Advanced Functions

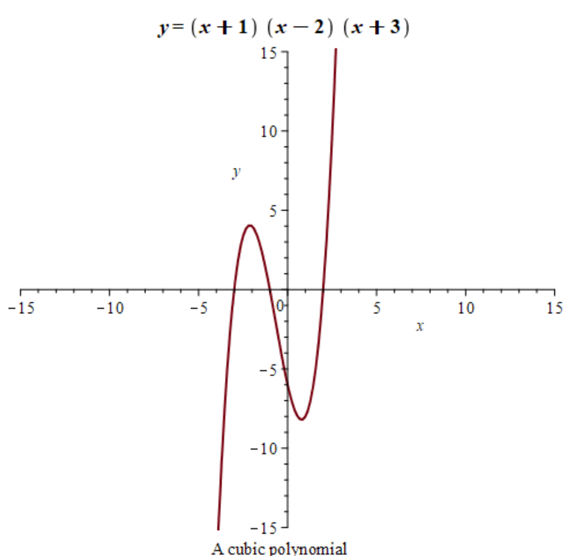
Fall 2017

Course Notes

Chapter 2 – Polynomial Functions

We will learn

- *about the algebraic and geometric structure of polynomial functions of degree three and higher*
- *algebraic techniques for dividing one polynomial by another*
- *use the techniques we learn for division to FACTOR polynomials*
- *solve problems involving Polynomial Equations and Inequalities*



Sketch made in Maple

A∞Ω
MATH@TD

Chapter 2 – Polynomial Functions

Contents with suggested problems from the Nelson Textbook (Chapter 3)

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2.1 Polynomial Functions: An Introduction

ex: $f(x) = 3x^3 - 2x^2 + 8$

Definition 2.1.1

A **Polynomial Function** is of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots a_2 x^2 + a_1 x^1 + a_0 \boxed{x^0} = 1$$

where a_i, $i = 0, 1, 2 \ldots n$, are coefficients.

Examples of Polynomial Functions

a) $f(x) = 8x^4 - 5x^3 + 2x^2 + 3x - 5$

$a_4 = 8$ $a_3 = -5$ $a_0 = -5$

b) $g(x) = 7x^6 - 4x^3 + 3x^2 + 2x$

$a_6 = 7$ $a_5 = 0$, $a_4 = 0$, $a_3 = -4$, $a_2 = 3$, $a_1 = 2$, $a_0 = 0$

Notes: The **TERM** $a_n x^n$ in any polynomial function (where n is the **highest power** we see) is

called the Leading term , and then we write all the following terms

in descending order .

The Leading term has two components:

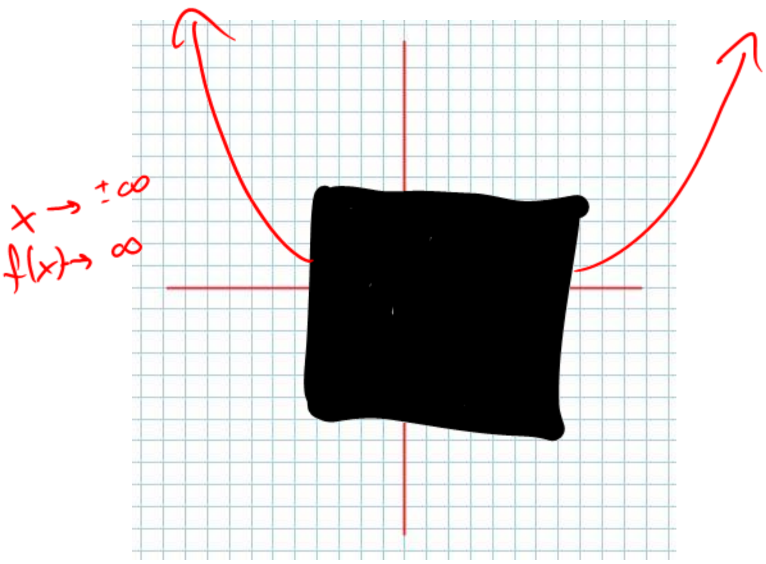
1) Leading coefficient, a_n, is positive or negative

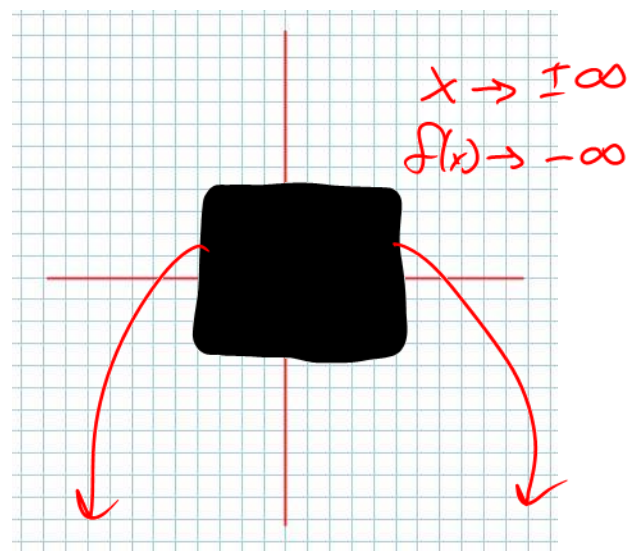
2) $n \rightarrow$ the highest power, it can be odd or even

The *leading term* tells us the **end behaviour** of the polynomial function.

