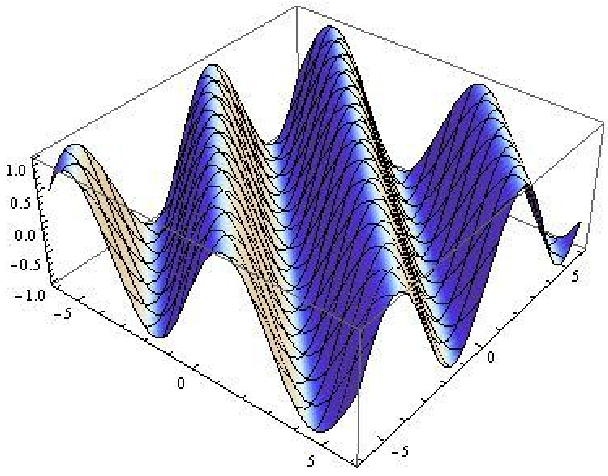
Advanced Functions

Fall 2017
Course Notes

Unit 5 – Trigonometric Functions

We will learn

- *about Radian Measure and its relationship to Degree Measure*
- *how to use Radian Measure with Trigonometric Functions*
- *about the connection between trigonometric ratios and the graphs of trigonometric functions*
- *how to apply our understanding of trigonometric functions to model and solve real world problems*



A∞Ω
MATH@TD

Chapter 5 – Trigonometric Functions

Contents with suggested problems from the Nelson Textbook (Chapter 5)

5.1 Radian Measure and Arc Length

Pg. 321 #2edfh, 3 – 9

5.2 Trigonometric Ratios and Special Triangles (Part 1)

Pg. 330 #1b – f, 2bcd, 3

5.3 Trigonometric Ratios and Special Triangles (Part 2 – Exact Values)

Pg. 330 – 331 #5, 7, 9

5.4 Trigonometric Ratios and Special Triangles (Pt 3 – Getting the Angles)

Pg. 331 #6, 11, 16

5.5 Sketching the Trigonometric Functions

~~Worksheet~~

5.6 Transformations of Trigonometric Functions

Pg. 343 - 345 #1, 4, 6 – 8, 13, 14ab

5.7 Applications of Trigonometric Functions

Pg. 360 – 362 #4, 6, 9, 10

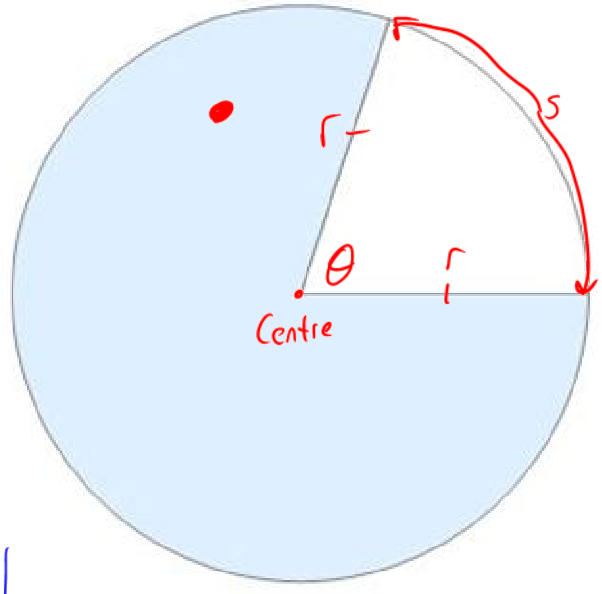
5.1 Radian Measure and Arc Length

Radian Measure

We are familiar with measuring angles using “degrees”, and now we will turn to another measure for angles: **Radians**.

Before getting to the notion of radians, we need to learn some notation.

Picture



θ = central angle

r = radius

s = arc length subtended by the central angle

→ part of the circumference.

There is a relationship between s, θ, and r.

$s = r\theta$ Arc Length Formula

↳ θ must be in radians.

Definition 5.1.1

In a circle of radius r, a central angle θ subtending an arc of length $\underline{\underline{s = r}}$ measures 1 radian.

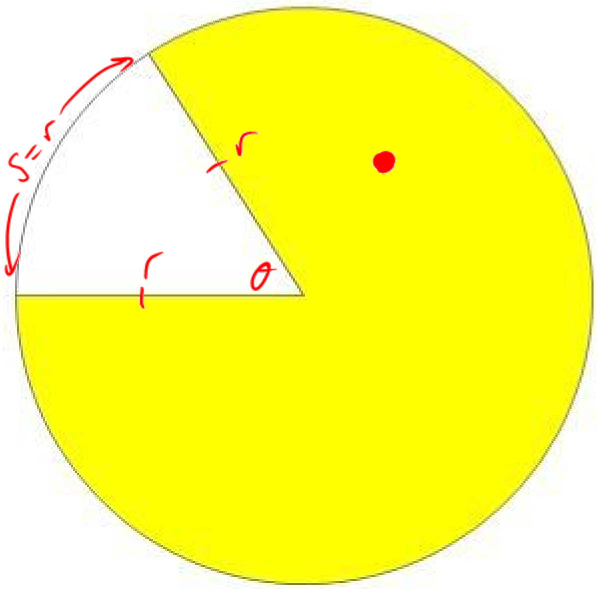
Picture

$$s = r\theta$$

↓

$$\frac{r}{r} = \frac{r\theta}{r}$$

$$1 \text{ radian} = \theta$$



Note: The circumference of a circle is given by $C = 2\pi r$

So, for a central angle of 360°, in a circle of radius $r = 1$, then

$$s = 2\pi r$$

$$\cancel{r}\theta = 2\pi \cancel{r}$$

$$\theta = 2\pi \text{ radians}$$

$$\therefore \quad 360^{\circ} = 2\pi \text{ radians}$$

$$\boxed{180^{\circ} = \pi \text{ radians}}$$ conversion factor.

