8}{\sqrt{x - 8}}$
$D - \{x \in \mathbf{R} \mid x > 8\}$

d) $(f \div g)(x) = \frac{7x^2}{\log x}$
$D = \{x \in \mathbf{R} \mid x > 0\}$

8. csc *x*, sec *x*, cot *x*

Lesson 9.5, pp. 552–554

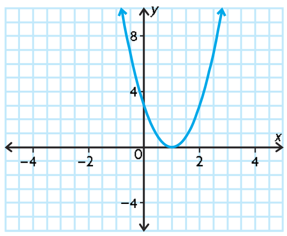
1. **a)** -1
b) -24
c) -129
d) $\frac{7}{16}$
e) 1
f) -8

2. **a)** 3
b) 5
c) 10
d) $(f \circ g)(0)$ is undefined.
e) 2
f) 4

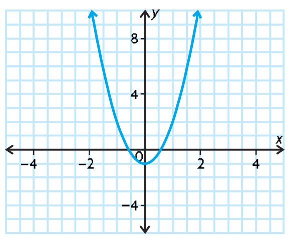
3. **a)** 5
b) 5
c) 4
d) $(f \circ f)(2)$ is undefined.

4. **a)** $C(d(5)) = 36$
It costs \$36 to travel for 5 h.
b) $C(d(t))$ represents the relationship between the time driven and the cost of gasoline.

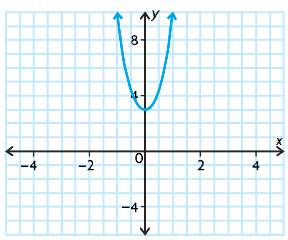
5. a) $f(g(x)) = 3x^2 - 6x + 3$
The domain is $\{x \in \mathbf{R}\}$.



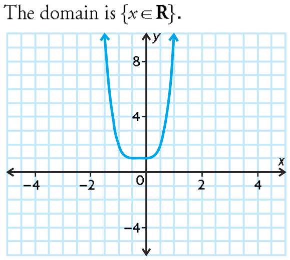
$g(f(x)) = 3x^2 - 1$
The domain is $\{x \in \mathbf{R}\}$.



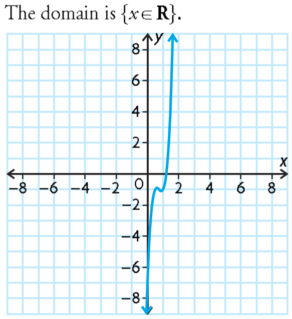
b) $f(g(x)) = 2x^4 + 5x^2 + 3$
The domain is $\{x \in \mathbf{R}\}$.



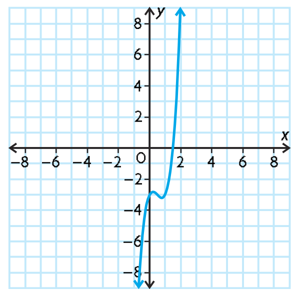
$g(f(x)) = 4x^4 + 4x^3 + x^2 + 1$
The domain is $\{x \in \mathbf{R}\}$.



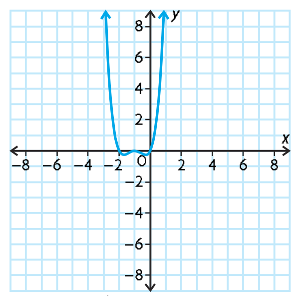
c) $f(g(x)) = 16x^3 - 36x^2 + 26x - 7$
The domain is $\{x \in \mathbf{R}\}$.



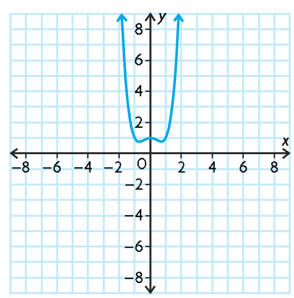
$g(f(x)) = 4x^3 - 6x^2 + 2x - 3$
The domain is $\{x \in \mathbf{R}\}$.



