Advanced Functions

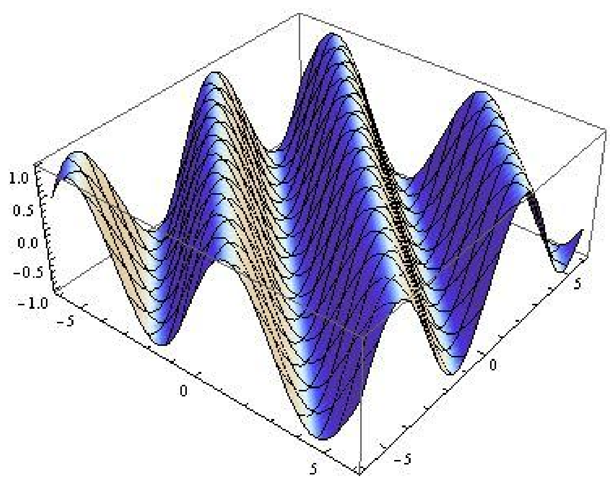
Course Notes

Unit 5 – Trigonometric Functions

Doing Trig with REAL Numbers

We will learn

- *about Radian Measure and its relationship to Degree Measure*
- *how to use Radian Measure with Trigonometric Functions*
- *about the connection between trigonometric ratios and the graphs of trigonometric functions*
- *how to apply our understanding of trigonometric functions to model and solve real world problems*



AΩΩ
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Chapter 5 – Trigonometric Functions

*Contents with suggested problems from the Nelson Textbook **(Chapter 6)***

You are welcome to ask for help, from myself or your peers, with any of the following problems. They will be handed in on the day of the Unit Test as a homework check.

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Pg. 321 #2edfh, 3 – 9

5.2 Trigonometric Ratios and Special Triangles (Part 1)
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5.5 Sketching the Trigonometric Functions
Worksheet

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5.1 Radian Measure and Arc Length

= real #'s

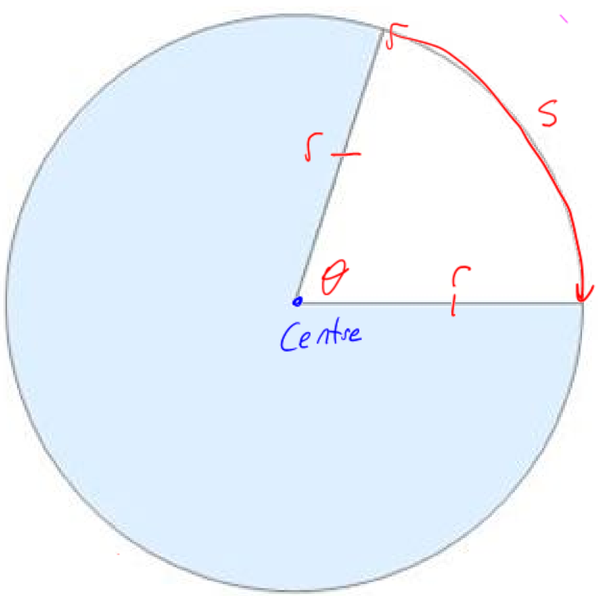
Learning Goal: We are learning to use radian measure to represent the size of an angle.

Radian Measure

We are familiar with measuring angles using “degrees”, and now we will turn to another measure for angles: **Radians**.

Before getting to the notion of radians, we need to learn some notation.

Picture



θ = central angle

r = radius

s = arc length subtended by the central angle

→ part of the circumference

The arc length formula is $\boxed{s = r\theta}$

this angle must be in radian measure

Definition 5.1.1

In a circle of radius r, a central angle θ subtending an arc of length $s = r$ measures 1 radian.

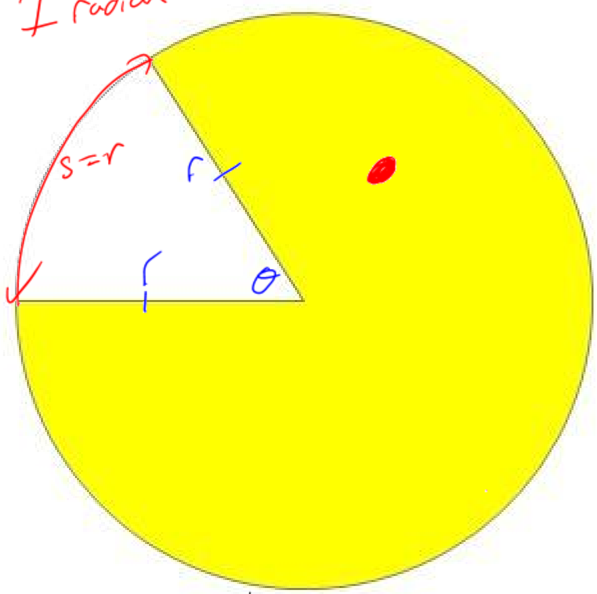
Picture

If $s = r$, $\theta = 1$ radian

$s = r\theta$

$\frac{r}{r} = \frac{r\theta}{r}$

$1 \text{ rad} = \theta$



When arc length is equal to the radius, $\theta = 1$ rad

Note: The circumference of a circle is given by $C = 2\pi r$

So, for a central angle of 360°, in a circle of radius $r = 1$, then

arc length is a whole circle.

$s = r\theta$

$2\pi \cancel{r} = \cancel{r}\theta$

$2\pi \text{ rad} = \theta$

$\therefore\ 360^\circ = 2\pi$ radians

$\boxed{180^\circ = \pi \text{ radians}}$ conversion factor

$$1^\circ = \frac{\pi}{180^\circ}$$

Example 5.1.1

Convert the following to radians:

a) $30^\circ \left(\frac{\pi \text{ rad}}{180^\circ}\right)$ $= \frac{\pi}{6}$ radians

b) $45^\circ \left(\frac{\pi}{180}\right)$ $= \frac{\pi}{4}$ rad

c) $\overset{2}{\cancel{120}}^\circ \left(\frac{\pi}{\underset{3}{\cancel{180}}}\right)$ $= \frac{2\pi}{3}$ rad

d) $315^\circ \left(\frac{\pi}{180}\right)$ $\div 45$ $= \frac{7\pi}{4}$

e) $161.3^\circ \left(\frac{\overset{3.14}{\cancel{\pi}}}{180}\right)$ $= 2.81$ rad

Example 5.1.2

Convert the following to degrees (round to two decimal places where necessary)

$180^\circ = \pi$ rad

$\frac{180^\circ}{\pi} = 1$ rad

a) $\frac{7\cancel{\pi}}{12}$ rad $\left(\frac{180}{\cancel{\pi}}\right)$ $= 105^\circ$

b) $\frac{10\cancel{\pi}}{9}$ rad $\left(\frac{180}{\cancel{\pi}}\right)$ $= 200^\circ$

c) 2.5 rad $\left(\frac{180}{3.14}\right)$ $= 143.31^\circ$

d) $\frac{\cancel{\pi}}{2}$ rad $\left(\frac{180}{\cancel{\pi}}\right)$ 90°

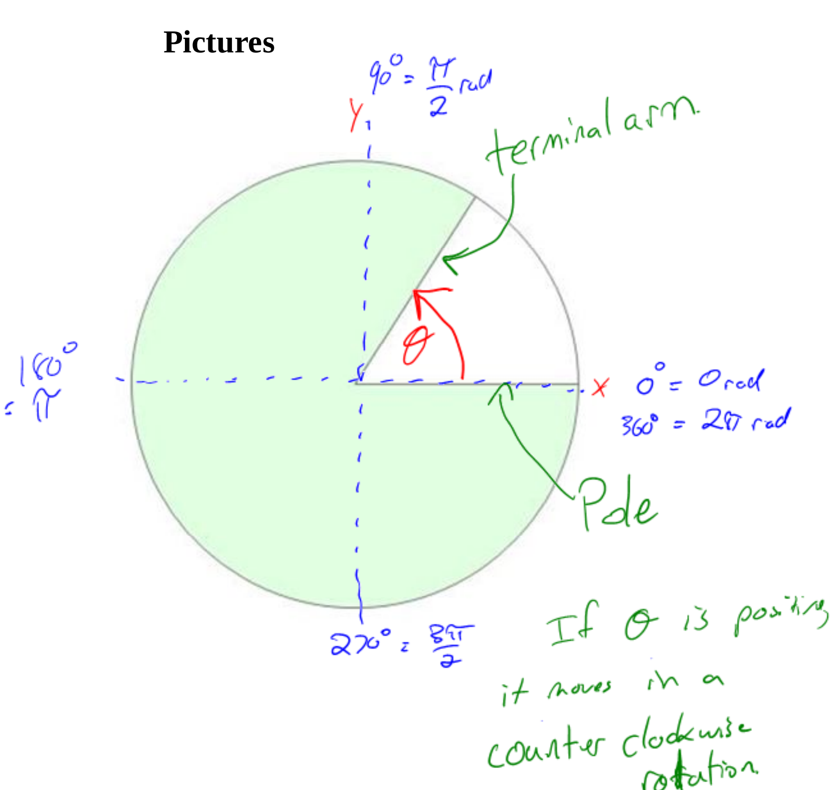
e) $-\frac{\cancel{\pi}}{3}$ rad $\left(\frac{180}{\cancel{\pi}}\right)$ $= -60^\circ$

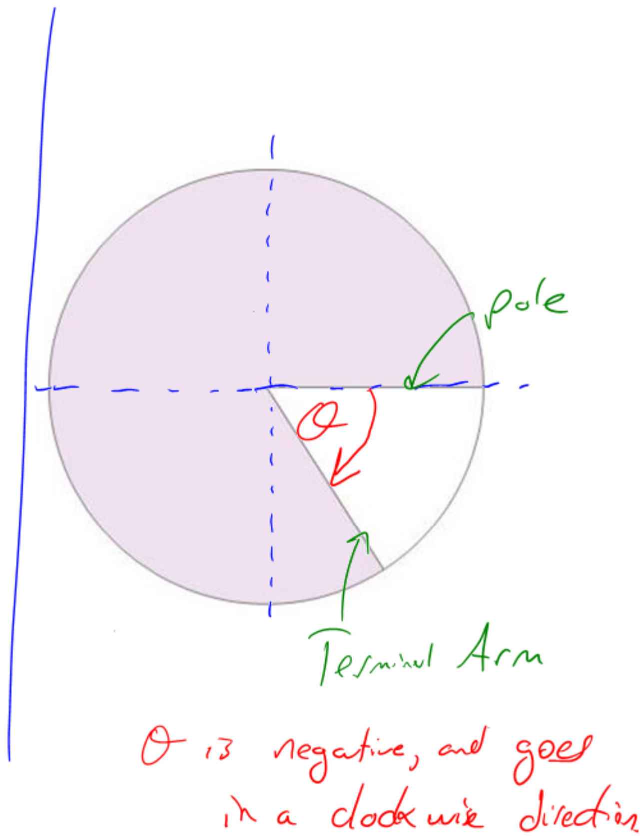
Q. What the rip is a negative degree?