$1^\circ = \frac{\pi}{180}$ radians

$\frac{12\cancel{0}}{18\cancel{0}} = \frac{12}{18} = \frac{2}{3}$

Example 5.1.1

Convert the following to radians:

a) $30^\circ \left(\frac{\pi}{180}\right)$

$= \frac{\pi}{6}$ rad

b) $45^\circ \left(\frac{\pi}{180}\right)$

$= \frac{\pi}{4}$ rad

c) $120^\circ \left(\frac{\pi}{180}\right)$

$= \frac{2\pi}{3}$ rad.

d) $\overset{7}{3\cancel{15}}^\circ \left(\frac{\pi}{\underset{4}{\cancel{180}}}\right)$

$= \frac{7\pi}{4}$

e) $161.3^\circ \left(\frac{\overset{3.14}{\cancel{\pi}}}{180}\right)$

just do it.

$= 2.81$ rad

Example 5.1.2

Convert the following to degrees (round to two decimal places where necessary)

$\boxed{\frac{180^\circ}{\pi} = 1 \text{ rad}}$

a) $\frac{7\cancel{\pi}}{12}$ rad $\left(\frac{180}{\cancel{\pi}}\right)$

$= 105^\circ$

b) $\frac{10\cancel{\pi}}{9}$ rad $\left(\frac{180}{\cancel{\pi}}\right)$

$= 200^\circ$

c) 2.5 ~~rad~~ $\left(\frac{180}{\pi}\right)$

$= 143.3^\circ$

d) $\frac{\pi}{2}$ ~~rad~~ $\left(\frac{180}{\pi}\right)$

$= 90^\circ$

e) $-\frac{\cancel{\pi}}{3}$ ~~rad~~ $\left(\frac{180}{\cancel{\pi}}\right)$

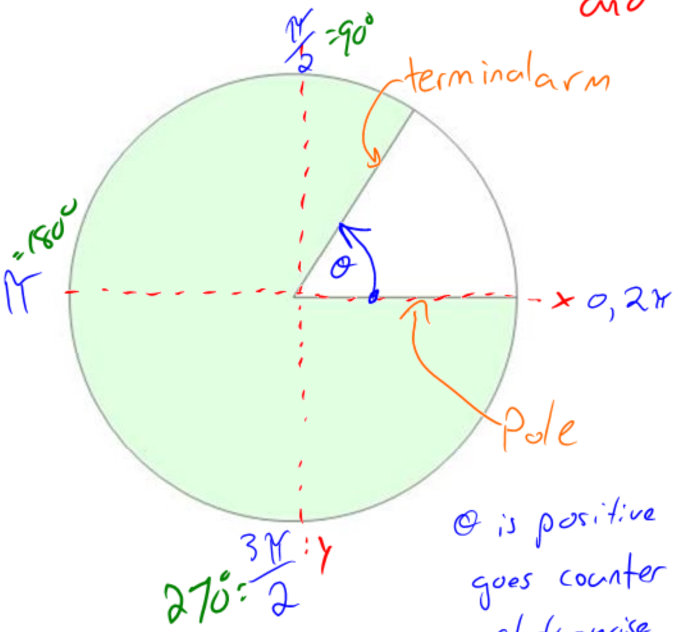
$= -60^\circ$

Q. What the rip is a negative degree?

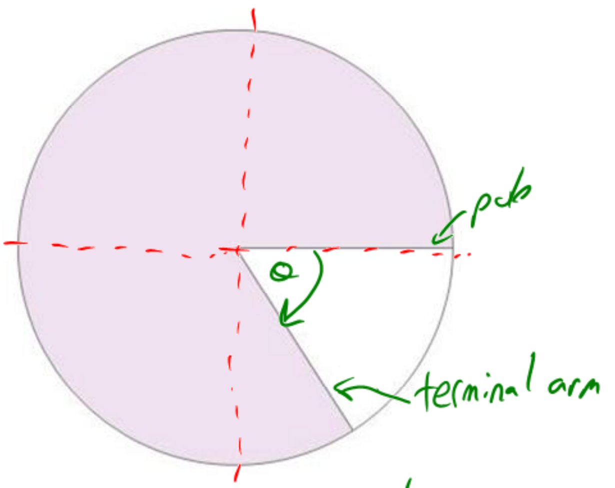
Angles of Rotation

The sign on an angle can be thought of as the direction of rotation (around a circle).

Pictures Angles of rotation always begin at the pole, and ends at the terminal arm.



θ is positive and goes counter clock-wise



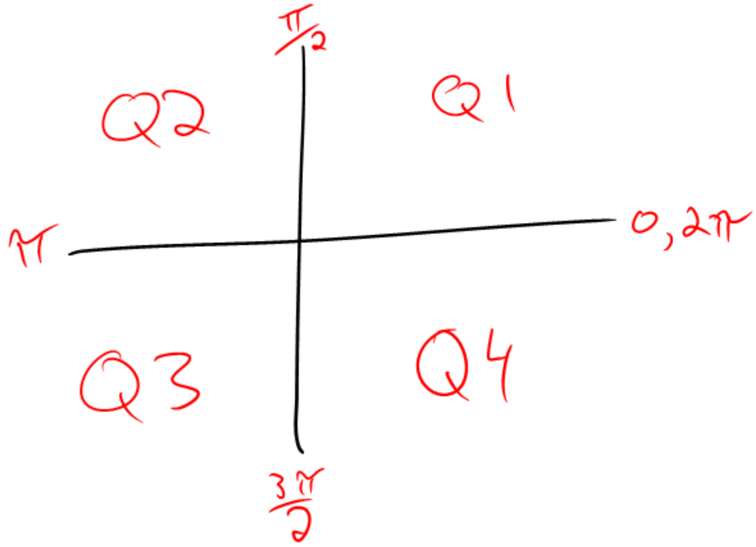
θ is negative in clock-wise rotation

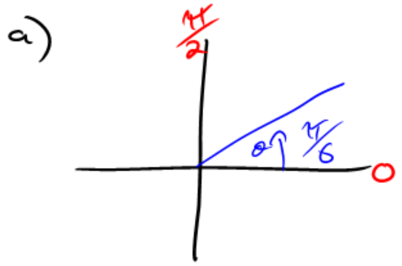
Example 5.1.3

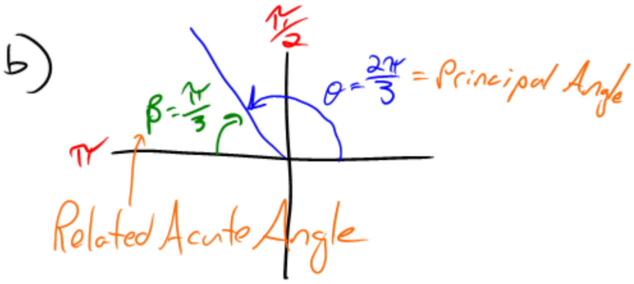
Sketch the following angles of rotation:

a) $\frac{\pi}{6}$ rad b) $\frac{2\pi}{3}$ rad c) $-\frac{3\pi}{4}$ rad d) $\frac{7\pi}{6}$

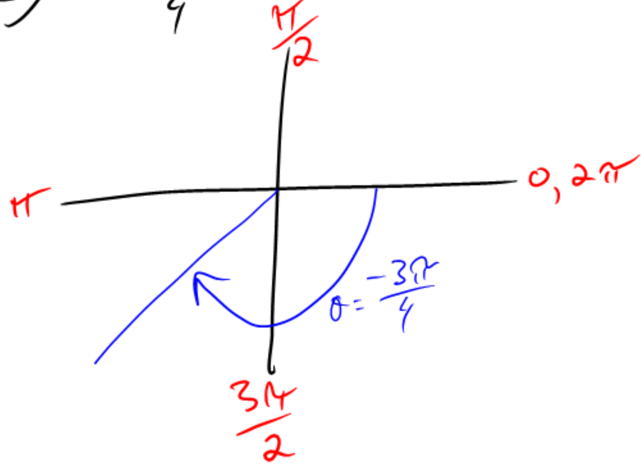
BUT FIRST: Consider the following picture:



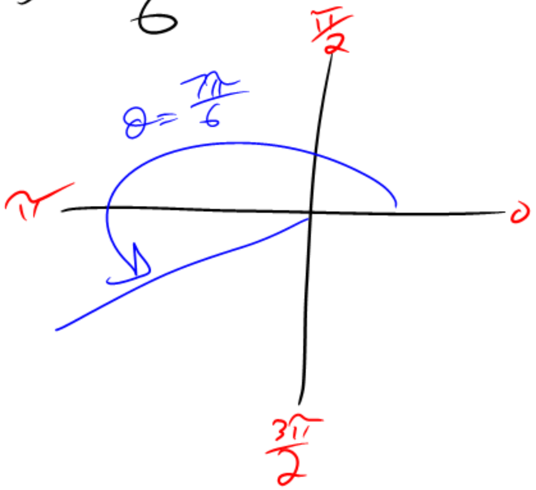




c) $\frac{-3\pi}{4}$



d) $\frac{7\pi}{6}$



Example 5.1.4

Determine the length of an arc, on a circle of radius $5cm$, subtended by an angle:

a) $\theta = 2.4$ rad

$S = r\theta$

$S = (5)(2.4)$

$S = 12$ cm

b) $\theta = 120^\circ\left(\frac{\pi}{180}\right)$

$\theta = \frac{2\pi}{3}$

$S = r\theta$

$S = 5\left(\frac{2\pi}{3}\right)$

$S = \frac{10\pi}{3}$ cm

$S \approx 10.5$ cm.

Class/Homework for Section 5.1

Pg. 321 #2edfh, 3 – 9

5.2 Trigonometric Ratios and Special Triangles

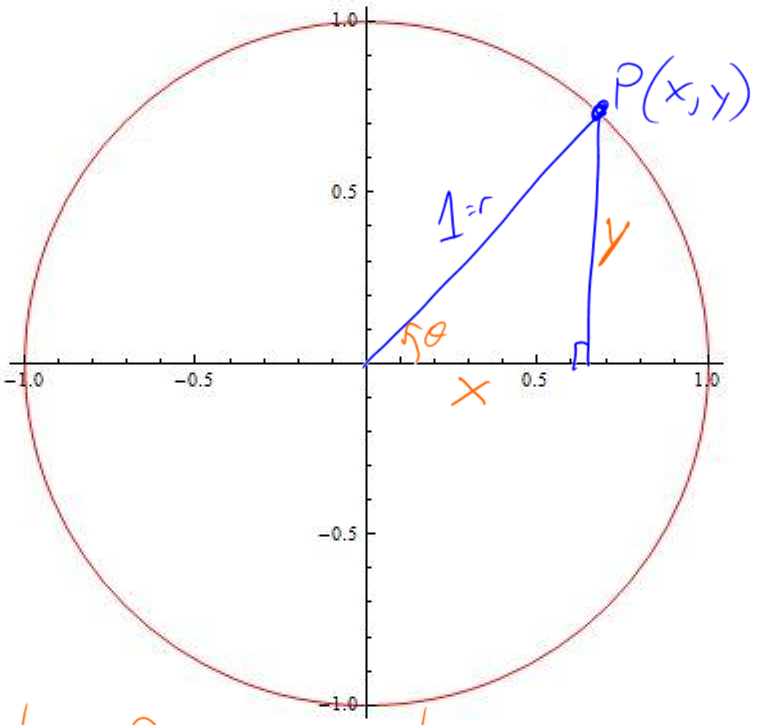
(*Part 1*)

- means NO calculators.

- exact answers

↳ fractions and √

Consider the circle of radius 1:

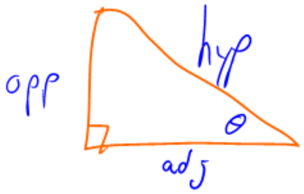


☆ Always go to the x-axis.