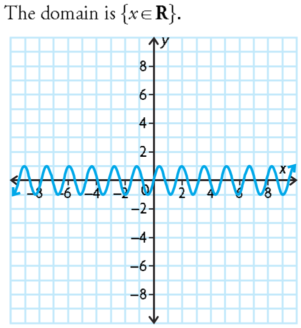
d) $f(g(x)) = x^4 + 4x^3 + 5x^2 + 2x$
The domain is $\{x \in \mathbf{R}\}$.



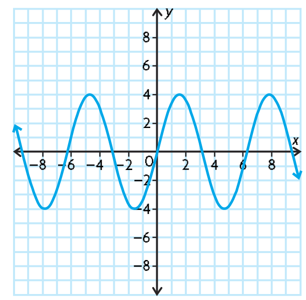
$g(f(x)) = x^4 - x^2 + 1$
The domain is $\{x \in \mathbf{R}\}$.



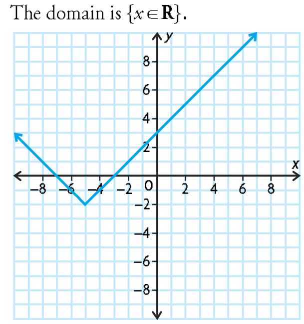
e) $f(g(x)) = \sin 4x$
The domain is $\{x \in \mathbf{R}\}$.



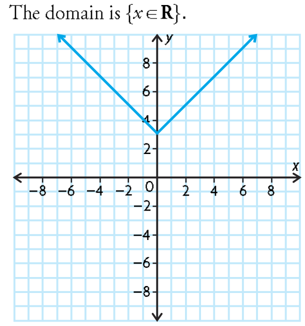
$g(f(x)) = 4 \sin x$
The domain is $\{x \in \mathbf{R}\}$.



f) $f(g(x)) = |x + 5| - 2$
The domain is $\{x \in \mathbf{R}\}$.



$g(f(x)) = |x| + 3$
The domain is $\{x \in \mathbf{R}\}$.



6. a) $f \circ g = 3\sqrt{x - 4}$
$D = \{x \in \mathbf{R} \mid x \geq 4\}$
$R - \{y \in \mathbf{R} \mid y \geq 0\}$
$g \circ f = \sqrt{3x - 4}$
$D = \left\{x \in \mathbf{R} \mid x \geq \frac{4}{3}\right\}$
$R - \{y \in \mathbf{R} \mid y \geq 0\}$

b) $f \circ g = \sqrt{3x + 1}$
$D = \left\{x \in \mathbf{R} \mid x \geq -\frac{1}{3}\right\}$
$R = \{y \in \mathbf{R} \mid y \geq 0\}$
$g \circ f = 3\sqrt{x} + 1$
$D = \{x \in \mathbf{R} \mid x \geq 0\}$
$R = \{y \in \mathbf{R} \mid y \geq 1\}$

c) $f \circ g = \sqrt{4 - x^4}$
$D = \{x \in \mathbf{R} \mid -\sqrt{2} \le x \le \sqrt{2}\}$
$R = \{y \in \mathbf{R} \mid y \ge 0\}$
$g \circ f = 4 - x^2$
$D = \{x \in \mathbf{R} \mid -2 \le x \le 2\}$
$R = \{y \in \mathbf{R} \mid 0 < y < 2\}$

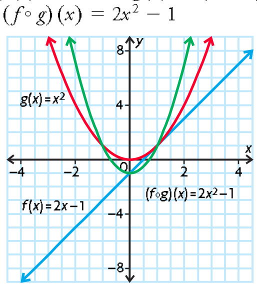
d) $f \circ g = 2\sqrt{x - 1}$
$D = \{x \in \mathbf{R} \mid x \ge 1\}$
$R = \{y \in \mathbf{R} \mid y \ge 1\}$
$g \circ f = \sqrt{2^x - 1}$
$D = \{x \in \mathbf{R} \mid x \ge 0\}$
$R = \{y \in \mathbf{R} \mid y \ge 0\}$

e) $f \circ g = x$
$D = \{x \in \mathbf{R} \mid x > 0\}$
$R = \{y \in \mathbf{R}\}$
$g \circ f = x$
$D = \{x \in \mathbf{R}\}$
$R = \{y \in \mathbf{R}\}$

f) $f \circ g = \sin(5^{2x} + 1)$
$D = \{x \in \mathbf{R}\}$
$R = \{y \in \mathbf{R} \mid -1 \le y \le 1\}$
$g \circ f = 5^{2\sin x} + 1$
$D = \{x \in \mathbf{R}\}$
$R = \left\{y \in \mathbf{R} \mid \frac{26}{25} \le y \le 26\right\}$