Note: all poly fns have 4 possible end behaviors

Pictures





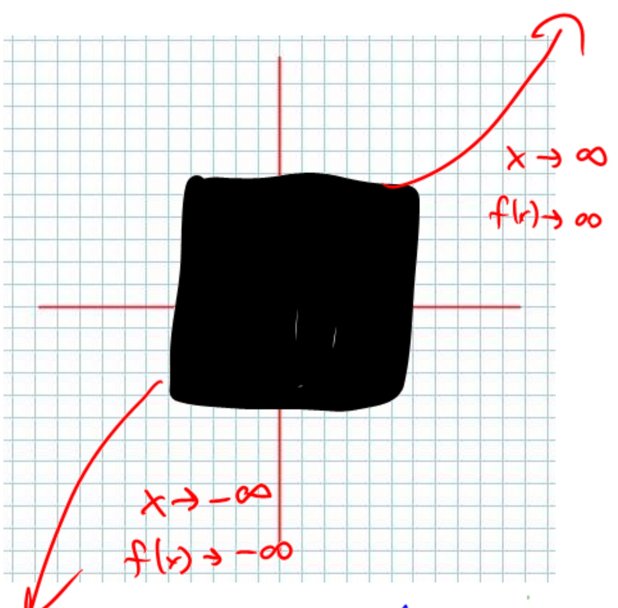
n is even

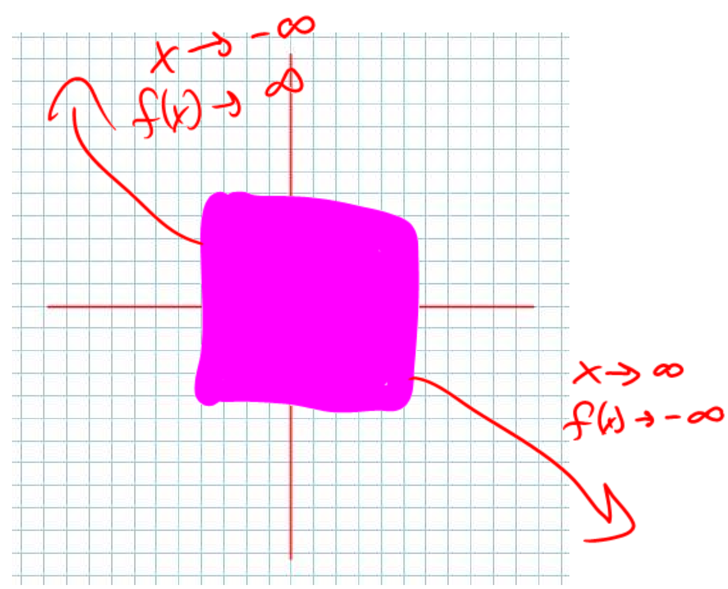
$a_n > 0$

☆ think of a parabola

n is even

$a_n < 0$





n is odd

$a_n > 0$

☆ think of a line

n is odd

$a_n < 0$

Definition 2.1.2

The **order** of a polynomial fn is the value of the highest power, or just the degree of the leading term.

ex: $g(x) = 2x^3 + 3x^2 - 8x^5 - 1$

The order of $g(x)$ is 5

> *Class/Homework for Section 2.1*
>
> *Pg. 122 #1 – 3 (Review on Quadratic Factoring)*
> *Pg. 127 – 128 #1, 2, 3cd, 5, 6*

2.2 Characteristics (Behaviours) of Polynomial Functions

Today we open, and look inside the black box of mystery

Consider the sketch of the graph of some function, $f(x)$:

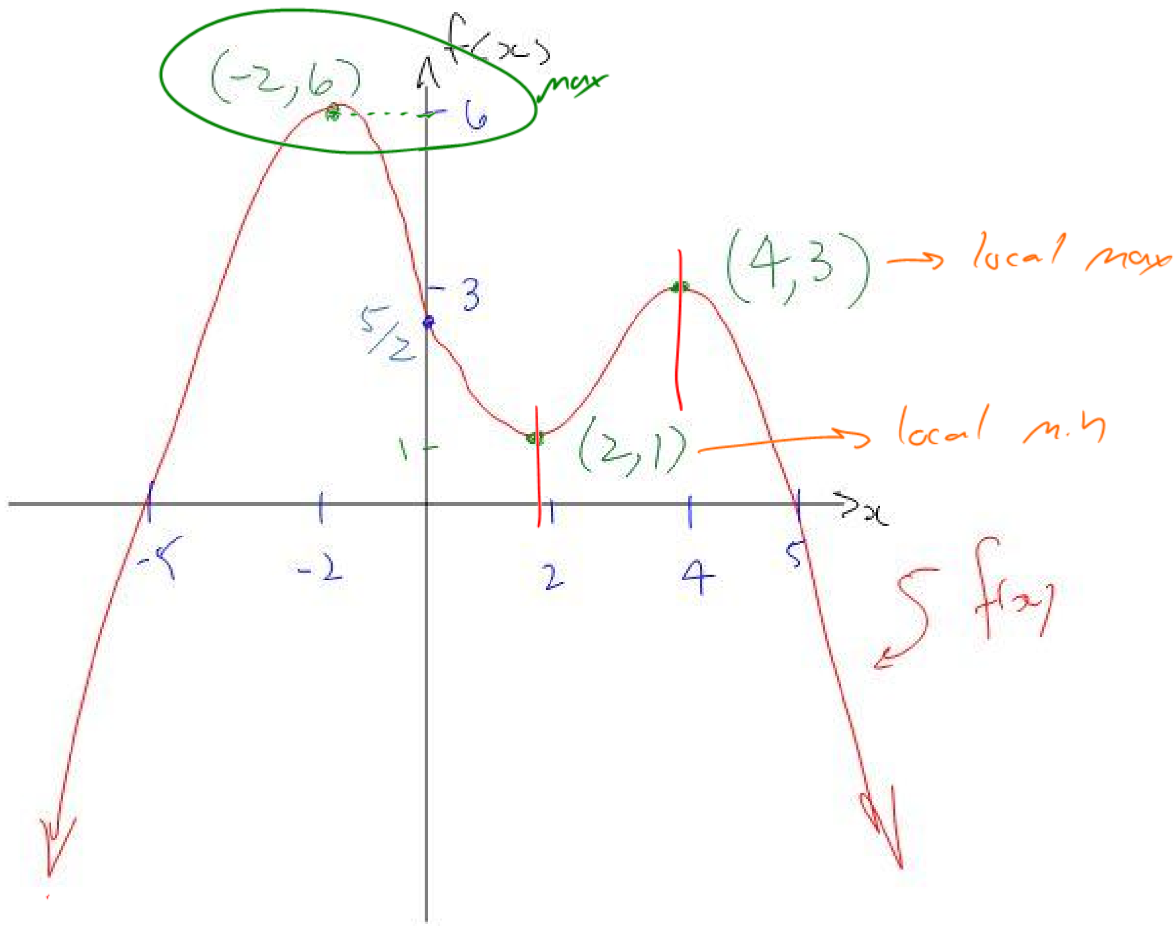


Figure 2.2.1

Observations about $f(x)$:

1) $f(x)$ is a polynomial of even order (degree). The end behaviors are the same.

2) The leading coefficient is negative.

3) $f(x)$ has 3 turning points (where the functional behaviour of INCREASING/DECREASING switches from one to the other.)

4) $f(x)$ has 2 zeros. $f(-5)=0$ and $f(5)=0$

$\rightarrow$ zeros at $x=-5$ and $x=5$

5) $f(x)$ is increasing on $x \in (-\infty,-2) \cup (2,4)$

$f(x)$ is decreasing on $x \in (-2,2) \cup (4,\infty)$

6) $f(x)$ has a maximum functional $\overset{\rightarrow y}{\text{value}}$, of 6.

The max occurs at $x=-2$.

This max is also called the global maximum, meaning it is the absolute highest value.

7) $f(x)$ has a local min at $(2,1)$.

In the neighbourhood of $x=2$, there is a local min of 1.

In the neighbourhood of $x=4$, there is a local max of 3.