By the Pythagorean theorem

$x^2+y^2=1$

Recall the six main Trigonometric Ratios:

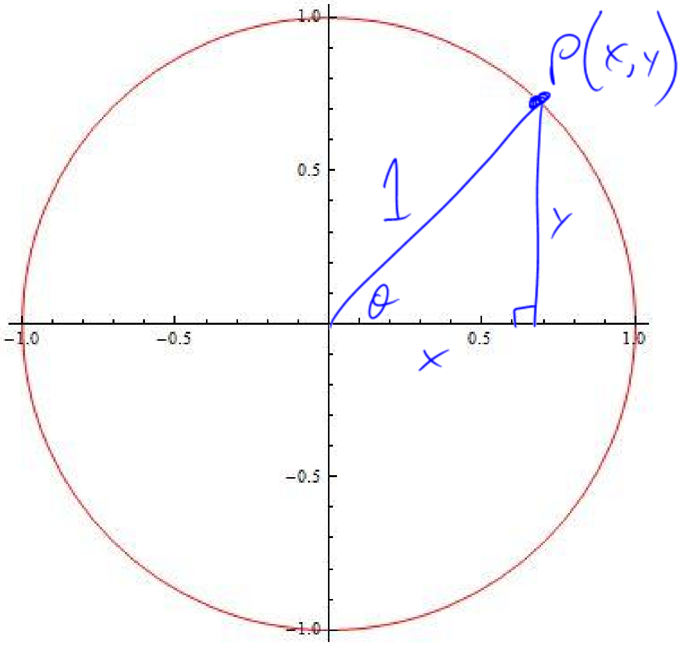


Primary Trig Ratios	Reciprocal Trig Ratios
$\sin\theta = \frac{opp}{hyp}$	$\frac{1}{\sin\theta} = \csc\theta = \frac{hyp}{opp}$
$\cos\theta = \frac{adj}{hyp}$	$\frac{1}{\cos\theta} = \sec\theta = \frac{hyp}{adj}$
$\tan\theta = \frac{opp}{adj}$	$\frac{1}{\tan\theta} = \cot\theta = \frac{adj}{opp}$

always less than one.

always greater than one.

Consider again the circle of radius 1 (but now keeping in mind SOH CAH TOA)



$$\sin\theta = \frac{y}{1} = y$$

$$\cos\theta = \frac{x}{1} = x$$

$$\tan\theta = \frac{y}{x}$$

Note: $P(x, y)$ can be represented by $P(\cos\theta, \sin\theta)$

The Pythagorean Identity

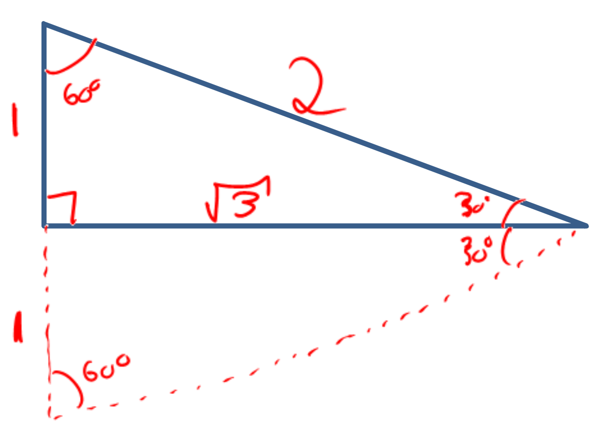
$x^2 + y^2 = 1$

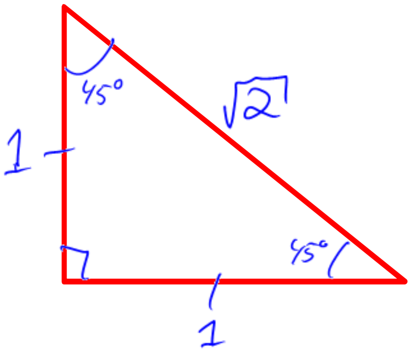
$\cos^2\theta + \sin^2\theta = 1$

$\boxed{\sin^2\theta + \cos^2\theta = 1}$ memorize!

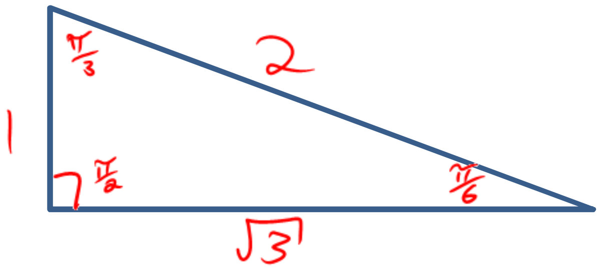
Special Triangles in Radians

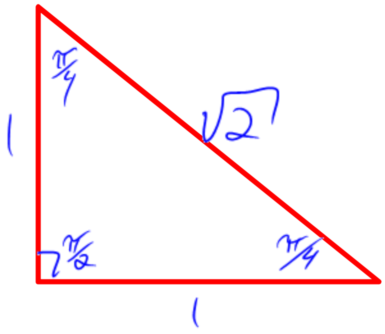
Recall: We have two “Special Triangles”. In **degrees** they are:





In radians we have $60°\left(\frac{\pi}{180}\right) = \frac{\pi}{3}$ $30°\left(\frac{\pi}{180}\right) = \frac{\pi}{6}$ $45°\left(\frac{\pi}{180}\right) = \frac{\pi}{4}$

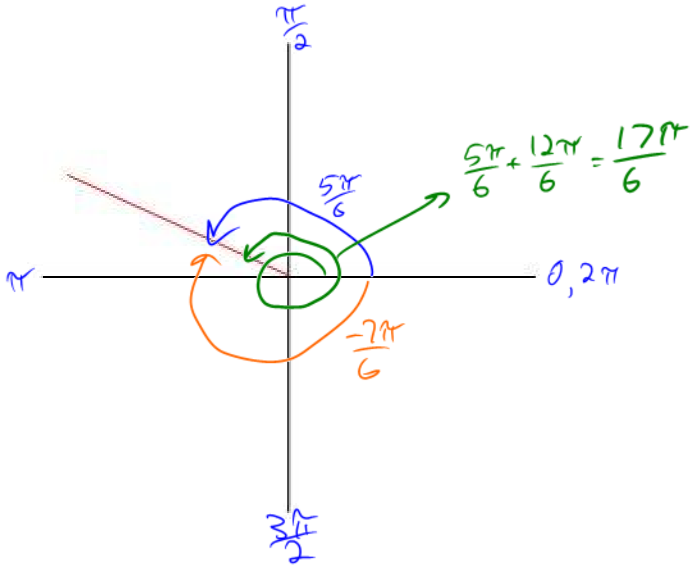




MEMORIZE THESE!