7. a) Answers may vary. For example, $f(x) = \sqrt{x}$ and $g(x) = x^2 + 6$
b) Answers may vary. For example, $f(x) = x^6$ and $g(x) = 5x - 8$
c) Answers may vary. For example, $f(x) = 2^x$ and $g(x) = 6x + 7$
d) Answers may vary. For example, $f(x) = \frac{1}{x}$ and $g(x) = x^3 - 7x + 2$
e) Answers may vary. For example, $f(x) = \sin^2 x$ and $g(x) = 10x + 5$
f) Answers may vary. For example, $f(x) = \sqrt[3]{x}$ and $g(x) = (x + 4)^2$

8. a) $(f \circ g)(x) = 2x^2 - 1$
b)



c) It is compressed by a factor of 2 and translated down 1 unit.

9. a) $f(g(x)) = 6x + 3$
The slope of $g(x)$ has been multiplied by 2, and the y-intercept of $g(x)$ has been vertically translated 1 unit up.
b) $g(f(x)) = 6x - 1$
The slope of $f(x)$ has been multiplied by 3.

10. $D(p) = 780 + 31.96p$

11. $f(g(x)) = 0.06x$

12. a) $d(s) = \sqrt{16 + s^2}$; $s(t) = 560t$
b) $d(s(t)) = \sqrt{16 + 313\,600t^2}$, where t is the time in hours and $d(s(t))$ is the distance in kilometres

13. $c(v(t)) = \left(\frac{40 + 3t + t^2}{500} - 0.1\right)^2 + 0.15$;
The car is running most economically 2 h into the trip.

14. Graph A(k); $f(x)$ is vertically compressed by a factor of 0.5 and reflected in the x-axis. Graph B(b); $f(x)$ is translated 3 units to the left.
Graph C(d); $f(x)$ is horizontally compressed by a factor of $\frac{1}{2}$.
Graph D(l); $f(x)$ is translated 4 units down.
Graph E(g); $f(x)$ is translated 3 units up.
Graph F(c); $f(x)$ is reflected in the y-axis.

15. **Sum:** $y = f + g$
$f(x) = \frac{4}{x - 3}$; $g(x) = 1$
Product: $y = f \times g$
$f(x) = x - 3$; $g(x) = \frac{x + 1}{(x - 3)^2}$
Quotient: $y = f \div g$
$f(x) = 1 + x$; $g(x) = x - 3$
Composition: $y = f \circ g$
$f(x) = \frac{4}{x} + 1$; $g(x) = x - 3$

16. a) $f(k) = 27k - 14$
b) $f(k) = 2\sqrt{9k - 16} - 5$

Lesson 9.6, pp. 560–562

1. a) i) $x = \frac{1}{2}$, 2, or $\frac{7}{2}$
ii) $x = -1$ or 2
b) i) $\frac{1}{2} < x < 2$ or $x > \frac{7}{2}$
ii) $-1 < x < 2$
c) i) $x \le \frac{1}{2}$; $2 \le x \le \frac{7}{2}$
ii) $x \le -1$ or $x \ge 2$
d) i) $\frac{1}{2} \le x \le 2$ or $x \ge \frac{7}{2}$
ii) $-1 \le x \le 2$

2. a) $x \doteq 0.8$
b) $x = 0$ and 3.5
c) $x \doteq -2.4$
d) $x \doteq 0.7$

3. $x = -1.3$ or 1.8

4. $f(x) < g(x)$: $1.3 < x < 1.6$
$f(x) = g(x)$: $x = 0$ or 1.3
$f(x) > g(x)$: $0 < x < 1.3$ or $1.6 < x < 3$

5. a) $x \doteq 2.5$
b) $x \doteq 2.2$
c) $x \doteq 1.8$
d) $x \doteq -2.1$
e) $x = 10$
f) $x = 1$ or 3