Angles of Rotation

The sign on an angle can be thought of as the direction of rotation (around a circle).

Pictures



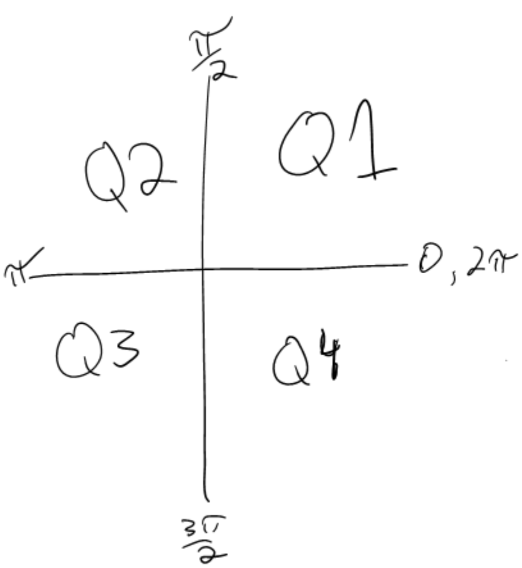


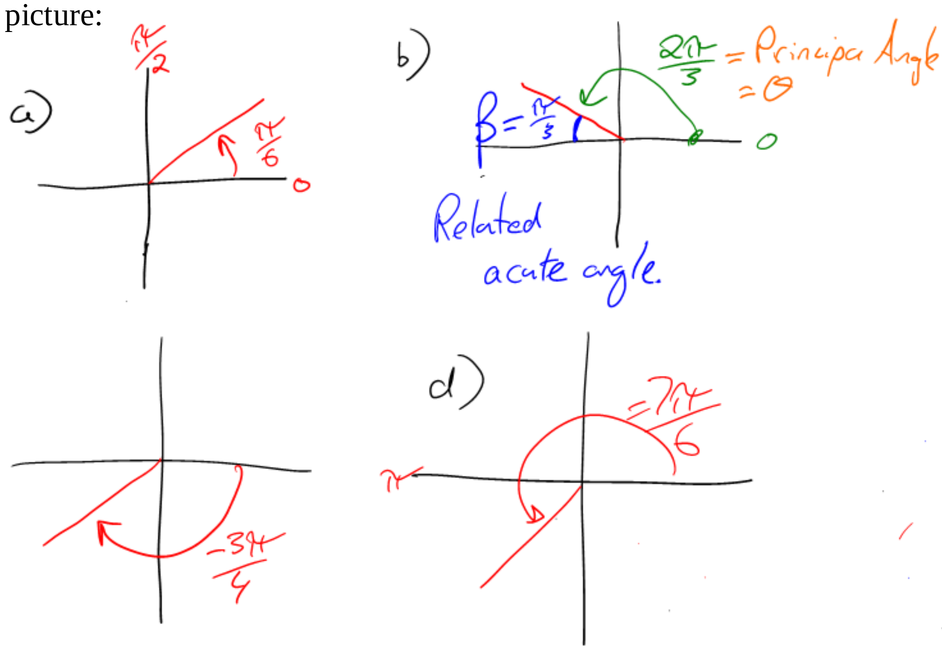
Example 5.1.3

Sketch the following angles of rotation:

a) $\frac{\pi}{6}$ rad b) $\frac{2\pi}{3}$ rad c) $-\frac{3\pi}{4}$ rad d) $\frac{7\pi}{6}$

BUT FIRST: Consider the following picture:





Example 5.1.4

Determine the length of an arc, on a circle of radius $5cm$ $=r$, subtended by an angle:

a) $\theta = 2.4$ rad

$S = r\theta$

$S = (5)(2.4)$

$S = 12\,cm$

b) $\theta = 120^{\circ}$

$120\left(\frac{\pi}{180}\right) = \frac{2\pi}{3}$

$S = r\theta$

$S = (5)\left(\frac{2\pi}{3}\right)$

$S = \frac{10\pi}{3}\,cm$

$S \approx 10.5\,cm$

Success Criteria:

- I can understand that a radian is a real number
- I can convert from degrees to radians by multiplying by $\frac{\pi}{180^{o}}$
- I can convert from radians to degrees by multiplying by $\frac{180^{o}}{\pi}$

5.2 Trigonometric Ratios and Special Triangles

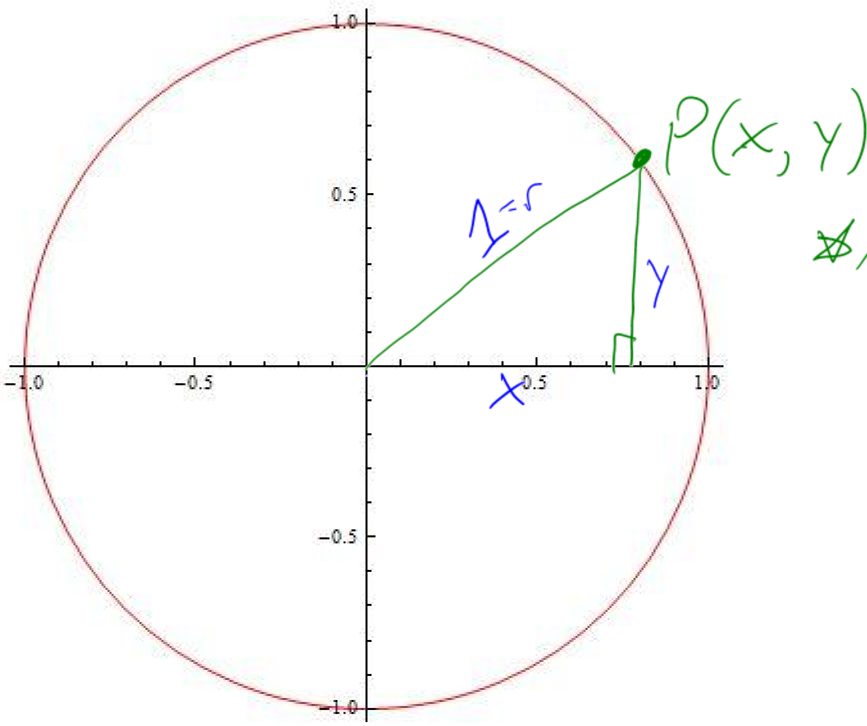
(*Part 1*)

- no calculators
- exact numbers
↳ √, π, fractions.

Learning Goal: We are learning to use the Cartesian plane to evaluate the trigonometric ratios for angles between 0 and 2π.

Consider the circle of radius 1:

unit circle.

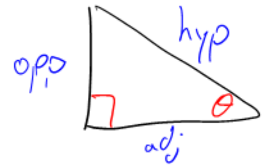


☆ Always connect to the pole
↳ x-axis

By Pythagorean Theorem:

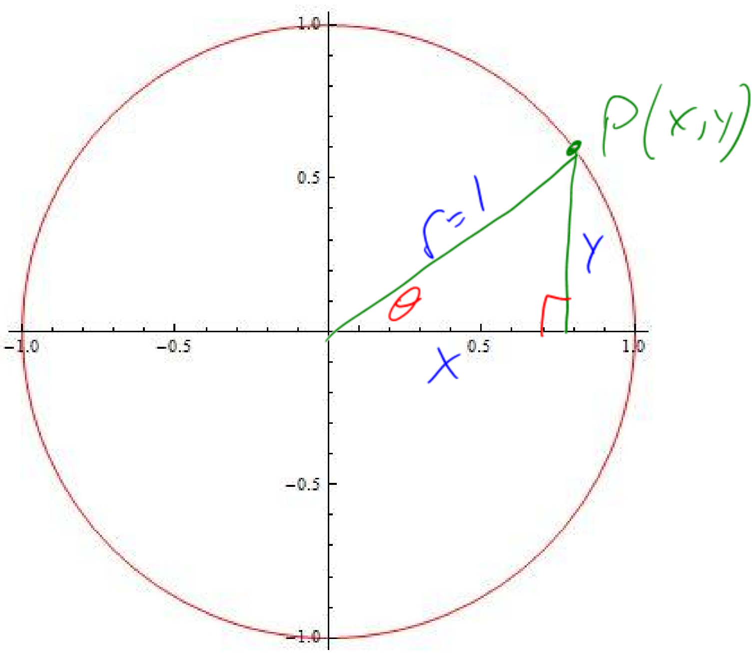
$$x^2 + y^2 = 1$$

Recall the six main Trigonometric Ratios:



Primary Trig Ratios	Reciprocal Trig Ratios
$\sin\theta = \frac{opp}{hyp}$	$\frac{1}{\sin\theta} = \csc\theta = \frac{hyp}{opp}$
$\cos\theta = \frac{adj}{hyp}$	$\frac{1}{\cos\theta} = \sec\theta = \frac{hyp}{adj}$
always in between 0 and 1	always be bigger than one
$\tan\theta = \frac{opp}{adj}$	$\frac{1}{\tan\theta} = \cot\theta = \frac{adj}{opp}$

Consider again the circle of radius 1 (but now keeping in mind SOH CAH TOA)



$\sin\theta = \frac{y}{1} = y$

$\cos\theta = \frac{x}{1} = x$

$\tan\theta = \frac{y}{x}$

Note: $P(x,y)$ can now be represented by $P(\overset{x}{\cos\theta}, \overset{y}{\sin\theta})$