Angles of Rotations and Trig Ratios

Consider the following sketch of the angle of rotation $\theta = \frac{5\pi}{6}$:



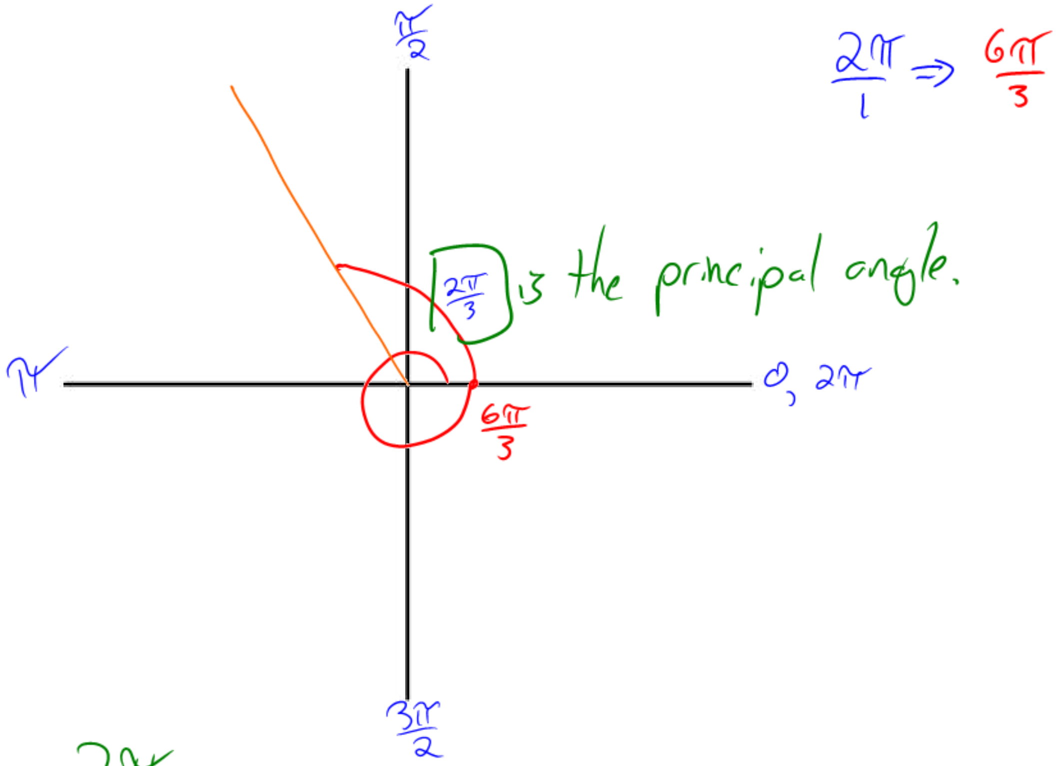
There are infinitely many angles of rotation for <u>each</u> terminal arm.

In this Example, we call $\theta = \frac{5\pi}{6}$ the **Principal Angle**, or the angle in standard position. We take this to mean the smallest <u>positive</u> angle of rotation

Note: All principal angles $\theta \in [0, 2\pi]$

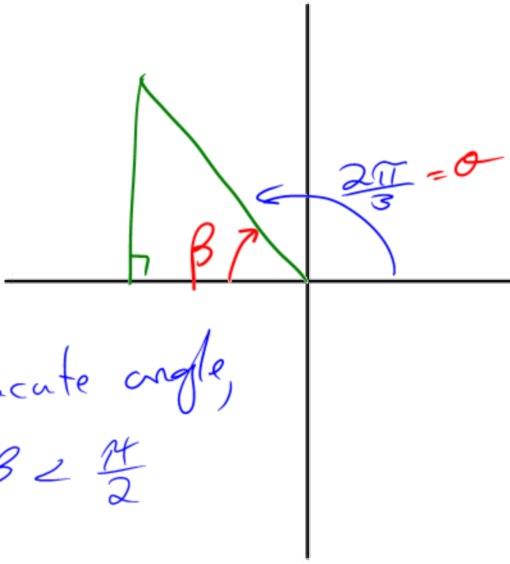
Example 5.2.1

Sketch the angle of rotation $\theta = \frac{8\pi}{3}$ and determine the principal angle.