6. a) $x = -1.81$ or 0.48
b) $x = -1.38$ or 1.6
c) $x = -1.38$ or 1.30
d) $x = -0.8$, 0, or 0.8
e) $x = 0.21$ or 0.74
f) $x = 0$, 0.18, 0.38, or 1

7. $(0.7, -1.5)$

8. They will be about the same in 2012.

9. a) $x \in (-0.57, 1)$
b) $x \in [0, 0.58]$
c) $x \in (-\infty, 0)$
d) $x \in (0.17, 0.83)$
e) $x \in (0.35, 1.51)$
f) $x \in (0.1, 0.5)$

10. Answers may vary. For example, $f(x) = x^3 + 5x^2 + 2x - 8$ and $g(x) = 0$.

11. Answers may vary. For example, $f(x) = -x^2 + 25$ and $g(x) = -x + 5$.

12. $a \doteq 7$, $b \doteq 2$

13. Answers may vary. For example:

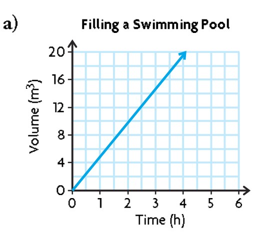
Perform the necessary algebraic operations to move all of the terms on the right side of the equation to the left side of the equation.
↓
Construct the function $f(x)$, such that $f(x)$ equals the left side of the equation.
↓
Graph the function $f(x)$.
↓
Determine the x-intercepts of the graph that fall within the interval provided, if applicable.
↓
The x-intercepts of the graph are the solutions to the equation.

14. $x = 0 \pm 2n$, $x = -0.67 \pm 2n$ or $x = 0.62 \pm 2n$, where $n \in \mathbf{I}$

15. $x \in (2n, 2n + 1)$, where $n \in \mathbf{I}$

Lesson 9.7, pp. 569–574

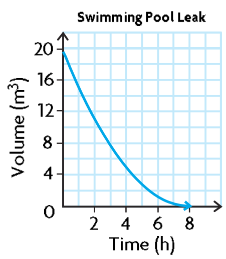
1. a)



b) $y = 6.25\pi\left(\frac{x}{4}\right)$

c) about 1.6 h

2. **a)** $y = \frac{6.25\pi}{64}(x - 8)^2$



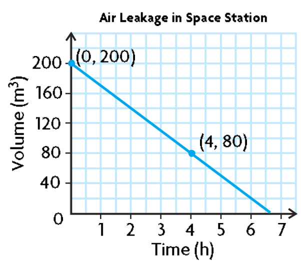
b) $V(t) = \frac{6.25\pi}{64}(t - 8)^2$

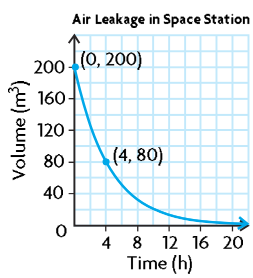
c) $V(2) \doteq 11$ m^3

d) -4.3 m^3/h

e) As time elapses, the pool is losing less water in the same amount of time.

3. **a)** Answers may vary. For example:

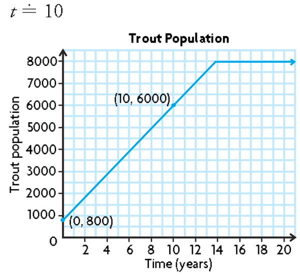




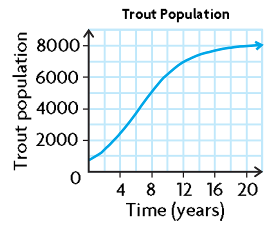
b) $V(t) = -30t + 200$;
$t \doteq 6.7$

c) $V(t) = 200(0.795)^t$;
$t \doteq 10$

4. **a)**



b) $P(t) = \frac{8000}{1 + 9(0.719)^t}$



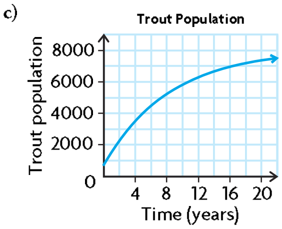
c) about 2349

d) 387.25 trout per year

5. **a)** the carrying capacity of the lake; 8000

b) Use (0, 800) and (10, 6000).
$a = 7200$, $b \doteq 0.88$

c)



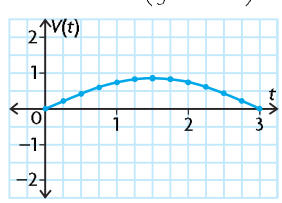
d) $P(4) \doteq 3682$

e) 720.5 trout per year

f) In the model in the previous problem, the carrying capacity of the lake is divided by a number that gets smaller and smaller, while in this model, a number that gets smaller and smaller is subtracted from the carrying capacity of the lake.