The Pythagorean Identity

$$x^2 + y^2 = 1$$

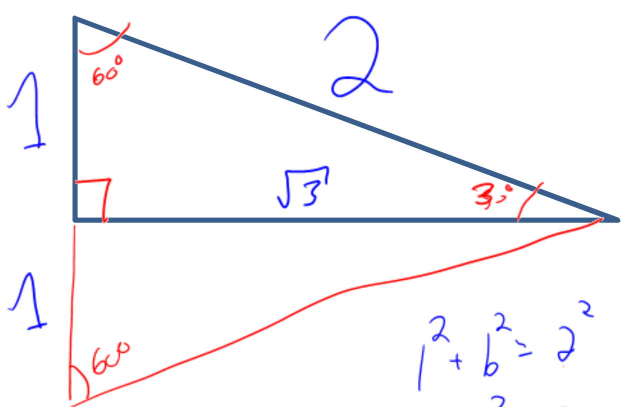
$$\cos^2\theta + \sin^2\theta = 1$$

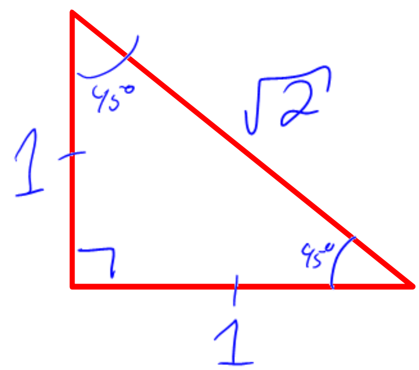
$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

Special Triangles in Radians

Recall: We have two “Special Triangles”. In **degrees** they are:

Equilateral Triangle

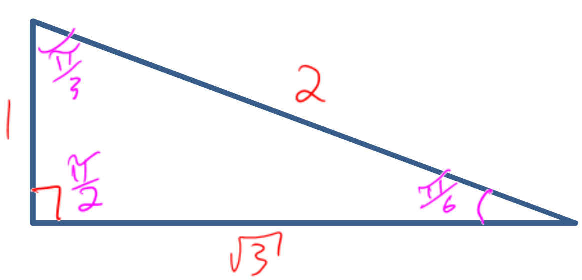


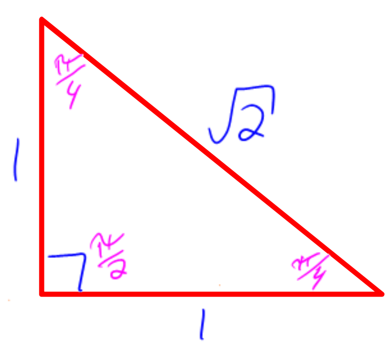


$$1^2 + b^2 = 2^2$$
$$b^2 = 3$$
$$b = \sqrt{3}$$

In radians we have

$$30°\left(\frac{\pi}{180}\right) = \frac{\pi}{6} \qquad 60°\left(\frac{\pi}{180}\right) = \frac{\pi}{3} \qquad 90°\left(\frac{\pi}{180}\right) = \frac{\pi}{2} \qquad 45°\left(\frac{\pi}{180}\right) = \frac{\pi}{4}$$

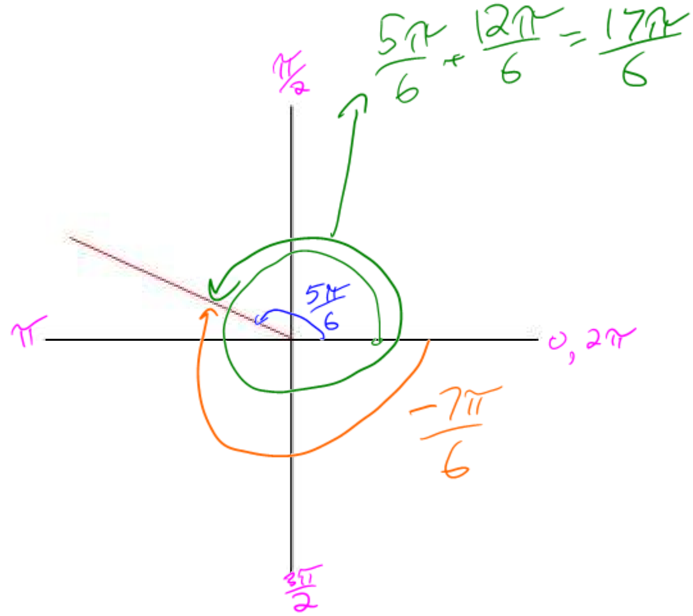




MEMORIZE THESE!

Angles of Rotations and Trig Ratios

Consider the following sketch of the angle of rotation $\theta = \frac{5\pi}{6}$:



There are infinitely many angles of rotation for each ~~angle~~ terminal arm

In this Example, we call $\theta = \frac{5\pi}{6}$ the **PRINCIPAL ANGLE**, or the angle in standard position.
This means the smallest positive angle.

Note: All principal angles $\theta \in [0, 2\pi]$

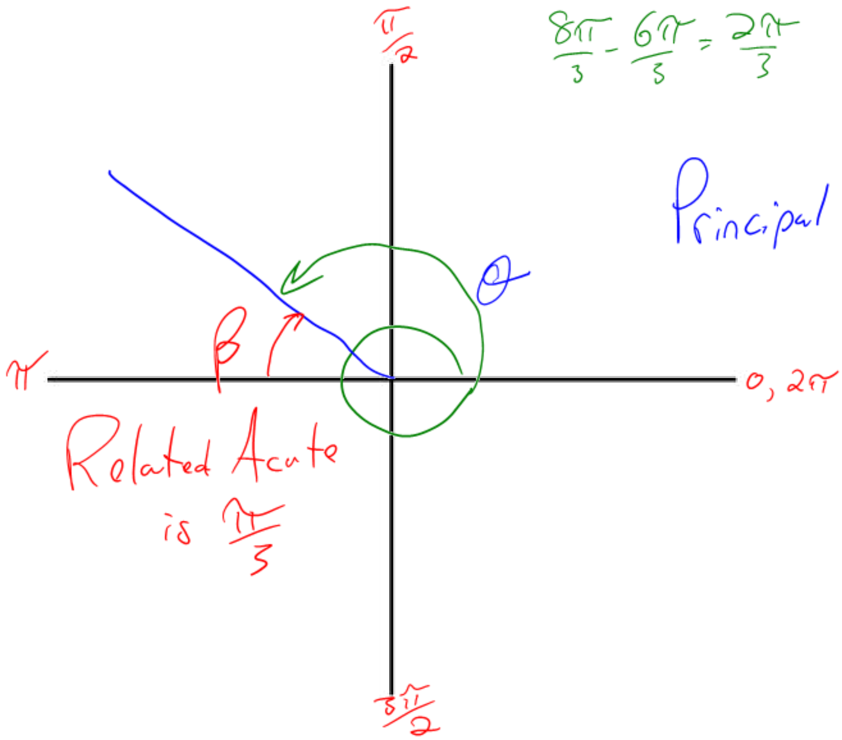
one circle

Example 5.2.1

Sketch the angle of rotation $\theta = \dfrac{8\pi}{3}$ and determine the principal angle.

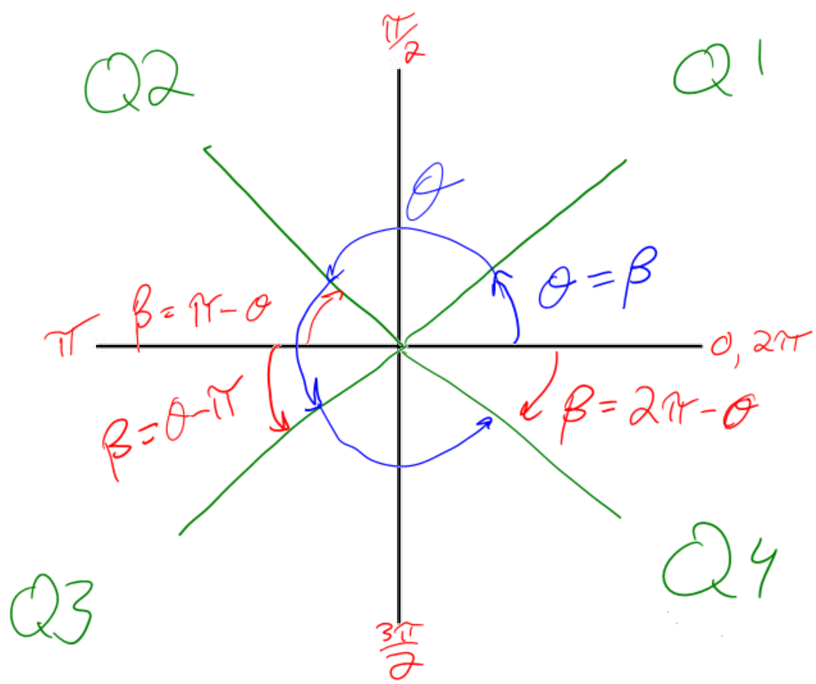
$\frac{8\pi}{3} - \frac{6\pi}{3} = \frac{2\pi}{3}$ $\quad$ $\frac{2\pi}{1} = \frac{6\pi}{3}$