Consider the sketch of the graph of some function $g(x)$:

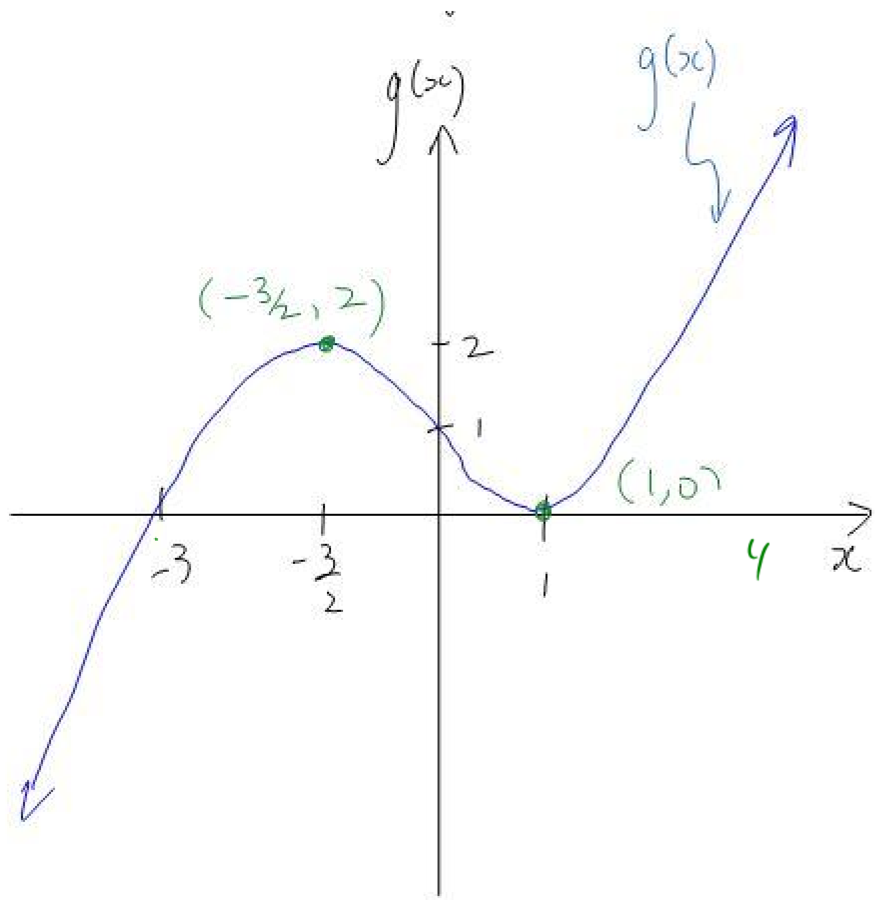


Figure 2.2.2

Observations about $g(x)$:

1. $g(x)$ is odd (degree, not symmetry). End behaviors are opposite.
2. $g(x)$ has two zeros at $x = -3$ and $x = 1$.
3. $g(x)$ has a local max of 2 around $x = \frac{-3}{2}$ $\left(\frac{-3}{2}, 2\right)$

   $g(x)$ has a local min of 0 around $x = 1$ $(1, 0)$
4. $g(x)$ has 2 turning points
5. $g(x)$ has a positive leading coefficient
6. $g(x)$ increasing on $(-\infty, -\frac{3}{2}) \cup (1, \infty)$, decreasing $(-\frac{3}{2}, 1)$

General Observations about the Behaviour of Polynomial Functions

1) The Domain of all Polynomial Functions is

$\{x \in \mathbb{R}\}$ or $(-\infty, \infty)$

2) The Range of ODD ORDERED Polynomial Functions is

$\{f(x) \in \mathbb{R}\}$ or $(-\infty, \infty)$

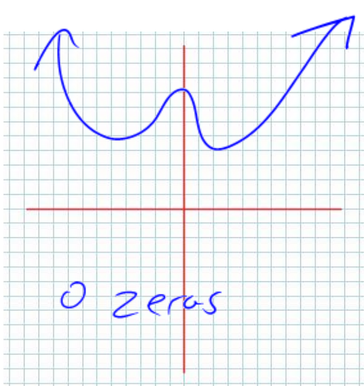
3) The Range of EVEN ORDERED Polynomial Functions depends on

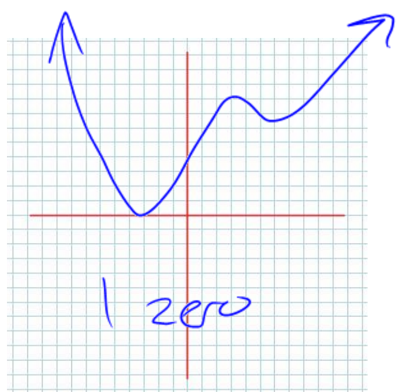
1. the sign of the L.C. → if positive, $\geq$
   → if negative, $\leq$
2. the value of the global max/min.

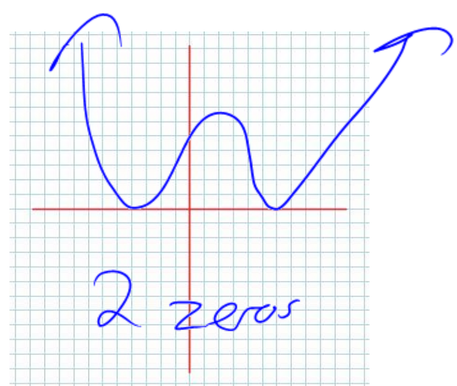
Even Ordered Polynomials

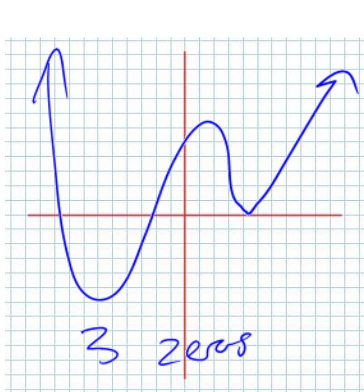
Zeros: A Polynomial Function, $f(x)$, with an even degree of "n" (i.e. $n = 2, 4, 6...$) can have 0 zeros, 1, 2 .... n zeros.