6. Answers may vary. For example, the first model more accurately calculates the current price of gasoline because prices are rising quickly.

7. **a)** $V(t) = 0.85\cos\left(\frac{\pi}{3}(t - 1.5)\right)$



b) The scatter plot and the graph are very close to being the same, but they are not exactly the same.

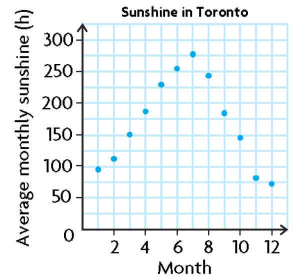
c) $V(6) = 0$ L/s

d) From the graph, the rate of change appears to be at its smallest at $t = 1.5$ s.

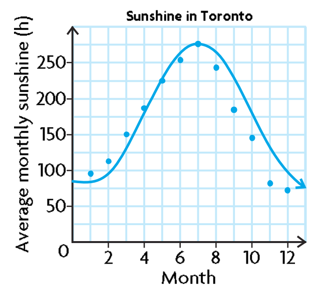
e) It is the maximum of the function.

f) From the graph, the rate of change appears to be greatest at $t = 0$ s.

8. **a)**



b)

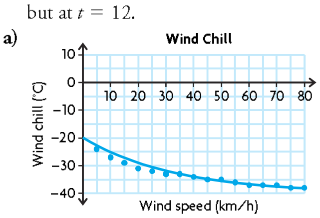


c) $S(t) = -97\cos\left(\frac{\pi}{6}(t - 1)\right) + 181$

d) From the model, the maximum will be at $t = 7$ and the minimum will be at $t = 1$.

e) It doesn't fit it perfectly, because, actually, the minimum is not at $t = 1$, but at $t = 12$.

9. **a)**



b) Answers may vary. For example,
$C(s) = -38 + 14(0.97)^s$

c) $C(0) = -24$ °C
$C(100) \doteq -37.3$ °C
$C(200) \doteq -38$ °C
These answers don't appear to be very reasonable, because the wind chill for a wind speed of 0 km/h should be -20 °C, while the wind chills for wind speeds of 100 km/h and 200 km/h should be less than -38 °C. The model only appears to be somewhat accurate for wind speeds of 10 to 70 km/h.

10. **a)** Answers will vary. For example, one polynomial model is
$P(t) = 1.4t^2 + 3230$, while an exponential model is
$P(t) = 3230(1.016)^t$. While neither model is perfect, it appears that the polynomial model fits the data better.

b) $P(155) = 1.4(155)^2 + 3230$
$\doteq 36\ 865$
$P(155) = 3230(1.016)^{155} \doteq 37\ 820$

c) A case could be made for either model. The polynomial model appears to fit the data better, but population growth is usually exponential.

d) According to the polynomial model, in 2000, the population was increasing at a rate of about 389 000 per year, while according to the exponential model, in 2000, the population was increasing at a rate of about 465 000 per year.

11. a) $P(t) = 3339.18(1.132\ 25)^t$
b) They were introduced around the year 1924.
c) rate of growth $\doteq$ 2641 rabbits per year
d) $P(65) \doteq 10\ 712\ 509.96$

12. a) $V(t) = 155.6 \sin\left(120\pi t + \frac{\pi}{2}\right)$
b) $V(t) = 155.6 \cos(120\pi t)$
c) The cosine function was easier to determine. The cosine function is at its maximum when the argument is 0, so no horizontal translation was necessary.