Principal angle is $\frac{2\pi}{3}$



~~Principal Angle:~~

θ = principal
β = related acute



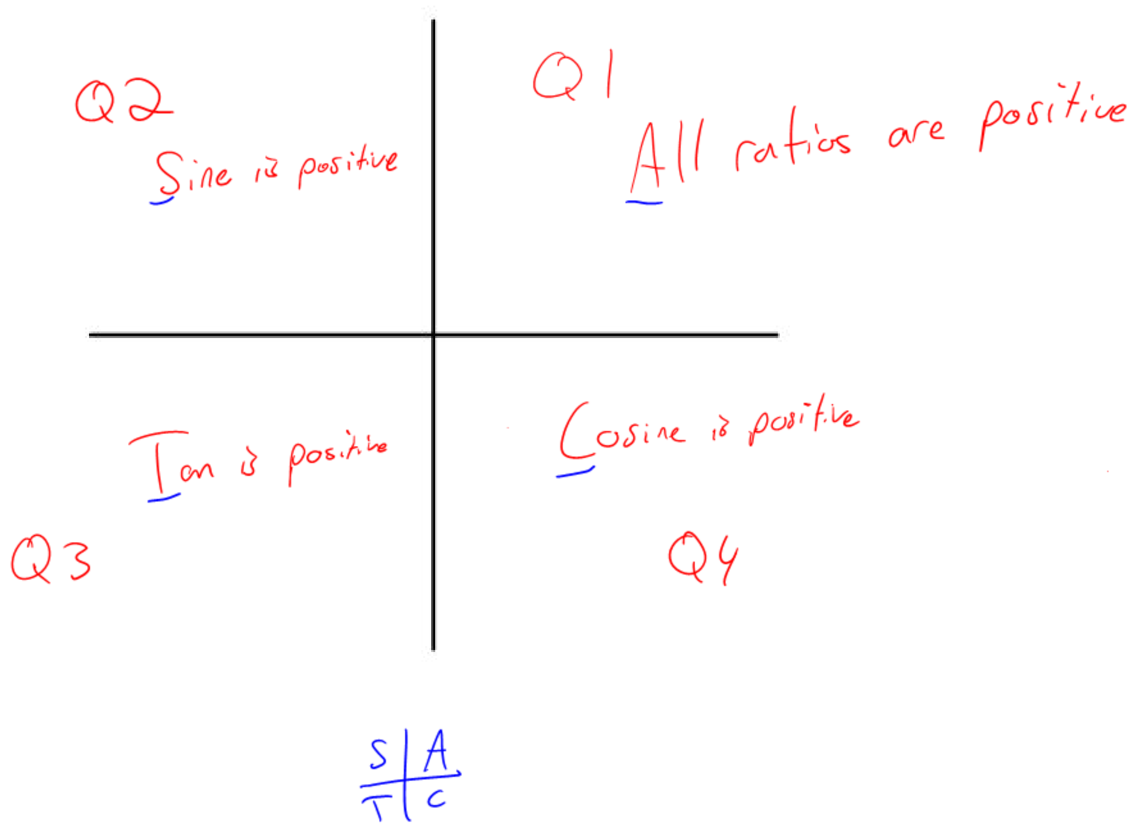
We now have enough tools to calculate the trigonometric ratios of any angle!

For any given angle θ (in radians from here on) we will:

1) Draw θ in standard position (i.e. draw the principal angle for θ)
2) Determine the related acute angle (between the terminal arm and the polar axis)
3) Use the related acute angle and the CAST RULE (and SOH CAH TOA) to determine the trig ratio in question

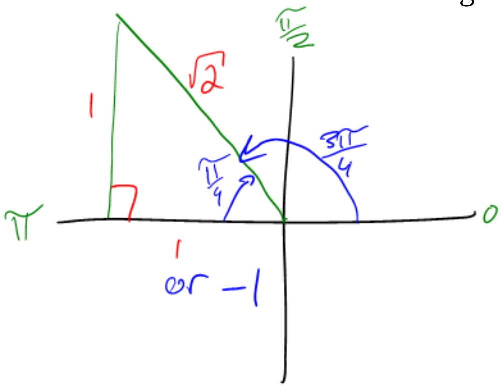
Recall the CAST RULE

Note: The CAST RULE determines the sign (+ or -) of the trig ratio



Example 5.2.2

Determine the trig ratio $\sin\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$



by CAST rule, sine is positive in Q2

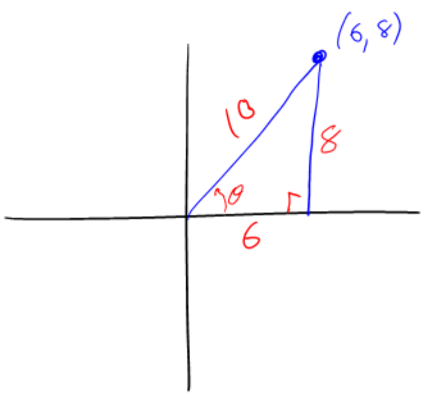
Example 5.2.3

The point $(6,8)$ lies on the terminal arm (of length *r*) of an angle of rotation. Sketch the angle of rotation.

Determine: a) the value of *r*

b) the primary trig ratios for the angle

c) the value of the angle of rotation in radians, to two decimal places



a) $6^2 + 8^2 = r^2$

$36 + 64 = r^2$

$100 = r^2$

$10 = r$

b) $\sin\theta = \frac{8}{10} = \frac{4}{5}$

$\cos\theta = \frac{6}{10} = \frac{3}{5}$

$\tan\theta = \frac{8}{6} = \frac{4}{3}$

c) $\tan\theta = \frac{4}{3}$

$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\theta = 0.93$ rad.

Success Criteria

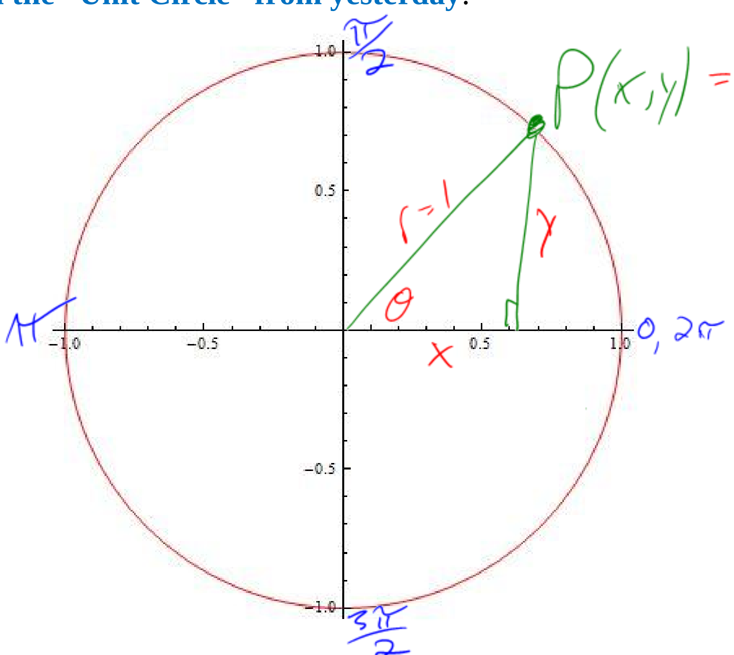
- I can write the angles in the special triangles using both radians (Unit Circle) and degrees
- I can find the principal angle θ, by first finding the related acute angle β
- I can write the trig ratios for any angle using “x, y, and r”
- I can use the CAST rule to determine where a ratio is positive or negative

5.3 Trigonometric Ratios and Special Triangles

(*Part 2 – Exact Values*)

Learning Goal: We are learning to use the Cartesian plane to evaluate the trigonometric ratios for angles between 0 and 2π.

Recall the "Unit Circle" from yesterday:



$P(x, y) = P(\cos\theta, \sin\theta)$

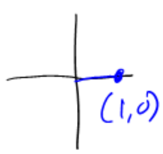
$\sin\theta = y$

$\cos\theta = x$

$\tan\theta = \frac{y}{x}$

$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$

With this circle (and without a calculator!) we can evaluate EXACTLY the trig ratios for the angles (in radians) $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ radians. Axis angles



$\sin(0) = 0 = \sin(2\pi)$

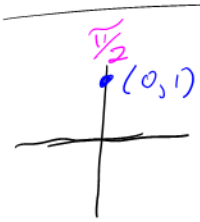
$\cos(0) = 1 = \cos(2\pi)$

$\tan(0) = \frac{0}{1} = 0 = \tan(2\pi)$

$\sin(\pi) = 0$

$\cos(\pi) = -1$

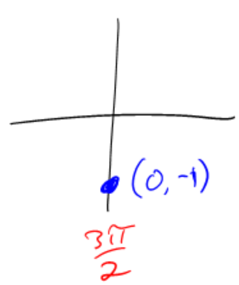
$\tan(\pi) = \frac{0}{-1} = 0$



$\sin(\frac{\pi}{2}) = 1$

$\cos(\frac{\pi}{2}) = 0$

$\tan(\frac{\pi}{2}) = \frac{1}{0}$ = undefined.

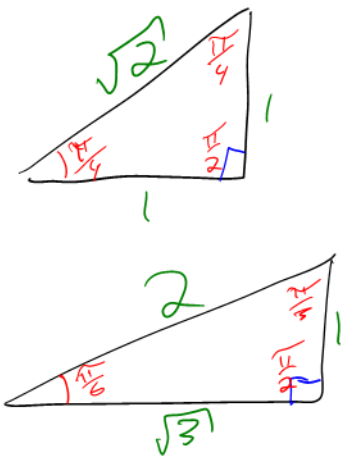


$\sin(\frac{3\pi}{2}) = -1$

$\cos(\frac{3\pi}{2}) = 0$

$\tan(\frac{3\pi}{2}) = \frac{-1}{0}$ = undefined.

Now, using **Special Triangles**, and **CAST** we can evaluate **EXACTLY** trig ratios for "special angles".



Note: A trig ratio is a NUMBER.
Numbers have 2 qualities
1) value
2) sign (+/−)

Thus a trig ratio has a value

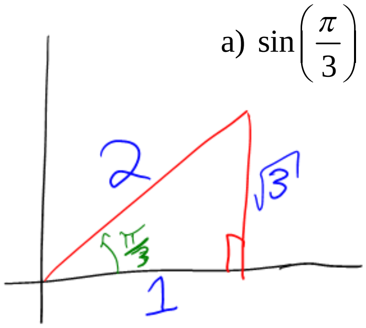
(which we **evalu**ate using the related acute angle and Special Triangles)

AND, a trig ratio has a sign from the cast rule.