Principal Angle: $\frac{2\pi}{3}$

Principal angles are outside the triangle



Note: In Q1 $\theta = \beta$

β is the related acute angle,

$0 < \beta < \frac{\pi}{2}$

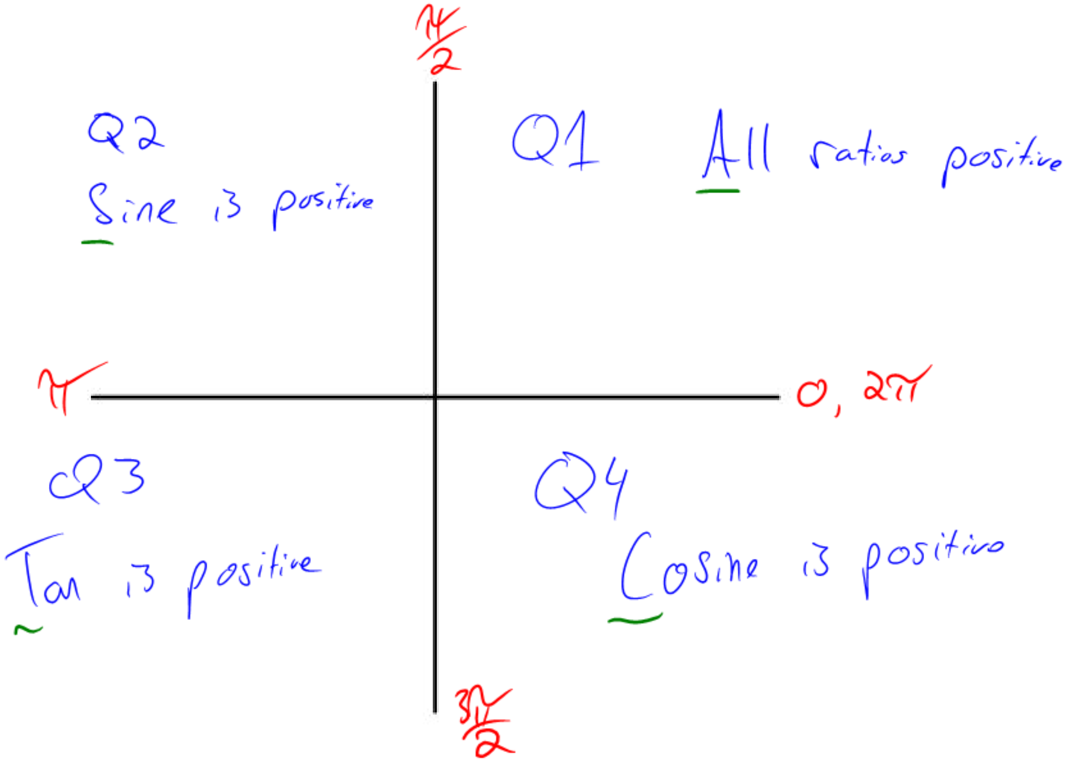
We now have enough tools to calculate the trigonometric ratios of any angle!

For any given angle θ (in radians from here on) we will:

1) Draw θ in standard position (i.e. draw the principal angle for θ)
2) Determine the related acute angle (between the terminal arm and the polar axis)
3) Use the related acute angle and the CAST RULE (and SOH CAH TOA) to determine the trig ratio in question

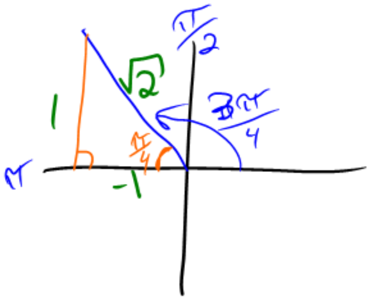
Recall the CAST RULE

Note: The CAST RULE determines the sign (+ or -) of the trig ratio



Example 5.2.2

Determine the trig ratio $\sin\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$



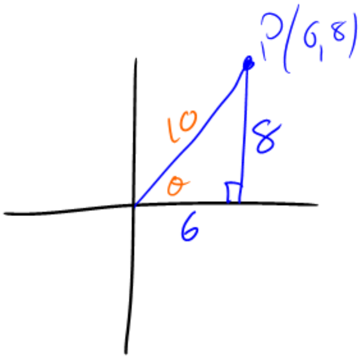
Example 5.2.3

The point $(6,8)$ lies on the terminal arm (of length *r*) of an angle of rotation. Sketch the angle of rotation.

Determine: a) the value of *r*

b) the primary trig ratios for the angle

c) the value of the angle of rotation in radians, to two decimal places



a) $6^2 + 8^2 = r^2$

$36 + 64 = r^2$

$100 = r^2$

$10 = r$

b) $\sin\theta = \frac{8}{10} = \frac{4}{5}$

$\cos\theta = \frac{6}{10} = \frac{3}{5}$

$\tan\theta = \frac{8}{6} = \frac{4}{3}$

c) $\sin\theta = \frac{4}{5}$

$\theta = \sin^{-1}\left(\frac{4}{5}\right) = 0.93$ rads

Class/Homework for Section 5.2

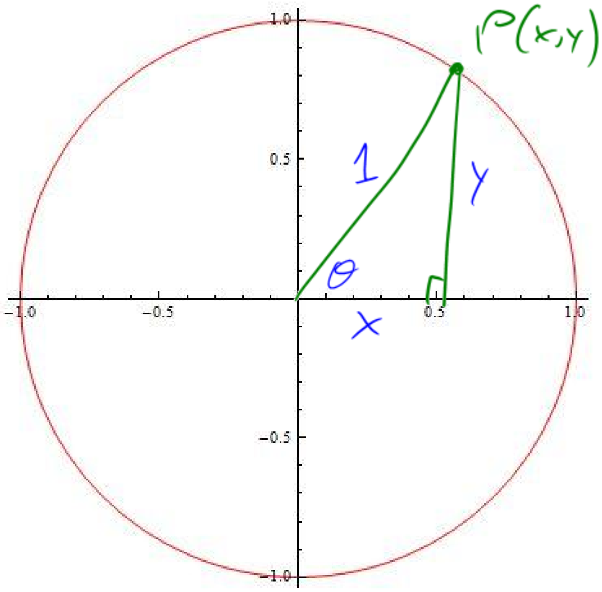
Pg. 330 #1b – f, 2bcd, 3

5.3 Trigonometric Ratios and Special Triangles

(*Part 2 – Exact Values*)

Recall the "Unit Circle" from yesterday:

↳ radius = 1.



$P(x,y) = P(\cos\theta, \sin\theta)$

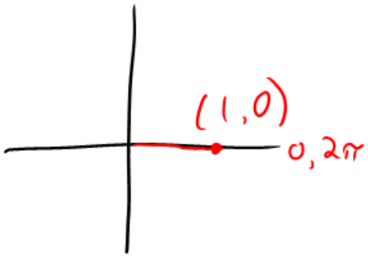
$\sin\theta = y$

$\cos\theta = x$

$\tan\theta = \frac{y}{x}$

$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$

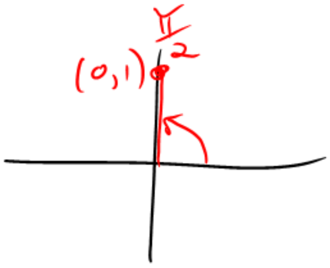
With this circle (and without a calculator!) we can evaluate EXACTLY the trig ratios for the angles (in radians) $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ radians.