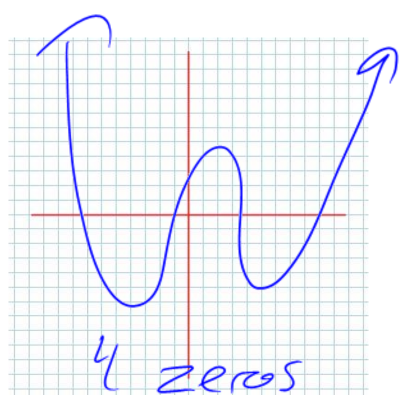
e.g. A degree 4 Polynomial Function (with a positive leading coefficient) can look like:





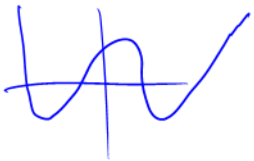






Turning Points:

The minimum number of turning points for an Even Ordered Polynomial Function is one, because you must turn



The maximum number of turning points for a Polynomial Function of (even) order n is $n-1$ (Even poly fn's turn odd # of times.

ex: if order is 6, max T.P. is 5

Odd Ordered Polynomials

Zeros: min # of zeros is one. It must go through at least once

max # of zeros is $\boxed{n}$

Turning Points:

min # of T.P. is zero

max # of T.P. is $n-1$

$f(x) = 2(x+3)(2x-1)(3x+5)$

$\rightarrow 12x^3$

Example 2.2.1 (*#2, for #1b, from Pg. 136*)

Determine the minimum and maximum number of zeros and turning points the given function may have: $g(x) = \boxed{2x^5} - 4x^3 + 10x^2 - 13x + 8$

order is $5^{=n}$ ∴ odd

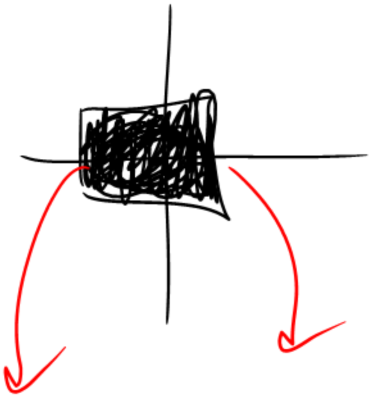
L.C. is 2 ∴ positive

Zeros: min = 1 max = 5^{n}

T.P.: min = 0 max = 4 $n-1$

Example 2.2.2 (*#4d from Pg. 136*)

Describe the end behaviour of the polynomial function using the order and the sign on the leading coefficient for the given function: $f(x) = \boxed{-2x^4} + 5x^3 - 2x^2 + 3x - 1$



even and negative

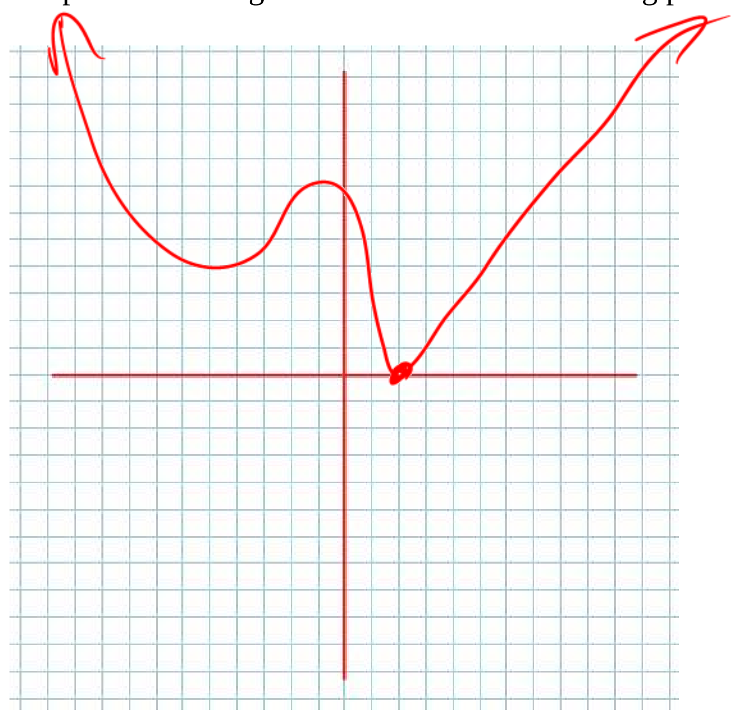
$\therefore\ x \to -\infty,\ f(x) \to -\infty$

$x \to \infty,\ f(x) \to -\infty$

Example 2.2.3 (*#7c from Pg. 137*)

Sketch a graph of a polynomial function that satisfies the given set of conditions:

Degree 4 - positive leading coefficient - 1 zero - 3 turning points.



Class/Homework for Section 2.2

Pg. 136 - 138 #1 – 5, 7, 8, 10, 11

2.3 Zeros of Polynomial Functions

(*Polynomial Functions in Factored Form*)

Today we take a deeper look inside the Box of Mystery, ***carefully examining Zeros of Polynomial Functions***

We'll begin with an **Algebraic Perspective**:

Consider the polynomial function in factored form:

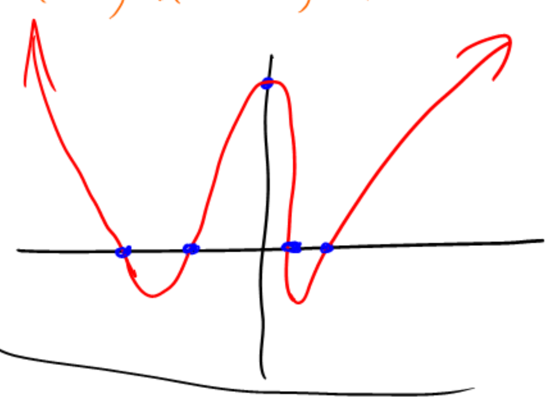
zeros form.

$$f(x) = (2x-3)(x-1)(x+2)(x+3)$$

$(2x)(x)(x)(x) = 2x^4$ = Leading Term

Observations:

1. $f(x)$ has order 4 (even)
2. Leading coefficient is 2 (positive)
3. end behaviors: $x \to -\infty,\ f(x) \to \infty$
   $x \to \infty,\ f(x) \to \infty$
4. $f(x)$ has 4 zeros at $x = \frac{3}{2}$, $x = 1$, $x = -2$, $x = -3$
5. y-int is $f(0) = (-3)(-1)(2)(3) = 18$

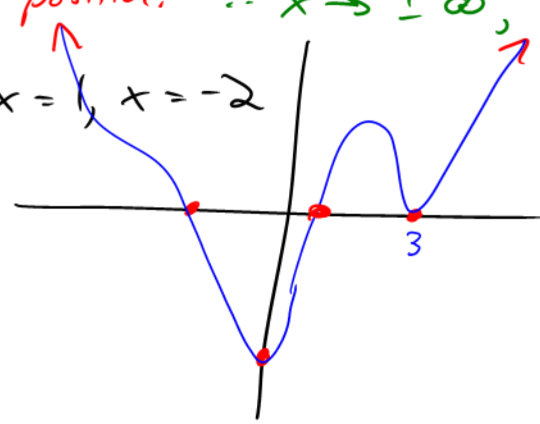


Now, consider the polynomial function $g(x) = (x-3)^2(x-1)(x+2)$

the zero is parabolic

Observations:

1. Leading Term: $(x)^2(x)(x) = x^4$
   ↳ order is 4 (even), L.C. is positive. $\therefore x \to \pm\infty,\ g(x) \to \infty$
2. $g(x)$ has 3 zeros at $x = 3$, $x = 1$, $x = -2$
3. y-int: $g(0) = (-3)^2(-1)(2)$
   $= -18$