Example 5.3.1

Determine **Exactly** (i.e. the **use of a calculator** means **MARKS OFF**)

a) $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

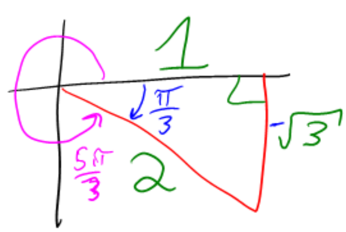


In Q1, $\theta = \beta$

Sine is positive in Q1

d) $\sec\left(\frac{5\pi}{3}\right) = \frac{2}{1} = 2$

Q4



$\cos = \frac{1}{2}$

In Q4, $\beta = 2\pi - \theta$

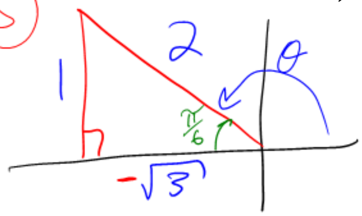
$\beta = 2\pi - \frac{5\pi}{3}$

$\beta = \frac{\pi}{3}$

5

Q2

b) $\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

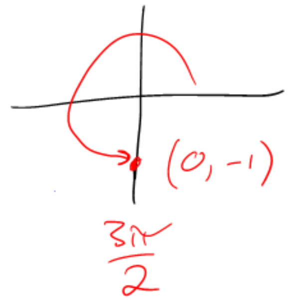


In Q2, $\beta = \pi - \theta$

Only sine positive in Q2, ∴

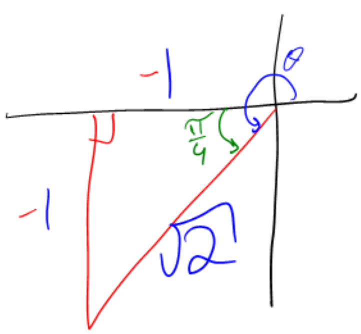
cosine is negative

e) $\tan\left(\frac{3\pi}{2}\right) = \frac{y}{x} = \frac{-1}{0} =$ undefined



Q3

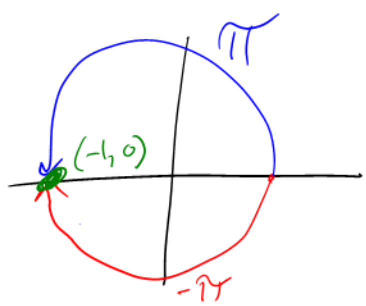
c) $\tan\left(\frac{5\pi}{4}\right) = \frac{1}{1} = 1$



Tan is positive in Q3

f) $\csc(-\pi) = \csc(\pi) = \frac{1}{\sin(\pi)} = \frac{1}{0}$

= undefined

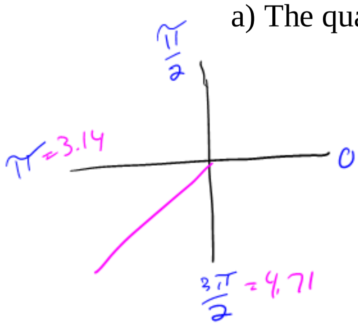


Example 5.3.2

Given $\sin(4)$ determine:

a) The quadrant $\theta = 4$ is in.

b) The sign of $\sin(4)$ (no calculators!)



$\therefore \theta = 4$ is in Q3

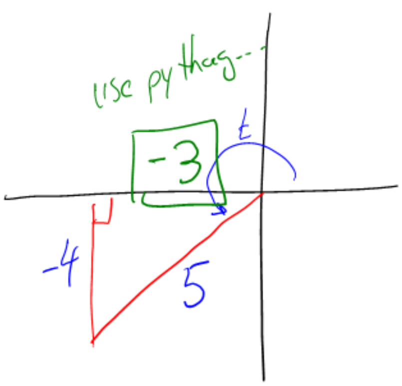
Negative because only tan is positive in Q3.
or cot

Example 5.3.3

Given $\sin(t) = -\frac{4}{5}$, $\pi \le t \le \frac{3\pi}{2}$, determine

Q3

a) $\cos(t)$ b) $\tan(t)$ c) t in radians, rounded to three decimal places.



a) $\cos(t) = \frac{-3}{5}$

b) $\tan(t) = \frac{-4}{-3} = \frac{4}{3}$

c) Always use a positive ratio to get the related acute angle, β. Then figure out θ.

$\beta = \theta - \pi$

$\beta + \pi = \theta$

$\tan(\beta) = \frac{4}{3}$

$\beta = \tan^{-1}\left(\frac{4}{3}\right)$

$\beta = 0.927$

$t = 3.141 + 0.927$

$t = 4.068$ rad

Success Criteria

- I can write the angles in the special triangles using both radians (Unit Circle) and degrees
- I can find the principal angle θ, by first finding the related acute angle β
- I can identify the trig ratios of the 4 axis angles
- I can write the trig ratios for any angle using "x, y, and r"
- I can use the CAST rule to determine where a ratio is positive or negative

5.4 Trigonometric Ratios and Special Triangles

(*Part 3 – Getting the Angles*)

Learning Goal: We are learning to determine the exact values for both angles between 0 and 2π, given a particular trig ratio.

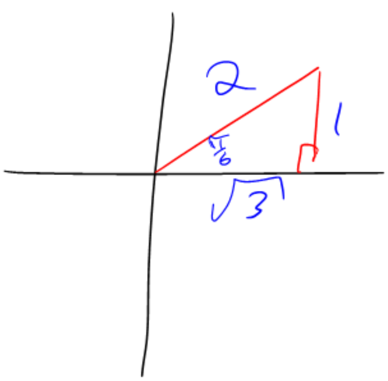
We have been looking at **evaluating exact values** for trigonometric ratios using special triangles and CAST, given an **angle of rotation**. We now turn our attention to the **inverse operation** – determining **angles of rotation** given a **trig ratio**.

Example 5.4.1

Determine exactly:

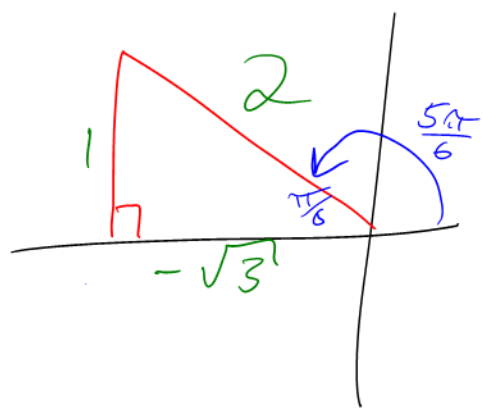
Q1

a) $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$



Q2

b) $\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$



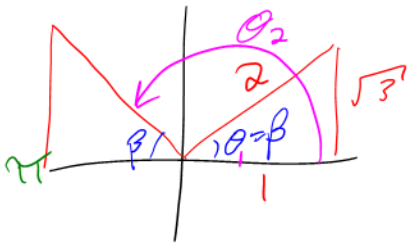
$\sin\theta = \frac{1}{2}$

Note: EVERY trig ratio has two angles of rotation in $[0, 2\pi]$, except for some axis angles

Example 5.4.2

Determine BOTH angles of rotation, θ, for $0 \le \theta \le 2\pi$ given

a) $\sin(\theta) = \frac{+\sqrt{3}}{2}$



Sin is positive in Q1 and Q2

$\beta = \frac{\pi}{3}$

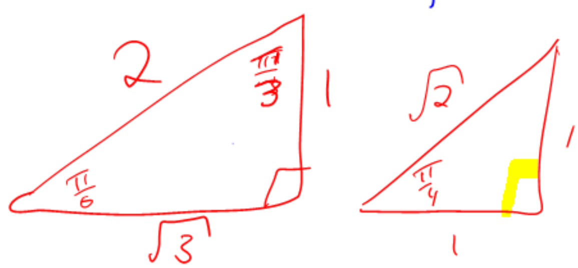
$$\boxed{\theta_1 = \frac{\pi}{3}}$$

$\theta_2 = \frac{3\pi}{3} - \frac{\pi}{3}$

$$\boxed{\theta_2 = \frac{2\pi}{3}}$$

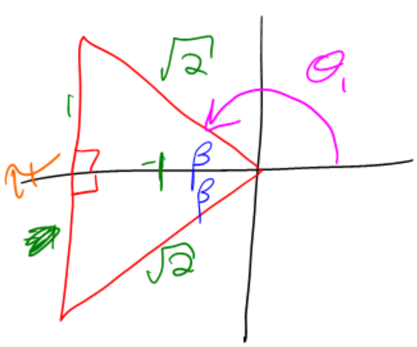
Procedure

1) Determine the quadrants θ is in.
2) Draw the angles of rotation.
3) Determine the related acute angle β and construct the appropriate special triangles.
4) Determine the angles of rotation.



b) $\cos(\theta) = -\frac{1}{\sqrt{2}}$

Q2 + Q3



$\beta = \frac{\pi}{4}$

$\theta_1 = \pi - \frac{\pi}{4}$

$$\boxed{\theta_1 = \frac{3\pi}{4}}$$

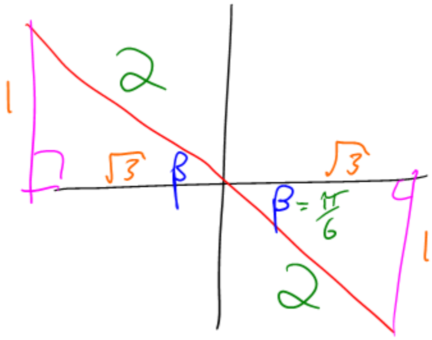
$\theta_2 = \pi + \frac{\pi}{4}$

$$\boxed{\theta_2 = \frac{5\pi}{4}}$$

tan

c) $\cot(\theta) = \frac{-\sqrt{3}}{1}$ $\frac{a}{o}$

Q2, Q4



$\beta = \frac{\pi}{6}$

$\theta_1 = \frac{6\pi}{6} - \frac{\pi}{6}$

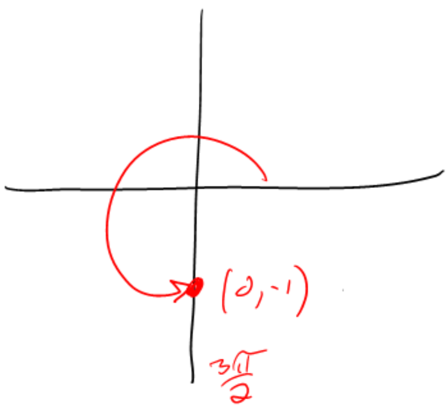
$\theta_1 = \frac{5\pi}{6}$

$\theta_2 = \frac{12\pi}{6} - \frac{\pi}{6}$

$\theta_2 = \frac{11\pi}{6}$

d) $\sin(\theta) = -1 = y$ Axis Angle | Recall: $\sin\theta = y$

$\cos\theta = x$



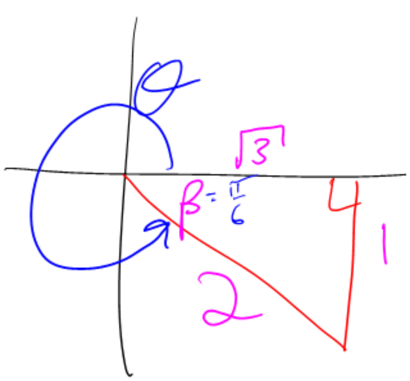
$\theta = \frac{3\pi}{2}$

Q4

Example 5.4.3

sine.