13. a) Answers will vary. For example, a linear model is $P(t) = -9t + 400$, a quadratic model is $P(t) = \frac{23}{90}(t - 30)^2 + 170$, and an exponential model is $P(t) - 400(0.972)^t$.
The exponential model fits the data far better than the other two models.
b) $P(t) = -9t + 400$
$P(60) = -140$ kPa
$P(t) = \frac{23}{90}(t - 30)^2 + 170,$
$P(60) - 400$ kPa
$P(t) = 400(0.972)^t$, $P(60) \doteq 73$ kPa
c) The exponential model gives the most realistic answer, because it fits the data the best. Also, the pressure must be less than 170 kPa, but it cannot be negative.

14. As a population procreates, the population becomes larger, and thus, more and more organisms exist that can procreate some more. In other words, the act of procreating enables even more procreating in the future.

15. a) linear, quadratic, or exponential
b) linear or quadratic
c) exponential

16. a) $T(n) = \frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n$
b) $47\ 850 = \frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n$
So, $n \doteq 64.975$. So, it is not a tetrahedral number because n must be an integer.

17. a) $P(t) - 30.75(1.008\ 418)^t$
b) In 2000, the growth rate of Canada was less than the growth rate of Ontario and Alberta.

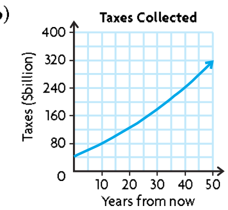
Chapter Review, pp. 576–577

1. division

2. a) Shop 2
b) $S_{1+2} = t^3 + 1.6t^2 + 1200$
c) 1 473 600
d) The owner should close the first shop, because the sales are decreasing and will eventually reach zero.

3. a) $C(x) = 9.45x + 52\ 000$
b) $I(x) - 15.8x$
c) $P(x) = 6.35x - 52\ 000$

4. a) $12 \sin(7x)$
b) $9x^2$
c) $121x^2 - 49$
d) $2a^2b^{3x}$

5. a) $C \times A = 42\ 750\ 000\ 000(1.01)^t + 3\ 000\ 000\ 000t(1.01)^t$
b)


d) about $156 402 200 032.31

6. a) $\frac{21}{x}$
b) $\frac{1}{2x + 9}$
c) $\frac{\sqrt{x + 15}}{x + 15}$
d) $\frac{x^3}{2 \log x}$

7. a) $\{x \in \mathbf{R} \mid x \neq 0\}$
b) $\left\{x \in \mathbf{R} \mid x \neq 4, x \neq -\frac{9}{2}\right\}$
c) $\{x \in \mathbf{R} \mid x > -15\}$
d) $\{x \in \mathbf{R} \mid x > 0\}$

8. a) Domain of $f(x)$: $\{x \in \mathbf{R} \mid x > -1\}$
Range of $f(x)$: $\{y \in \mathbf{R} \mid y > 0\}$
Domain of $g(x)$: $\{x \in \mathbf{R}\}$
Range of $g(x)$: $\{y \in \mathbf{R} \mid y \geq 3\}$
b) $f(g(x)) = \frac{1}{\sqrt{x^2 + 4}}$
c) $g(f(x)) = \frac{3x + 4}{x + 1}$
d) $f(g(0)) = \frac{1}{2}$
e) $g(f(0)) = 4$
f) For $f(g(x))$: $\{x \in \mathbf{R}\}$
For $g(f(x))$: $\{x \in \mathbf{R} \mid x > -1\}$

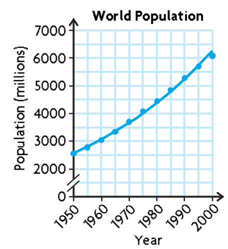
9. a) $x - 6$
b) $x - 9$
c) $x - 12$
d) $x - 3(1 + n)$

10. a) $A(r) = \pi r^2$
b) $r(C) = \frac{C}{2\pi}$
c) $A(r(C)) = \frac{C^2}{4\pi}$
d) $\frac{C^2}{4\pi} \doteq 1.03$ m

11. $f(x) < g(x)$: $-1.2 < x < 0$ or $x > 1.2$
$f(x) - g(x)$: $x - -1.2, 0,$ or 1.2
$f(x) > g(x)$: $x < -1.2$ or $0 < x < 1.2$

12. a) $x \doteq 4.0$
b) $x \doteq 2.0$
c) $x \doteq -0.8$
d) $x \doteq 0.7$

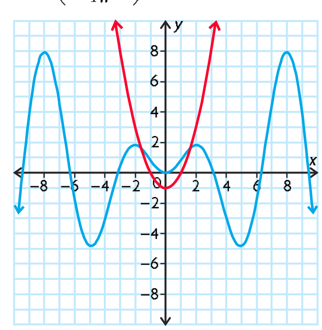
13. a) $P(t) = 600t - 1000$. The slope is the rate that the population is changing.
b) $P(t) = 617.6(1.26)^t$, 617.6 is the initial population and 1.26 represents the growth.