Geometric Perspective on Repeated Roots (zeros) of order **2**

Consider the quadratic in factored form: $f(x) = (x-1)^2$

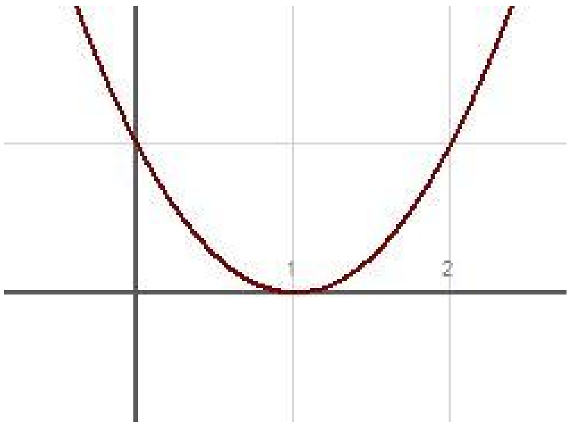


Figure 2.3.1

All order 2 zeros act like parabolas.

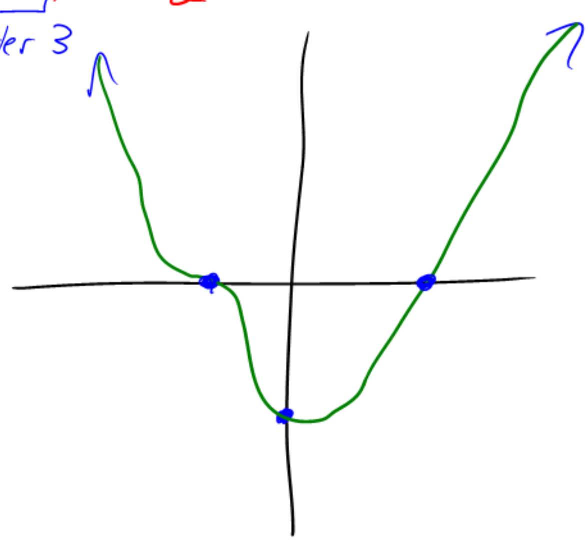
Consider the polynomial function in factored form: $h(t) = (t+1)^3(2t-5)$

Observations:

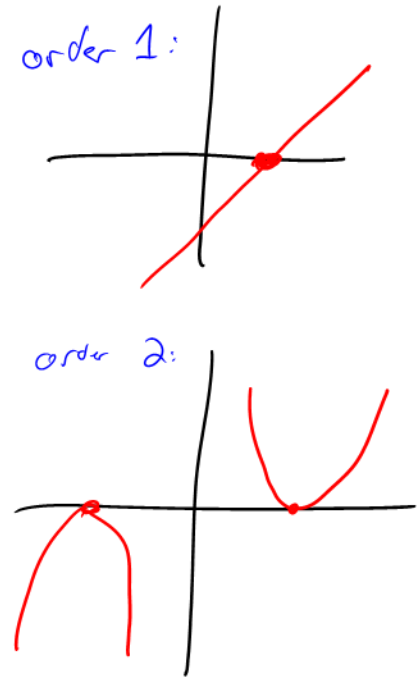
1. Leading term is $(t)^3(2t) = 2t^4$
   ↳ order is 4 (even), positive and $x \to \pm\infty$, $h(t) \to \infty$

2. $h(t)$ has $\underline{2}$ zeros, $\boxed{t=-1}$, $t = \frac{5}{2}$
   order 3

3. y-int: $h(0) = (1)^3(-5) = -5$



Geometric Perspective on Repeated Roots (zeros) of order **3**



Consider the function $f(x) = (x-1)^3$

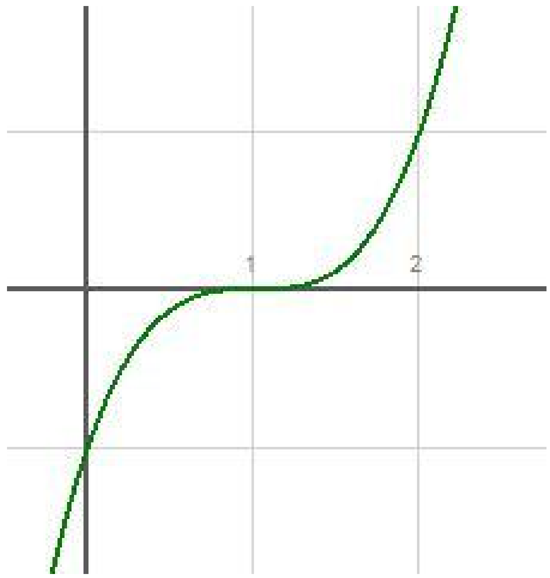
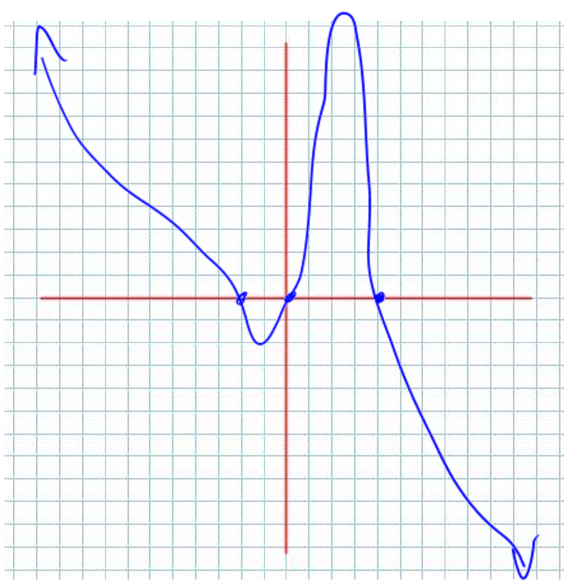


Figure 2.3.2

Example 2.3.1

Sketch a (**possible**) graph of $f(x) = -2x(x+1)(x-2)$



Leading term: $(-2x)(x)(x)$

$= -2x^{3}$ → odd

negative

$x \to -\infty$, $f(x) \to \infty$

$x \to \infty$, $f(x) \to -\infty$

zeros: $x = -1$, $x = 2$, $x = 0$

y-int: $f(0) = 0$

Families of Functions

highest exponent.

Polynomial functions which share the same order are "broadly related" (e.g. **all** quadratics are in the "order 2 family").

Polynomial Functions which share the same order and zeros are more tightly related.

Polynomial Functions which share the same order, zeros, and end behaviors are like siblings.

