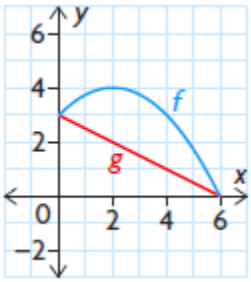
Chapter 8 Problem Set – Combinations of Functions

8.2 Sums and Differences of Functions #1c, 4 – 7, 9, 11 (pg 528 in textbook)

1. Let $f = \{(-4, 4), (-2, 4), (1, 3), (3, 5), (4, 6)\}$ and $g = \{(-4, 2), (-2, 1), (0, 2), (1, 2), (2, 2), (4, 4)\}$. Determine:

a) $f + g$ c) $f - g$ e) $f + f$
b) $g + f$ d) $g - f$ f) $g - g$

4. Make a reasonable sketch of the graph of $f + g$ and $f - g$, where $0 \le x \le 6$, for the functions shown.



5. a) Given the function $f(x) = |x|$ (which is even) and $g(x) = x$ (which is odd), determine $f + g$.
b) Is $f + g$ even, odd, or neither?

6. K $f = \{(-9, -2), (-8, 5), (-6, 1), (-3, 7), (-1, -2), (0, -10)\}$ and $g = \{(-7, 7), (-6, 6), (-5, 5), (-4, 4), (-3, 3)\}$. Calculate:

a) $f + g$ c) $f - g$ e) $f - f$
b) $g + f$ d) $g - f$ f) $g + g$

7. a) If $f(x) = \frac{1}{3x + 4}$ and $g(x) = \frac{1}{x - 2}$, what is $f + g$?
b) What is the domain of $f + g$?
c) What is $(f + g)(8)$?
d) What is $(f - g)(8)$?

9. For each pair of functions, determine the equations of $f(x) + g(x)$ and $f(x) - g(x)$. Using graphing technology, graph these new functions and discuss each of the following characteristics of the resulting graphs: symmetry, intervals of increase/decrease, zeros, maximum and minimum values, period (where applicable), and domain and range.

a) $f(x) = 2^x, g(x) = x^3$
b) $f(x) = \cos(2\pi x), g(x) = x^4$
c) $f(x) = \log(x), g(x) = 2x$
d) $f(x) = \sin(2\pi x), g(x) = 2\sin(\pi x)$
e) $f(x) = \sin(2\pi x) + 2, g(x) = \frac{1}{x}$
f) $f(x) = \sqrt{x - 2}, g(x) = \frac{1}{x - 2}$

11. A Recall, from Example 3, the function $P(t) = 5000 - 1000\cos\left(\frac{\pi}{6}t\right)$, which models the deer population in a provincial park. A disease in the deer population has caused it to decline. Biologists have discovered that the deer population is decreasing by 25 deer each month.

a) Assuming that this pattern continues, determine the new function that will model the deer population over time and discuss its characteristics.

b) Estimate when the deer population in this park will be extinct.

8.3 Product and Quotient Combinations

#1bd,3,8bd,10,15 (pg 537 in textbook)

#1aef, 2 (for 1aef) (pg 542 in textbook)

1. For each of the following pairs of functions, determine $(f \times g)(x)$.

b) $f(x) = \{(0, 3), (1, 6), (2, 10), (3, -5)\}$, $g(x) = \{(0, 4), (2, -2), (4, 1), (6, 3)\}$

d) $f(x) = x, g(x) = 2x$

3. If $f(x) = \sqrt{1 + x}$ and $g(x) = \sqrt{1 - x}$, determine the domain of $y = (f \times g)(x)$.

8. For each of the following pairs of functions, state the domain of $(f \times g)(x)$.

b) $f(x) = 99^x, g(x) = \log(x - 8)$

d) $f(x) = \log(x^2 + 6x + 9), g(x) = \sqrt{x^2 - 1}$

10. A An average of 20 000 people visit the Lakeside Amusement Park each day in the summer. The admission fee is \$25.00. Consultants predict that, for each \$1.00 increase in the admission fee, the park will lose an average of 750 customers each day.

a) Determine the function that represents the projected daily revenue if the admission fee is increased.

b) Is the revenue function a product function? Explain.

c) Estimate the ticket price that will maximize revenue.

