Chapter 9 Problem Set – Rates of Change and the Tangent Problem

9.1 The Average Rate of Change: The AROC #1, 2, 4, 9, 10 (pg 76 in textbook)

1. Calculate the average rate of change for the function $g(x) = 4x^2 - 5x + 1$ over each interval.

a) $2 \leq x \leq 4$
b) $2 \leq x \leq 3$
c) $2 \leq x \leq 2.5$
d) $2 \leq x \leq 2.25$
e) $2 \leq x \leq 2.1$
f) $2 \leq x \leq 2.01$

2. An emergency flare is shot into the air. Its height, in metres, above the ground at various times in its flight is given in the following table.

Time (s)	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Height (m)	2.00	15.75	27.00	35.75	42.00	45.75	47.00	45.75	42.00

a) Determine the average rate of change in the height of the flare during each interval.
i) $1.0 \leq t \leq 2.0$
ii) $3.0 \leq t \leq 4.0$

b) Use your results from part a) to explain what is happening to the height of the flare during each interval.

4. This table shows the growth of a crowd at a rally over a 3 h period.
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Time (h)	0.0	0.5	1.0	1.5	2.0	2.5	3.0
Number of People	0	176	245	388	402	432	415

a) Determine the average rate of change in the size of the crowd for each half hour of the rally.
b) What do these numbers represent?
c) What do positive and negative rates of change mean in this situation?

9. During the Apollo 14 mission, Alan Shepard hit a golf ball on the Moon. The function $h(t) = 18t - 0.8t^2$ models the height of the golf ball's trajectory on the Moon, where $h(t)$ is the height, in metres, of the ball above the surface of the Moon and t is the time in seconds. Determine the average rate of change in the height of the ball over the time interval $10 \leq t \leq 15$.

10. A company that sells sweatshirts finds that the profit can be modelled by $P(s) = -0.30s^2 + 3.5s + 11.15$, where $P(s)$ is the profit, in thousands of dollars, and s is the number of sweatshirts sold (expressed in thousands).

a) Calculate the average rate of change in profit for the following intervals.

i) $1 \leq s \leq 2$ ii) $2 \leq s \leq 3$ iii) $3 \leq s \leq 4$ iv) $4 \leq s \leq 5$

b) As the number of sweatshirts sold increases, what do you notice about the average rate of change in profit on each sweatshirt? What does this mean?

c) Predict if the rate of change in profit will stay positive. Explain what this means.

9.2 Instantaneous Rate of Change: The IROC #4ac, 6, 8, 9, 10 (pg 86 in textbook)

4. For the function $f(x) = 6x^2 - 4$, estimate the instantaneous rate of change for the given values of x.

a) $x = -2$ c) $x = 4$

6. A family purchased a home for \$125 000. Appreciation of the home's value, in dollars, can be modelled by the equation $H(t) = 125\,000(1.06)^t$, where $H(t)$ is the value of the home and t is the number of years that the family owns the home. Estimate the instantaneous rate of change in the home's value at the start of the eighth year of owning the home.

8. Jacelyn purchased a new car for \$18 999. The yearly depreciation of the value of the car can be modelled by the equation $V(t) = 18\,999(0.93)^t$, where $V(t)$ is the value of the car and t is the number of years that Jacelyn owns the car. Estimate the instantaneous rate of change in the value of the car when the car is 5 years old. What does this mean?

9. **A** A diver is on the 10 m platform, preparing to perform a dive. The diver's height above the water, in metres, at time t can be modelled using the equation $h(t) = 10 + 2t - 4.9t^2$.

a) Determine when the diver will enter the water.

b) Estimate the rate at which the diver's height above the water is changing as the diver enters the water.

10. **T** To make a snow person, snow is being rolled into the shape of a sphere. The volume of a sphere is given by the function $V(r) = \frac{4}{3}\pi r^3$, where r is the radius in centimetres. Use two different methods to estimate the instantaneous rate of change in the volume of the snowball with respect to the radius when $r = 5$ cm.