Example 2.3.2

The family of functions of order 4, with zeros $x = -1, 0, 3, 5$ can be expressed as:

$f(x) = a(x+1)(x+0)(x-3)(x-5)$

↳ the L.C. distinguishes from family members

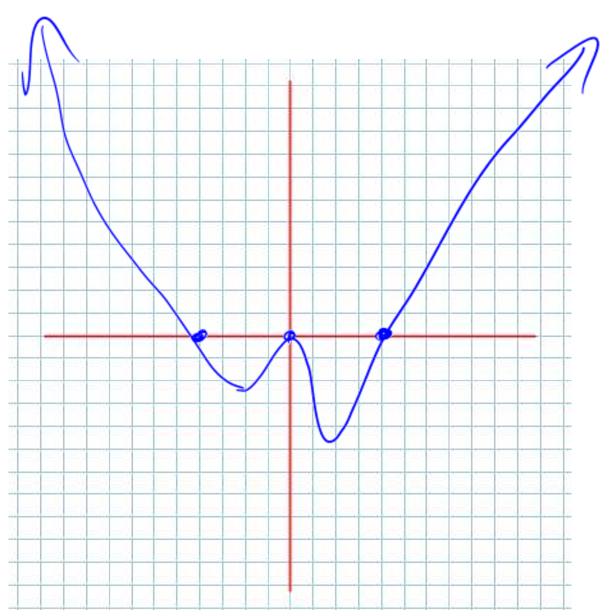
Example 2.3.3

Sketch a graph of $g(x) = 4x^4 - 16x^2$

Factor first

$g(x) = 4x^2(x^2 - 4)$

$g(x) = 4x^2(x+2)(x-2)$



1. L.T. of $4x^4$

↳ positive and even

$\therefore$ $x \to -\infty$, $f(x) \to \infty$

$x \to \infty$, $f(x) \to \infty$

2. Zeros: $x = 0$ (order 2)

$x = 2$

$x = -2$

3. y-int: $g(0) = 0$

Example 2.3.4

Sketch a (possible) graph of $h(t) = (t-1)^3(t+2)^2$

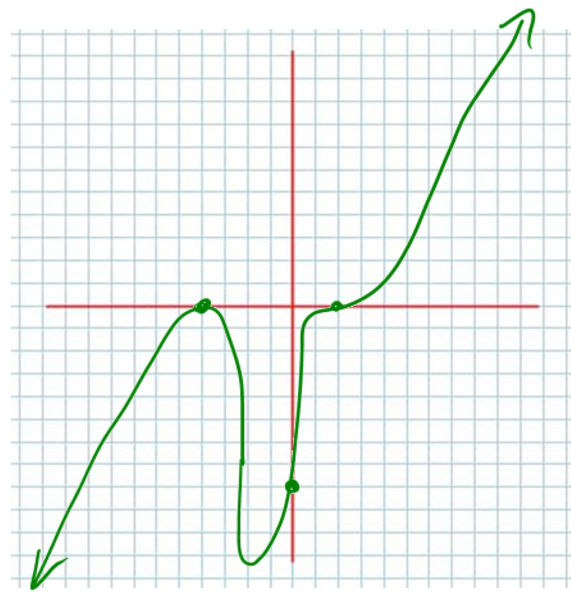
1. L.T. is $(t)^3(t)^2 = t^5$

→ positive and odd

∴ $x \to -\infty$, $f(x) \to -\infty$
$x \to \infty$, $f(x) \to \infty$

2. Zeros at $t = 1$ (order 3)
$t = -2$ (order 2)

3. y-int: $h(0) = (-1)^3(2)^2$
$= -4$



Example 2.3.5

Determine **the** quartic function, $f(x)$, with zeros at $x = -2, 0, 1, 3$, if $f(-1) = -2$.

$f(x) = a(x+2)(x+0)(x-1)(x-3)$

$-2 = a(-1+2)(-1+0)(-1-1)(-1-3)$

$-2 = a(1)(-1)(-2)(-4)$

$\frac{-2}{-8} = \frac{-8a}{-8}$

$\frac{1}{4} = a$

∴ $f(x) = \frac{1}{4}x(x+2)(x-1)(x-3)$

Class/Homework for Section 2.3

READ ex 3, 4, 5 on Pg 141 - 144
Pg. 146 - 148 #1, 2, 4, 6, 8ab, 10, 12, 13b

2.4a Dividing a Polynomial by a Polynomial

(*The Hunt for Factors*)

Note: In this course we will almost always be dividing a polynomial by a ~~monomial~~ linear divisor

$x+1$ or $2x-5$

Before embarking, we should consider some “basic” terms (and notation):

the thing you are dividing

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient} + \frac{\text{remainder}}{\text{divisor}}$$

the stuff left over

the thing you are dividing by

the “answer”

$$\boxed{\text{Dividend} = (\text{quotient})(\text{divisor}) + \text{remainder}}$$

The division statement.

Note: The Divisor and the Quotient will both be

FACTORS

IF

remainder is zero

Example 2.4.1

Use **LONG DIVISION** for the following division problem:

$$\frac{5x^4+3x^3-2x^2+6x-7}{x-2}$$

Please read Example 1 (Part A) on Pgs. 162 – 163 in your textbook.

$$
\begin{array}{r|l}
 & 5x^3+13x^2+24x+54 \\
\hline
x-2 & 5x^4+3x^3-2x^2+6x-7 \\
 & -(5x^4-10x^3) \\
\hline
 & 13x^3-2x^2 \\
 & -(13x^3-26x^2) \\
\hline
 & 24x^2+6x \\
 & -(24x^2-48x) \\
\hline
 & 54x-7 \\
 & -(54x-108) \\
\hline
 & 101
\end{array}
$$

$(x)(5x^3) = 5x^4$

$(x)(13x^2) = 13x^3$

$(x)(24x) = 24x^2$

$(x)(54) = 54x$

$\therefore\ 5x^4+3x^3-2x^2+6x-7 = (5x^3+13x^2+24x+54)(x-2)+101$

KEY OBSERVATION:

$(x-2)$ is not a factor

Example 2.4.2

Using Long Division, divide $\dfrac{2x^5 + 3x^3 - 4x - 1}{x - 1}$.

$0x^4$ $0x^2$

Always include all x-terms even if they are not there

$$
\begin{array}{r}
2x^4 + 2x^3 + 5x^2 + 5x + 1 \\
x-1 \overline{\big) \, 2x^5 + 0x^4 + 3x^3 + 0x^2 - 4x - 1} \\
-(2x^5 - 2x^4) \\
\hline
2x^4 + 3x^3 \\
-(2x^4 - 2x^3) \\
\hline
5x^3 + 0x^2 \\
-(5x^3 - 5x^2) \\
\hline
5x^2 - 4x \\
-(5x^2 - 5x) \\
\hline
x - 1 \\
-(x - 1) \\
\hline
0
\end{array}
$$

$$\therefore 2x^5 + 3x^3 - 4x - 1 = (x-1)(2x^4 + 2x^3 + 5x^2 + 5x + 1)$$

KEY OBSERVATION:

$(x-1)$ is a factor!