Determine θ where $\boxed{\frac{3\pi}{2} \le \theta \le 2\pi}$ for $\csc(\theta) = \frac{-2}{1}$ $\frac{h}{o}$



$\beta = \frac{\pi}{6}$

$\theta = 2\pi - \frac{\pi}{6}$

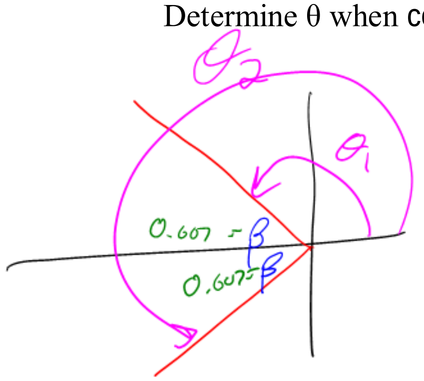
$\theta = \frac{11\pi}{6}$

This is not a special Δ.

Example 5.4.4

Determine θ when $\cos\theta = -0.8213$.

Q2, Q3



To calculate β, use the positive ratio

$$\cos\beta = 0.8213$$

$$\beta = \cos^{-1}(0.8213)$$

$$\beta = 0.607_{\text{rad}}$$

$$\theta_1 = 3.141 - 0.607$$

$$\theta_1 = 2.534$$

$$\theta_2 = 3.141 + 0.607$$

$$\theta_2 = 3.748$$

Practice Problems (Homework)

Determine the angles of rotation, θ, for $0 \le \theta \le 2\pi$:

a) $\sin(\theta) = -\frac{\sqrt{3}}{2}$

b) $\sec(\theta) = \sqrt{2}$

c) $\tan(\theta) = \frac{1}{\sqrt{3}}$

d) $\cot(\theta) = -1$

e) $\csc(\theta) = \frac{2}{\sqrt{3}}$

f) $\cos(\theta) = 0$

g) $\sin(\theta) = 1$

h) $\sqrt{3}\cos(\theta) - 2\cos(\theta) \cdot \sin(\theta) = 0$

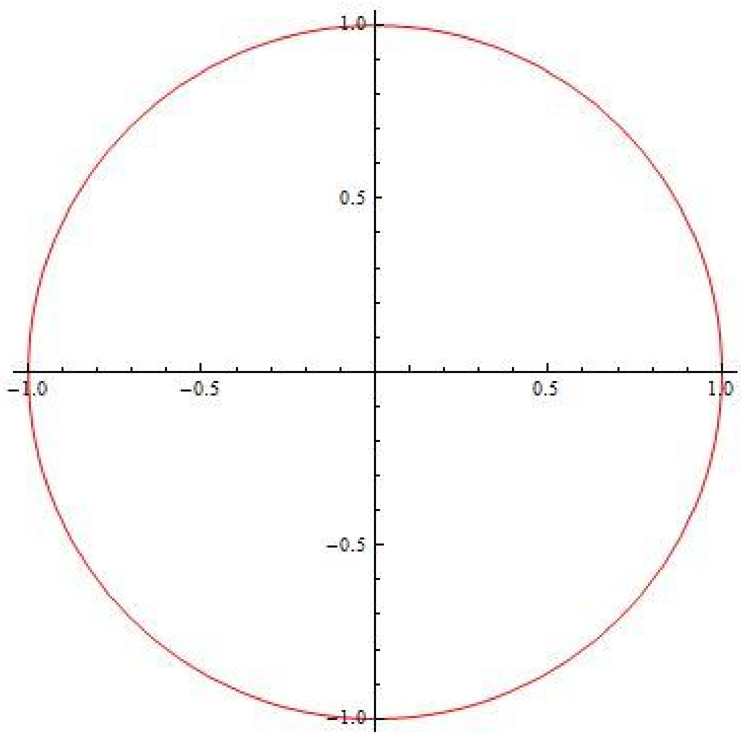
Success Criteria

- I can write the angles in the special triangles using both radians (Unit Circle) and degrees
- I can find the principal angle θ, by first finding the related acute angle β
- I can, given a trig ratio, determine the exact values for both angles between 0 and 2π
- I can identify the trig ratios of the 4 axis angles
- I can write the trig ratios for any angle using "x, y, and r"
- I can use the CAST rule to determine where a ratio is positive or negative

5.5 Sketching the Trigonometric Functions

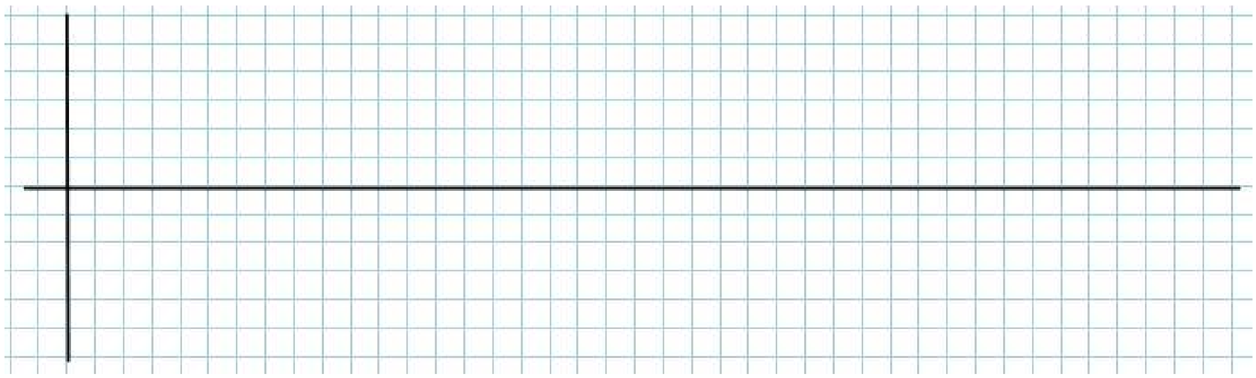
Learning Goal: We are learning to sketch the graphs of 6 trigonometric functions.

Before beginning the sketches, recall the diagram of the unit circle that we have been using to explore the basic ideas in trigonometry:

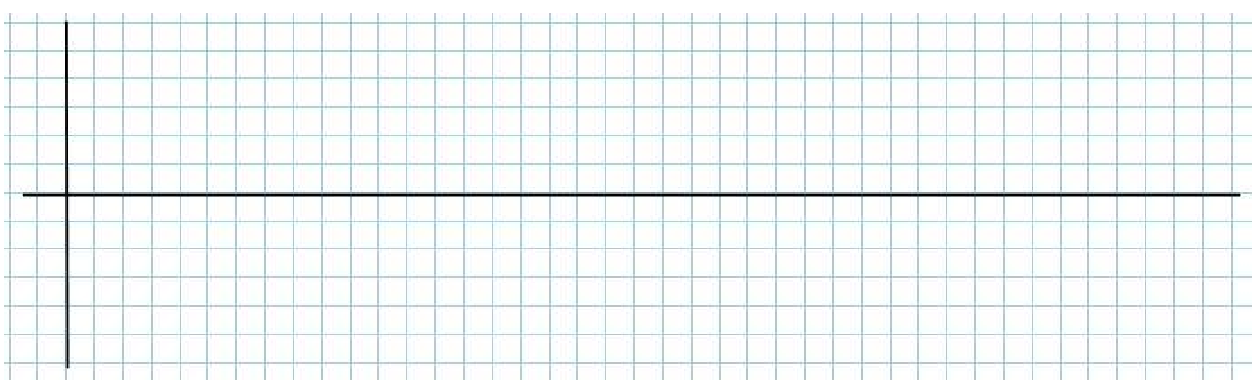


The Primary Trigonometric Functions

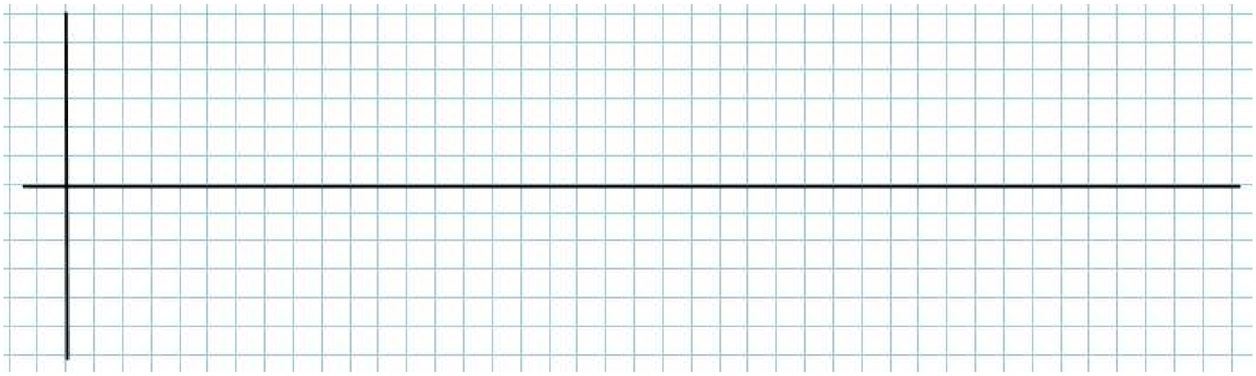
$f(\theta) = \sin(\theta), \quad \theta \in [0, 4\pi]$