15. a) If $f(x) = x^2 - 25$, determine the equation of the product function $f(x) \times \frac{1}{f(x)}$.

b) Determine the domain, and sketch the graph of the product function you found in part a).

c) If $f(x)$ is a polynomial function, explain how the domain and range of $f(x) \times \frac{1}{f(x)}$ changes as the degree of $f(x)$ changes.

1. For each of the following pairs of functions, write the equation of $y = (f \div g)(x)$.

a) $f(x) = 5, g(x) = x$ e) $f(x) = 8, g(x) = 1 + \left(\frac{1}{2}\right)^x$ f) $f(x) = x^2, g(x) = \log(x)$

2. a) Graph each pair of functions in question 1 on the same grid.
b) State the domains of f and g.
c) Use your graphs to make an accurate sketch of $y = (f \div g)(x)$.
d) State the domain of $f \div g$.

8.3 Composition of Functions

#1abf, 2bdf, 5bcdf (use tech to graph), 6bcd, 7bcdef, 13 (pg 552 in textbook)

1. Use $f(x) = 2x - 3$ and $g(x) = 1 - x^2$ to evaluate the following expressions.

a) $f(g(0))$ b) $g(f(4))$ f) $(g \circ g)(2)$

2. Given $f = \{(0, 1), (1, 2), (2, 5), (3, 10)\}$ and $g = \{(2, 0), (3, 1), (4, 2), (5, 3), (6, 4)\}$, determine the following values.

b) $(f \circ f)(1)$ d) $(f \circ g)(0)$ f) $(g^{-1} \circ f)(1)$

5. K In each case, functions f and g are defined for $x \in \mathbf{R}$. For each pair of functions, determine the expression and the domain of $f(g(x))$ and $g(f(x))$. Graph each result.

a) $f(x) = 3x^2, g(x) = x - 1$
b) $f(x) = 2x^2 + x, g(x) = x^2 + 1$
c) $f(x) = 2x^3 - 3x^2 + x - 1, g(x) = 2x - 1$
d) $f(x) = x^4 - x^2, g(x) = x + 1$
e) $f(x) = \sin x, g(x) = 4x$
f) $f(x) = |x| - 2, g(x) = x + 5$

6. For each of the following,
- determine the defining equation for $f \circ g$ and $g \circ f$
- determine the domain and range of $f \circ g$ and $g \circ f$

b) $f(x) = \sqrt{x}, g(x) = 3x + 1$ c) $f(x) = \sqrt{4 - x^2}, g(x) = x^2$ d) $f(x) = 2^x, g(x) = \sqrt{x - 1}$

7. For each function h, find two functions, f and g, such that $h(x) = f(g(x))$.

a) $h(x) = \sqrt{x^2 + 6}$ d) $h(x) = \dfrac{1}{x^3 - 7x + 2}$
b) $h(x) = (5x - 8)^6$ e) $h(x) = \sin^2(10x + 5)$
c) $h(x) = 2^{(6x+7)}$ f) $h(x) = \sqrt[3]{(x + 4)^2}$

13. In a vehicle test lab, the speed of a car, v kilometres per hour, at a time of t hours is represented by $v(t) = 40 + 3t + t^2$. The rate of gasoline consumption of the car, c litres per kilometre, at a speed of v kilometres per hour is represented by $c(v) = \left(\frac{v}{500} - 0.1\right)^2 + 0.15$. Determine algebraically $c(v(t))$, the rate of gasoline consumption as a function of time. Determine, using technology, the time when the car is running most economically during a 4 h simulation.