$\sin(0) = 0$

$\cos(0) = 1$

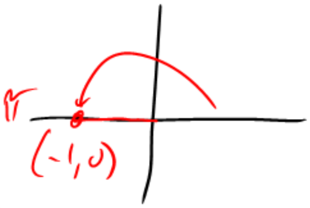
$\tan(0) = \frac{0}{1} = 0$



$\sin(\frac{\pi}{2}) = 1$

$\cos(\frac{\pi}{2}) = 0$

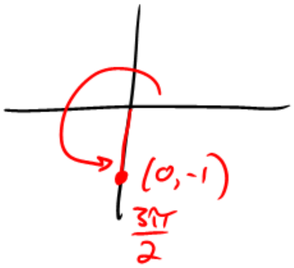
$\tan(\frac{\pi}{2}) = \frac{1}{0}$ = undefined.



$\sin(\pi) = 0$

$\cos(\pi) = -1$

$\tan(\pi) = \frac{0}{-1} = 0$



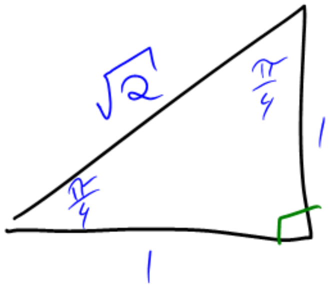
$\sin(\frac{3\pi}{2}) = -1$

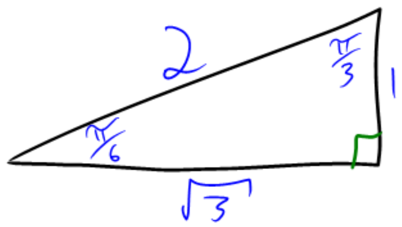
$\cos(\frac{3\pi}{2}) = 0$

$\tan(\frac{3\pi}{2}) = \frac{-1}{0}$

= undefined

Now, using **Special Triangles**, and **CAST** we can evaluate **EXACTLY** trig ratios for “special angles”.





Note: A trig ratio is a NUMBER.

Numbers have 2 qualities

1) Value

2) sign (+ or −)

Thus a trig ratio has a Value

(which we **evalu**ate using the related acute angle and Special Triangles)

AND, a trig ratio has a sign, which we get by using the CAST rule or by graphing it on x-y axis.

Example 5.3.1

Determine **Exactly** (i.e. the **use of a calculator** means **MARKS OFF**)

a) $\sin\left(\frac{\pi}{3}\right)$

b) $\cos\left(\frac{5\pi}{6}\right)$

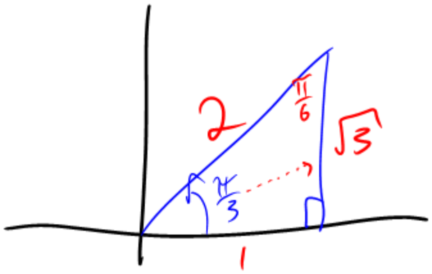
c) $\tan\left(\frac{5\pi}{4}\right)$

d) $\sec\left(\frac{5\pi}{3}\right)$

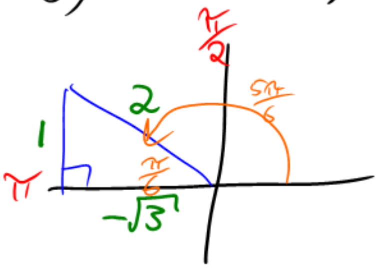
e) $\tan\left(\frac{3\pi}{2}\right)$

f) $\csc(-\pi)$

a) $\sin\left(\overset{\theta}{\frac{\pi}{3}}\right) = \frac{\sqrt{3}}{2}$

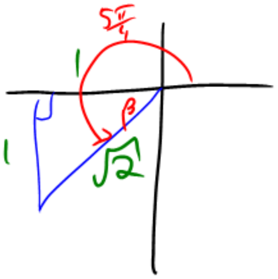


b) $\cos\left(\frac{5\pi}{6}\right) = \frac{-\sqrt{3}}{2}$



Using CAST RULE for c).

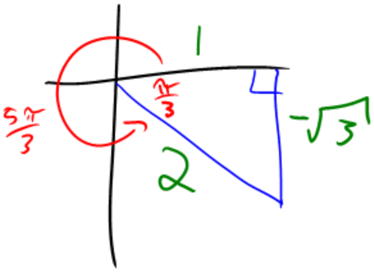
c) $\tan\left(\frac{5\pi}{4}\right) = \frac{1}{1} = 1$



$\theta = \frac{5\pi}{4}$

$\beta = \frac{\pi}{4}$

d) $\sec\left(\frac{5\pi}{3}\right) = \frac{hyp}{adj} = \frac{2}{1} = 2$



① figure out the β

② Get the value of the ratio

③ Use CAST rule to get the sign.

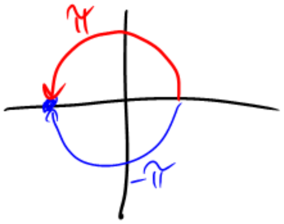
e) $\tan\left(\frac{3\pi}{2}\right) = \frac{\#}{0} = \text{undefined}$

f) $\csc(-\pi) = \csc(\pi)$

$= \frac{1}{\sin(\pi)}$

$= \frac{1}{0}$

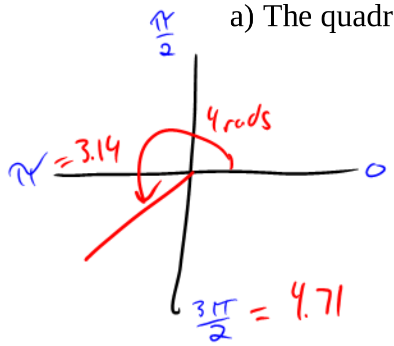
$= \text{undefined.}$



Example 5.3.2

Given sin(4) determine:

a) The quadrant $\theta = 4$ is in. b) The sign of sin(4) (no calculators!)