Classwork: Pg. 169 #5 **(Yep, that's it for today)**

2.4b Dividing a Polynomial by a Polynomial

(The Hunt for Factors – Part 2)

Here we will examine an alternative form of polynomial division called **Synthetic Division**. Don't be fooled! This is not "fake division". You're thinking with the wrong meaning for "synthetic". (Do a search online and see if you can come up with the meaning I am taking!)

In Synthetic Division we concern ourselves with the coefficients of the dividend and the zero of the divisor.

Synthetic Division uses

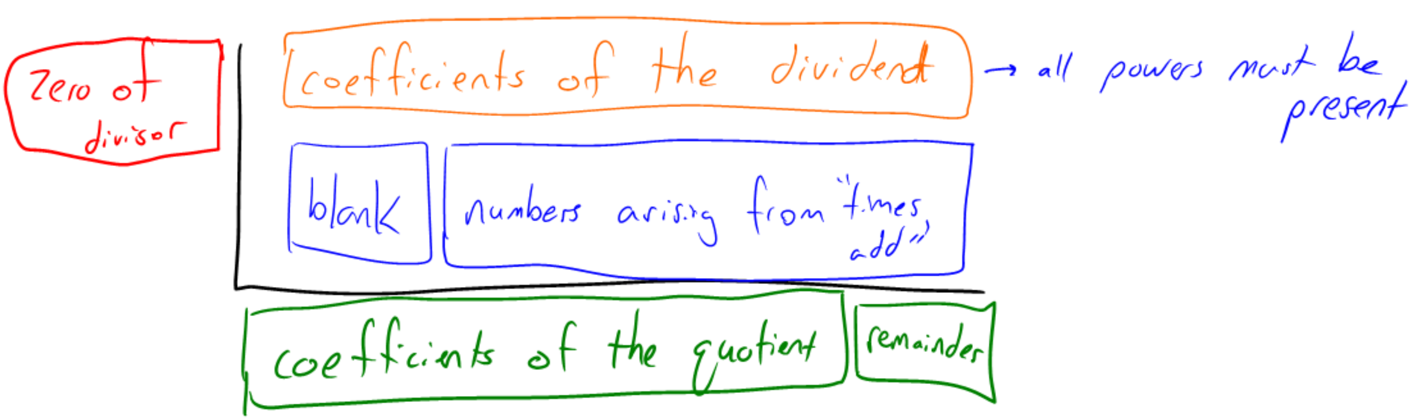
numbers, no variables, and three operations.

→ bring down, times, and add.

Note: Synthetic division only uses linear divisors

ex: $2x-5$, $x+3$, $\cancel{x^2-4}$

The Set-up



Example 2.4.3

Divide using synthetic division:

zero

$$(4x^3 - 5x^2 + 2x - 1) \div (x - \boxed{2})$$

① Bring Down

② Times

③ Add.

2	4	−5	2	−1
+	↓	8	6	16
	4	3	8	$\boxed{15}$ remainder
	x^2	x^1	x^0	

$$\therefore\ 4x^3 - 5x^2 + 2x - 1 = (x-2)(4x^2 + 3x + 8) + 15$$

Example 2.4.4

Divide using synthetic division:

$0x^3$

$$\frac{4x^4 + 3x^2 - 2x + 1}{x+1}$$

⓪ Bring Down.

① Times

② Add

−1	4	0	3	−2	1
+	↓	−4	4	−7	9
	4	−4	7	−9	10
					remainder

$$\frac{x^4}{x^1} = x^3$$

$$\therefore\ 4x^4 + 3x^2 - 2x + 1 = (x+1)(4x^3 - 4x^2 + 7x - 9) + 10$$

Example 2.4.5

Divide using your choice of method *(and you choose synthetic division…amen)*

$$(2x^3 - 9x^2 + x + 12) \div (2x - 3)$$

$$\begin{array}{r|rrrr} 3/2 & 2 & -9 & 1 & 12 \\ & \downarrow & 3 & -9 & -12 \\ \hline & 2 & -6 & -8 & 0 \\ & 1 & -3 & -4 & \end{array}$$

when you have a fraction, divide the quotient by the denominator

$$\therefore 2x^3 - 9x^2 + x + 12 = (2x-3)(x^2 - 3x - 4)$$
$$= (2x-3)(x-4)(x+1)$$

Example 2.4.6

Is $3x - 1$ a factor of the function $f(x) = 6x - x^3 + 2 + 3x^4$?

$x = 1/3$

$$= 3x^4 - x^3 + 0x^2 + 6x + 2$$

$$\begin{array}{r|rrrrr} 1/3 & 3 & -1 & 0 & 6 & 2 \\ & \downarrow & 1 & 0 & 0 & 2 \\ \hline & 3 & 0 & 0 & 6 & \boxed{4} \end{array}$$

$\therefore$ $3x-1$ is not a factor because there is a remainder

Example 2.4.7 (OK…this is a lot of examples!)

Consider again (from **Example 2.4.6**) $f(x)=3x^4-x^3+6x+2$, and calculate $f\left(\frac{1}{3}\right)$.

$$f\left(\tfrac{1}{3}\right)=3\left(\tfrac{1}{3}\right)^4-\left(\tfrac{1}{3}\right)^3+6\left(\tfrac{1}{3}\right)+2$$

$$=\cancel{3}\left(\frac{1}{\cancel{81}\,\cancel{27}}\right)-\frac{1}{27}+2+2$$

$$=4.$$

Wait!!! This is the SAME remainder when dividing by $3x-1$.!!

Example 2.4.8

Consider **Example 2.4.5**. Let $g(x)=2x^3-9x^2+x+12$, and calculate $g\left(\frac{3}{2}\right)$.

$$g\left(\tfrac{3}{2}\right)=2\left(\tfrac{3}{2}\right)^3-9\left(\tfrac{3}{2}\right)^2+\frac{3}{2}+12$$

$$=\cancel{2}^{1}\left(\frac{27}{\cancel{8}\,4}\right)-9\left(\frac{9}{4}\right)+\frac{3}{2}+\frac{12}{1}$$

$$=\frac{27}{4}-\frac{81}{4}+\frac{6}{4}+\frac{48}{4}=\frac{0}{4}=0$$

The Remainder Theorem

Given a polynomial function, $f(x)$, divided by a linear binomial, $x-k$, then the remainder of the division is the value $f(k)$.

Proof of the Remainder Theorem

Consider $f(x) \div (x-k)$

then the division statement is:

quotient

$$f(x) = (x-k) \cdot q(x) + r$$

now,

$$f(k) = (k-k) \cdot q(x) + r$$

$$f(k) = r \; \square$$

Example 2.4.9

Determine the remainder of $\dfrac{5x^4 - 3x^3 - 50}{x - \boxed{2}}$. $= f(x)$

WAIT!!!! We MUST have a FUNCTION

$$f(2) = 5(2)^4 - 3(2)^3 - 50$$

$$= 80 - 24 - 50$$

$$= 6$$

$\therefore$ the remainder is 6.