$g(\theta) = \cos(\theta)$

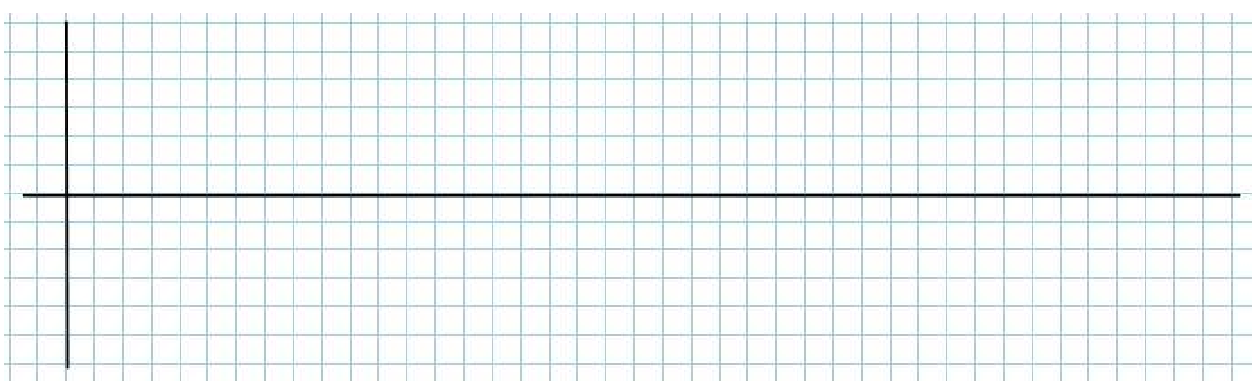


$h(\theta) = \tan(\theta)$

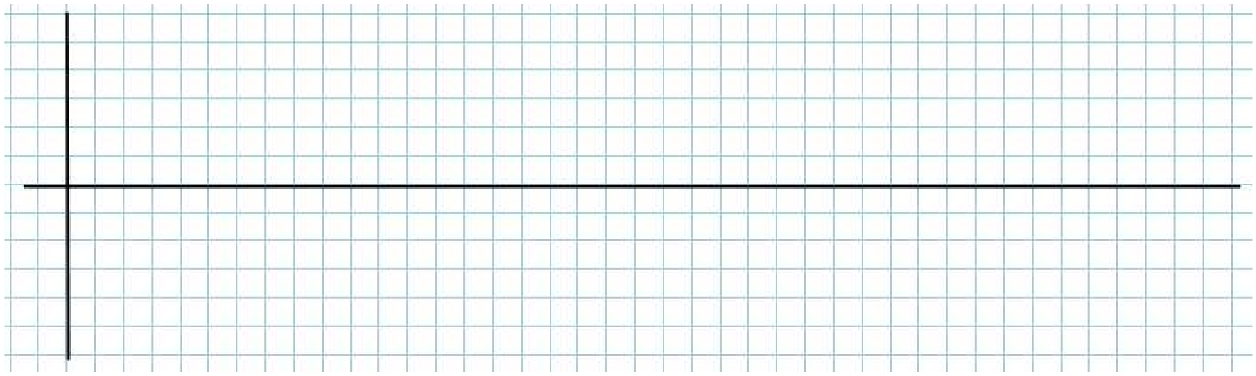


The Reciprocal Trig Functions

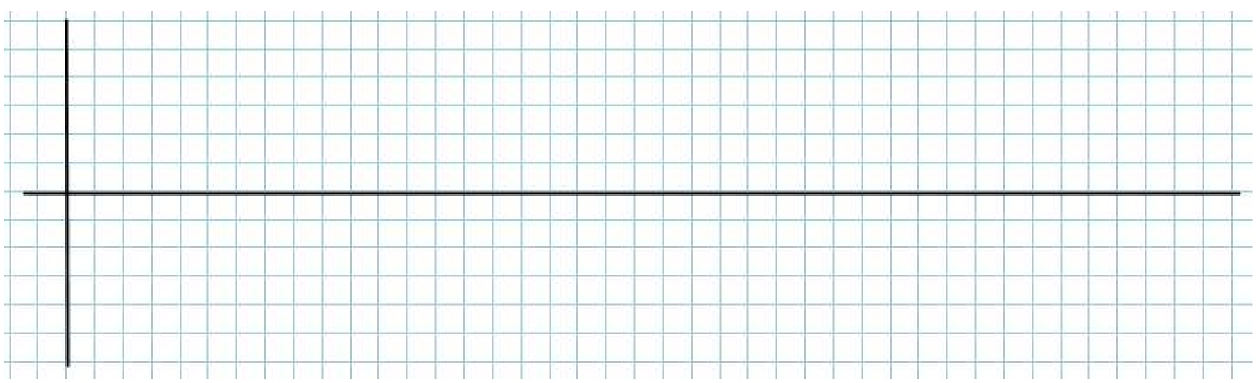
$f(\theta) = \csc(\theta)$



$g(\theta) = \sec(\theta)$



$h(\theta) = \cot(\theta)$



Success Criteria:

- I can recognize the graphs of sin, cos, tan, csc, sec, and cot

5.6 Transformations of Trigonometric Functions

Learning Goal: We are learning to use transformations to sketch the graphs of trigonometric functions in radians.

By this point in your illustrious High School careers, you have a solid understanding of Transformations of Functions in general. In terms of the trig functions Sine and Cosine in particular, the concepts are as you expect, but the transformations have specific meanings relating to nature of the sinusoidal "wave".

General Form of the Sine and Cosine Functions

$$f(\theta) = a\sin(k(\theta - d)) + c \qquad g(\theta) = a\cos(k(\theta - d)) + c$$

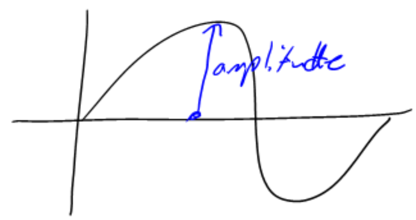
$|a|$ = amplitude

Vertical stretch

$$a = \frac{\overset{\text{peak}}{\max} - \overset{\text{trough}}{\min}}{2}$$

$a = \max - \text{middle}$

$a = \text{middle} - \min$



k = Period factor

$\frac{1}{k}$ = Horizontal Stretch

Period = $\frac{2\pi}{k}$

one cycle

$k = \frac{2\pi}{\text{Period}}$

d = Phase shift

Horizontal Shift

Note: To determine d you MUST isolate d from θ or x.

factor

this is the starting point when graphing.

c = central axis / middle

vertical shift

$$c = \frac{\max + \min}{2}$$

Example 5.6.1

Determine the amplitude, period, phase shift and the equation of the central axis for:

a) $f(\theta)=\overset{a}{2}\sin\left(\overset{k}{1}\left(\theta\overset{d}{+}\frac{\pi}{3}\right)\right)\overset{c}{+1}$

$\left(\theta-\left(-\frac{\pi}{3}\right)\right)$

amp is 2

Period is $\frac{2\pi}{1}=2\pi$

Phase Shift: $-\frac{\pi}{3}$

Central axis: $y=1$

b) $g(\theta)=\overset{a}{3}\cos\left(\overset{k}{2}\theta-\frac{\pi}{2}\right)$ c ?

$2\left(\theta-\frac{\pi}{4}\right)$ d

amp $=3$

Period $=\frac{2\pi}{2}=\pi$

Phase Shift $=\frac{\pi}{4}$

Central axis: $y=0$

Example 5.6.2

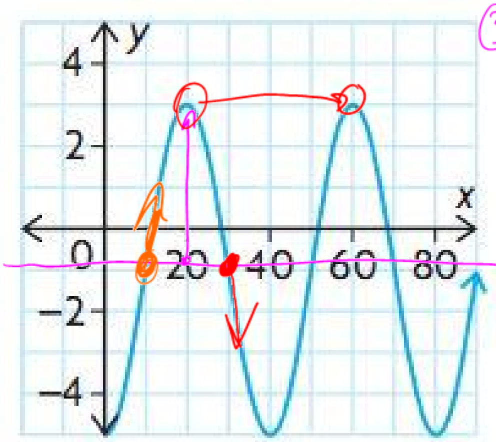
From your text: Pg. 346 #14c

Determine a ***sinusoidal*** function for the given sketch of a graph

↳ sine or cosine

First: max $=3$

min $=-5$



(3) Amplitude: $y=a$

amp $=\frac{3-(-5)}{2}=\frac{8}{2}=4$

Period:

↳ horizontal distance from peak to peak

Period is 40 $\quad k=\frac{2\pi}{40}=\frac{\pi}{20}$

Phase Shift: As Cosine

↓ x-coordinate

Peak → $d=20, 60, 100$

Trough → $d=0, 40, 80$ → $a=$ negative

As Sine

$d=10$ ↗ ∴ $a=+$

$d=30$ ↘ ∴ $a=$ negative

(2) Equation of Central Axis: $y=\frac{3+-5}{2}=\frac{-2}{2}=-1=c$

Equation as a Cosine Wave

Peak: $f(x)=4\cos\left(\frac{\pi}{20}(x-20)\right)-1$

Trough: $f(x)=-4\cos\left(\frac{\pi}{20}(x-0)\right)-1$

middle ↗

middle ↘

Equation as a Sine Wave

$f(x)=4\sin\left(\frac{\pi}{20}(x-10)\right)-1$

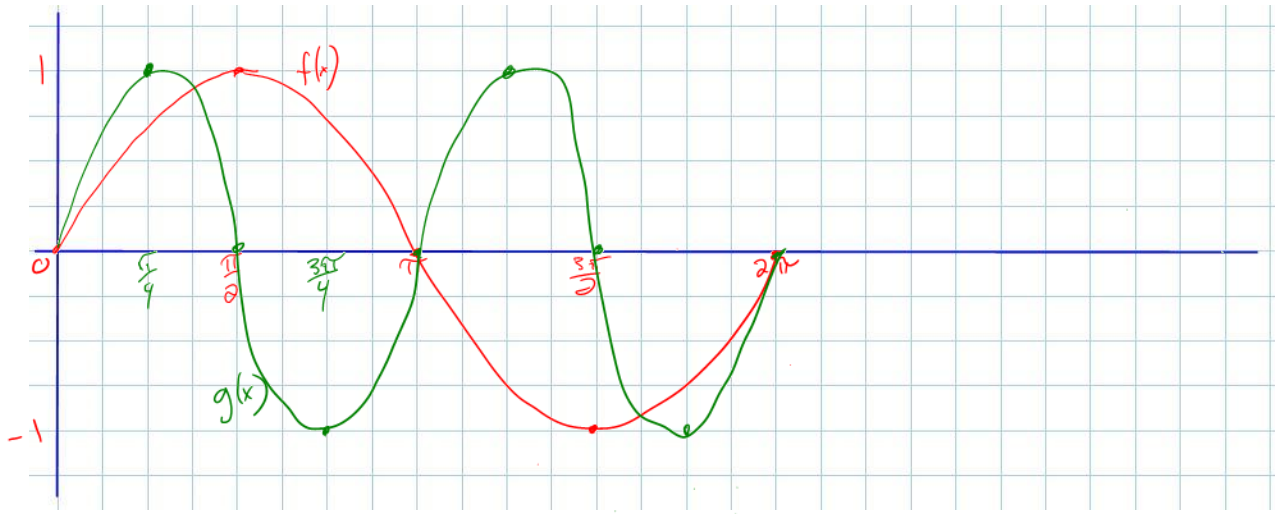
$f(x)=-4\sin\left(\frac{\pi}{20}(x-30)\right)-1$

Period $= \frac{2\pi}{2} = \pi$

k

Example 5.6.3

Sketch $f(x) = \sin(x)$ and $g(x) = \sin(2x)$ for $0 \le x \le 2\pi$ on the same set of axes.