14. $P(t) = 2570.99(1.018)^t$


When $t = 13$, $P(t) = 3242$.
When $t = 23$, $P(t) = 3875$.
When $t = 90$, $P(t) = 12\ 806$.

Chapter Self-Test, p. 578

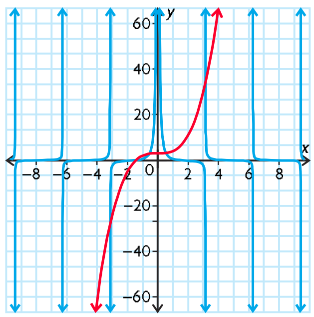
1. a) $A(r) = 4\pi r^2$
b) $r(V) = \sqrt[3]{\frac{3V}{4\pi}}$
c) $A(r(V)) = 4\pi\left(\frac{3V}{4\pi}\right)^{\frac{2}{3}}$
d) $4\pi\left(\frac{3(0.75)}{4\pi}\right)^{\frac{2}{3}} \doteq 4\ \text{m}^2$

2.


From the graph, the solution is $-1.62 \leq x \leq 1.62$.

3. Answers may vary. For example, $g(x) = x^7$ and $h(x) = 2x + 3$, $g(x) = (x + 3)^7$ and $h(x) = 2x$

4. a) $N(n) = 1n^3 + 8n^2 + 40n + 400$
   b) $N(3) = 619$
5. $(f \times g)(x) = 30x^3 + 405x^2 + 714x - 4785$
6. a) There is a horizontal asymptote of $y = 275$ cm. This is the maximum height this species will reach.
   b) when $t \doteq 21.2$ months
7. $x = 4.5$ or 4500 items
8.



The solutions are $x = -3.1, -1.4, -0.6, 0.5$, or 3.2.

9. Division will turn it into a tangent function that is not sinusoidal.

Cumulative Review Chapters 7–9, pp. 580–583

1.	(d)	**10.**	(d)	**19.**	(c)	**28.**	(a)
2.	(b)	**11.**	(a)	**20.**	(d)	**29.**	(d)
3.	(a)	**12.**	(b)	**21.**	(b)	**30.**	(d)
4.	(a)	**13.**	(d)	**22.**	(a)	**31.**	(c)
5.	(d)	**14.**	(d)	**23.**	(c)	**32.**	(d)
6.	(c)	**15.**	(c)	**24.**	(c)	**33.**	(d)
7.	(d)	**16.**	(a)	**25.**	(c)	**34.**	(b)
8.	(b)	**17.**	(b)	**26.**	(b)		
9.	(c)	**18.**	(b)	**27.**	(a)		

35. 27° or 63°
36. a) Answers may vary. For example, Niagara: $P(x) = (414.8)(1.0044^x)$; Waterloo: $P(x) = (418.3)(1.0117^x)$
    b) Answers may vary. For example, Niagara: 159 years; Waterloo: 60 years
    c) Answers may vary. For example, Waterloo is growing faster. In 2025, the instantaneous rate of change for the population in Waterloo is about 6800 people/year, compared to about 2000 people/year for Niagara.
37. $m(t) = 30\,000 - 100t$,
    $a(t) = \dfrac{T}{30\,000 - 100t} - 10$,
    $v(t) = -\dfrac{\log\left(1 - \frac{t}{300}\right)}{\log 2.72} - gt$;
    at $t = 0$, $\dfrac{T}{30\,000} - 10$ must be greater than 0 m/s^2, so T must be greater than $300\,000 \text{ kg} \times \text{m/s}^2$ (or 300 000 N)

Answers