Class/Homework for Section 2.4

Pg. 168 - 170 #2, 6acdef, 10acef, 12, 13

2.5 The Factor Theorem

(Factors have been FOUND)

> **The Factor Theorem**
>
> **Given a polynomial function,** $f(x)$, then $x-a$ is a factor of $f(x)$ if and only if
>
> $f(a)=0$

Example 2.5.1

Use the Factor Theorem to factor x^3+2x^2-5x-6.

WAIT!!!! We need a FUNCTION

$f(x)=x^3+2x^2-5x-6$

Test the possible factors of -6

$\pm 1, \ \pm 2, \ \pm 3, \ \pm 6$

$f(x)=(x-a)(x-b)(x-c)$

$(a)(b)(c)=-6$

$\therefore$ the factors must divide -6.

Try $x-1$, $\therefore x=1$

$f(1)=1^3+2(1)^2-5(1)-6=-8$

Try $x+1$, $\therefore x=-1$

$f(-1)=(-1)^3+2(-1)^2-5(-1)-6$

$=-1+2+5-6=0$

$$\begin{array}{c|cccc} -1 & 1 & 2 & -5 & -6 \\ \oplus & \downarrow & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & 0 \\ & x^2 & x & x^0 & \end{array}$$

$\therefore x^3+2x^2-5x-6=(x+1)(x^2+x-6)$

$=(x+1)(x+3)(x-2)$

Example 2.5.2

Factor **fully** $x^4 - x^3 - 16x^2 + 4x + 48$ → $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16, \pm 24, \pm 48$

Try $x+2$, $x=-2$

$$f(-2) = (-2)^4 - (-2)^3 - 16(-2)^2 + 4(-2) + 48$$
$$= 16 + 8 - 64 - 8 + 48$$
$$= 0$$

$$\begin{array}{r|rrrrr} -2 & 1 & -1 & -16 & 4 & 48 \\ & & -2 & 6 & 20 & -48 \\ \hline & 1 & -3 & -10 & 24 & 0 \end{array}$$

$\therefore x^3 - 3x^2 - 10x + 24 = g(x)$

Try $x-2$, $x=2$

$$g(2) = 2^3 - 3(2)^2 - 10(2) + 24$$
$$= 8 - 12 - 20 + 24$$
$$= 0$$

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -10 & 24 \\ & & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & 0 \\ & x^2 & x & x^0 & \end{array}$$

$$\therefore x^4 - x^3 - 16x^2 + 4x + 48 = (x-2)(x+2)(x^2 - x - 12)$$
$$= (x-2)(x+2)(x-4)(x+3)$$

Example 2.5.3 (*Pg 177 #6c in your text***)**

Factor fully $x^4+8x^3+4x^2-48x$

$$f(x)=x(\underbrace{x^3+8x^2+4x-48}_{g(x)})$$

Try $x-2$, $f(2)=2^3+8(2)^2+4(2)-48$

$$=8+32+8-48=0!$$

$$\begin{array}{r|rrrr} 2 & 1 & 8 & 4 & -48 \\ & & 2 & 20 & 48 \\ \hline & 1 & 10 & 24 & 0 \end{array}$$

$\therefore x^4+8x^3+4x^2-48x$

$$=x(x-2)(x^2+10x+24)$$

$$=x(x-2)(x+4)(x+6)$$

Example 2.5.4 (*Pg 177 #10*)

$f(1)=10$

When ax^3-x^2+2x+b is divided by $x-1$ the remainder is 10. When it is divided by $x-2$ the remainder is 51. Find a and b.

This problem is very instructive.

$f(x)=ax^3-x^2+2x+b$

$f(1)=a(1)^3-(1)^2+2(1)+b=10$

$a+1+b=10$

$a+b=9$

$-(8a+b=51)$

$-7a=-42$

$\boxed{a=6 \therefore b=3}$

$f(2)=a(2)^3-(2)^2+2(2)+b=51$

$8a-4+4+b=51$

$8a+b=51$

Use the Remainder Theorem!!

Class/Homework for Section 2.5

Pg. 176 - 177 #1, 2, 5 – 7 abcd, 8ac, 9, 12 (angels sing over 9 & 12)

2.6 Factoring Sums and Differences of Cubes

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Knowing how to factor a sum or difference of cubes is a simple matter of remembering patterns.

Example 2.6.1 (*Recalling the pattern for factoring a Difference of Squares*)

Factor $4x^2 - 25$

$= (2x - 5)(2x + 5)$

Note: Sums of Squares DO NOT factor!!

e.g. Simplify $x^2 + 4$

$8x^3 \pm 27$

Differences of Cubes

Pattern

Same Opposite Always Positive

$(cube_1 - cube_2) = (cuberoot_1 - cuberoot_2)(cuberoot_1^2 + cuberoot_1 \times cuberoot_2 + cuberoot_2^2)$

$8x^3 - 27 = (2x - 3)(4x^2 + 6x + 9)$

TWO POSITIVES and ONE NEGATIVE

Sums of Cubes (These DO factor!!)

Pattern

$(cube_1 + cube_2) = (cuberoot_1 + cuberoot_2)(cuberoot_1^2 - cuberoot_1 \times cuberoot_2 + cuberoot_2^2)$

$8x^3 + 27 = (2x + 3)(4x^2 - 6x + 9)$

$$\begin{array}{c|cccc} 2 & 1 & 0 & 0 & -8 \\ & & 2 & 4 & +8 \\ \hline & 1 & 2 & 4 & 0 \end{array}$$

Example 2.6.2

Factor $x^3 - 8$ $= (x-2)(x^2+2x+4)$

Example 2.6.3

Factor $27x^3 + 125y^3$ $= (\underline{3x} + \underline{5y})(9x^2 - 15xy + 25y^2)$

$(3x)(5y)$

Example 2.6.4

Factor $1 - 64z^3$ $= (1-4z)(1+4z+16z^2)$

Example 2.6.5

Factor $1000x^3 + 27$

$27^{\wedge(1/3)}$

$= (10x+3)(100x^2 - 30x + 9)$

$(10x)^2 \quad (10x)(3) \quad (3)^2$

Example 2.6.6

Factor $x^6 - 729$

$\sqrt[3]{x^6}$

$= (x^6)^{1/3}$

$= x^2$

$= (x^2-9)(x^4+9x^2+81)$

$(x^2)^2 \quad (x^2)(9) \quad (9)^2$

$= (x-3)(x+3)(x^4+9x^2+81)$

Class/Homework for Section 2.6

Pg 182 #2aei, 3, 4
If you finish early, begin the review
Pgs. 184 – 185 (skip #8, 9)