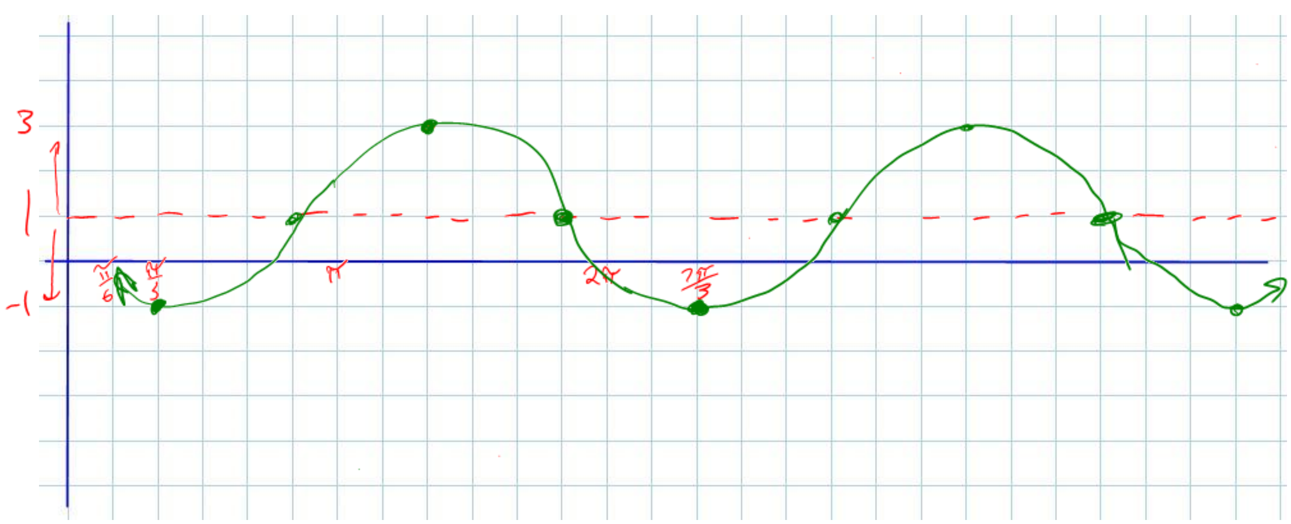
begins at min/trough

Example 5.6.4

Sketch $f(\theta) = -2\cos\left(\theta - \frac{\pi}{3}\right) + 1$ on $0 \le \theta \le 2\pi$

Period $= 2\pi$
Middle $= y = 1$
Amp $= 2$
Phase $= \frac{\pi}{3}$

Start $\left(\frac{\pi}{3}, -1\right)$
$+2\pi$
End $\left(\frac{7\pi}{3}, -1\right)$



use $\frac{\pi}{6}$ as scale is period is 2π

Example 5.6.5

start at middle ?

Amp = 3

Central y = -1

Period = $\frac{2\pi}{2} = \pi$

Phase Shift = $\frac{\pi}{3}$

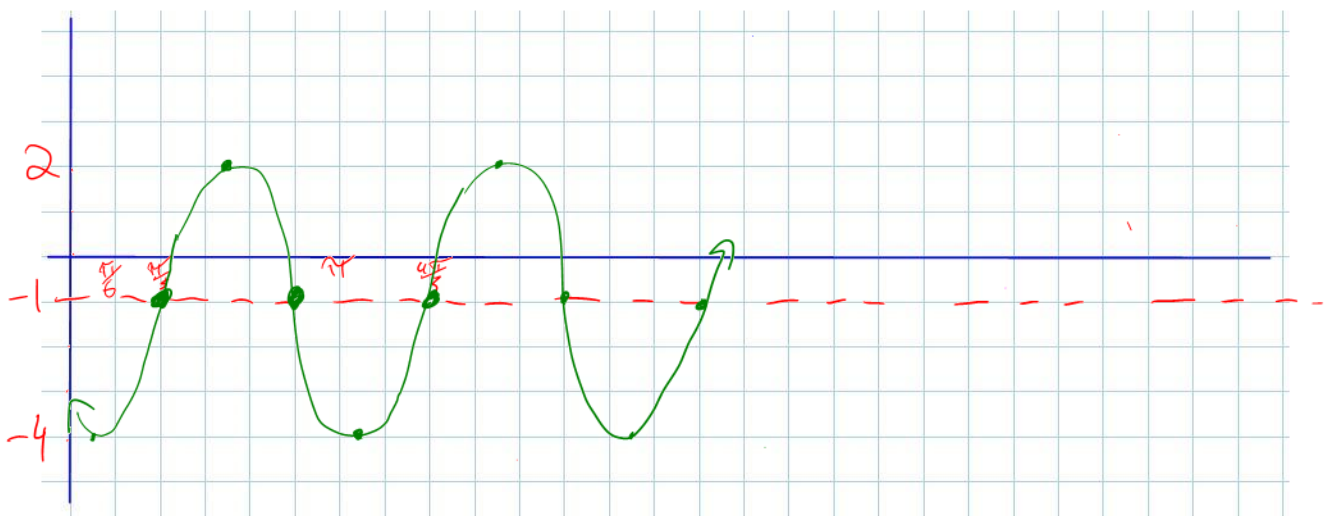
Begin at $\left(\frac{\pi}{3}, -1\right)$

$\downarrow +\pi$

End at $\left(\frac{4\pi}{3}, -1\right)$

Sketch $f(\theta) = 3\sin\left(2\theta - \frac{2\pi}{3}\right) - 1$

$\underset{k}{2}\left(\theta - \frac{\pi}{3}\right)$



Success Criteria:

- I can sketch the graph of a trigonometric function that has undergone transformations
- I can recognize the properties (amplitude, central axis, maximum, minimum, period, and phase shift) of a trigonometric function from its graph or equation

5.7 Applications of Trigonometric Functions

Learning Goal: We are learning to solve real-world problems that can be modeled with a trigonometric function.

For any phenomenon in the real world which has a periodically repeating behaviour, Trigonometric Functions can be used to describe and analyze that behaviour. There are a myriad of such phenomena. From the rise and fall of tides to computer gaming habits, Trigonometric Functions have a say.

Figure 5.7.1 A periodic rise and fall in online gamers

Example 5.7.1

From your text: Pg. 345 #9

9. Each person's blood pressure is different, but there is a range of pressure values that is considered healthy. The function $P(t) = -20\cos\frac{5\pi}{3}t + 100$ models the blood pressure, p, in millimetres of mercury, at time t, in seconds, of a person at rest.

a) What is the period of the function? What does the period represent for an individual?

b) How many times does this person's heart beat each minute?

c) Sketch the graph of $y = P(t)$ for $0 \le t \le 6$.

d) What is the range of the function? Explain the meaning of range in terms of a person's blood pressure.

a) $\text{Period} = \frac{2\pi}{k}$

$= \frac{2\pi}{\frac{5\pi}{3}}$

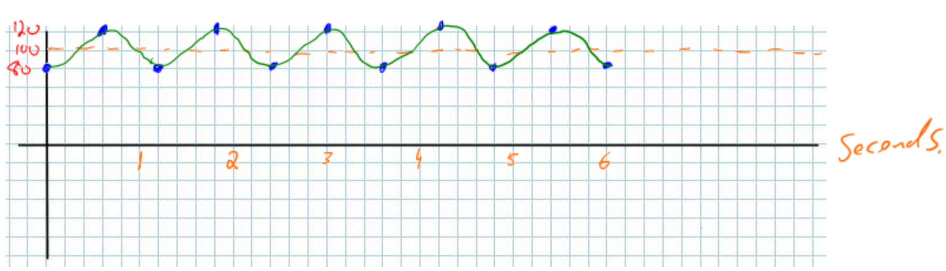
$= 2\cancel{\pi} \times \frac{3}{5\cancel{\pi}}$

$= \frac{6}{5}$

$= 1.2$ seconds/beat

b) $\frac{1 \text{ beat}}{1.2 \text{ seconds}} \times \frac{60 \text{ seconds}}{\text{minutes}} = \frac{50 \text{ beats}}{\text{min}}$

$-\cos$ starts a min.

c) 

d) Range $P(t) \in [80, 120]$

Example 5.7.2

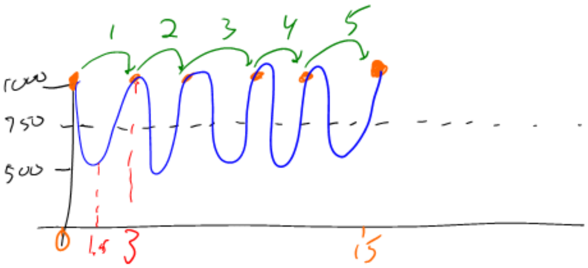
From your text Pg. 361 #7

7. A person who was listening to a siren reported that the frequency of the sound fluctuated with time, measured in seconds. The minimum frequency that the person heard was 500 Hz, and the maximum frequency was 1000 Hz. The maximum frequency occurred at $t = 0$ and $t = 15$. The person also reported that, in 15, she heard the maximum frequency 6 times (including the times at $t = 0$ and $t = 15$). What is the equation of the cosine function that describes the frequency of this siren?

Min = 500 Max = 1000

$\therefore$ central axis $= \dfrac{500 + 1000}{2}$

$= 750$ c



$\therefore$ amplitude $= 1000 - 750 = 250$

or $\dfrac{1000 - 500}{2} = 250$ a

There are 5 periods in 15 seconds

$\therefore$ a period is 3 seconds.

$\therefore k = \dfrac{2\pi}{3}$

You have the point (0, 1000)

d max

$$\therefore f(t) = \overset{a}{250}\cos\left(\overset{k}{\frac{2\pi}{3}}t\right) + \overset{c}{750}$$

d

Success Criteria:

- I can model a real-world situation using a trigonometric function
- I can use the trigonometric model to solve problems