∴ in Q3

only tan is positive in Q3,

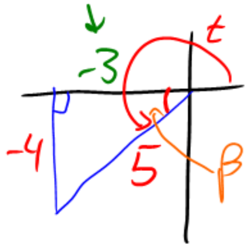
∴ sin(4) is negative

Example 5.3.3

Given $\sin(t) = -\frac{4^{o}}{5_{h}}$, $\overset{Q3}{\boxed{+\pi \le t \le \frac{3\pi}{2}}}$, determine

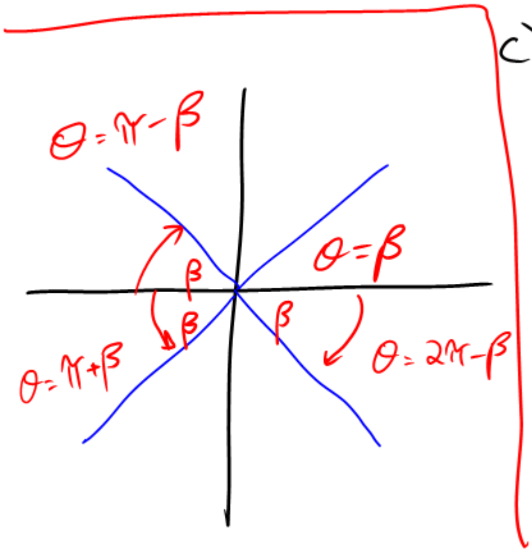
by Pythag...

a) $\cos(t)$ b) $\tan(t)$ c) t in radians, rounded to three decimal places.



a) $\cos(t) = \frac{-3}{5}$

b) $\tan(t) = \frac{-4}{-3} = \frac{4}{3}$



c) $\tan(\beta) = \frac{4}{3}$

$\beta = \tan^{-1}\left(\frac{4}{3}\right)$

$\beta = 0.927$

$t = \pi + \beta$

$t = 3.14 + 0.927$

$t = 4.067$ rad.

Class/Homework for Section 5.3

Pg. 330 – 331 #5, 7, 9

5.4 Trigonometric Ratios and Special Triangles

(Part 3 – Getting the Angles)

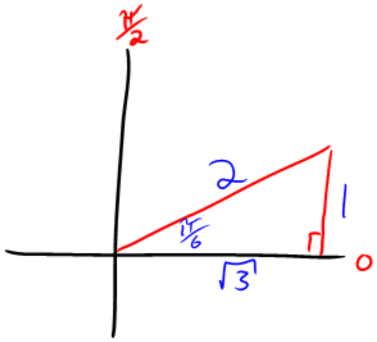
We have been looking at **evaluating exact values** for trigonometric ratios using special triangles and CAST, given an **angle of rotation**. We now turn our attention to the **inverse operation** – determining **angles of rotation** given a **trig ratio**.

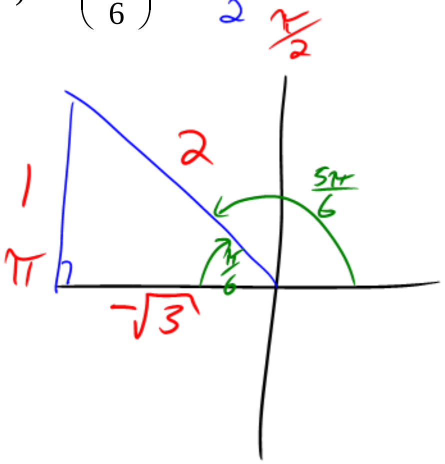
Example 5.4.1

Determine exactly:

a) $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

b) $\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$





Note: EVERY Trig ratio has two angles of rotation in $[0, 2\pi]$, except for <u>some</u> axis angle.

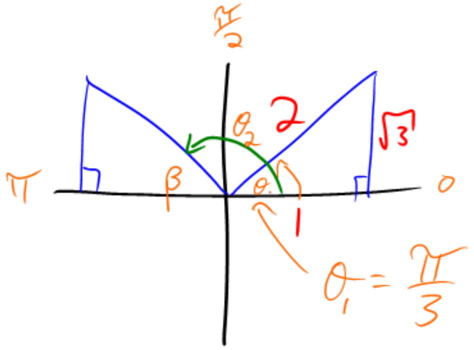
Example 5.4.2

Determine BOTH angles of rotation, θ, for $0 \le \theta \le 2\pi$ given

a) $\sin(\theta)=\frac{\sqrt{3}}{2}$ o h

Procedure

1) Determine the quadrants θ is in.
2) Draw the angles of rotation.
3) Determine the related acute angle β and construct the appropriate special triangles.
4) Determine the angles of rotation.



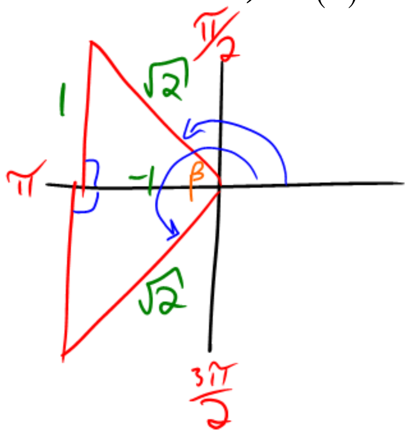
$\theta_1 = \frac{\pi}{3}$

$\beta = \frac{\pi}{3}$

$\therefore \theta_2 = \frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3}$

$\therefore \theta = \frac{\pi}{3}$ and $\frac{2\pi}{3}$

b) $\cos(\theta)=-\frac{1}{\sqrt{2}}$ a h



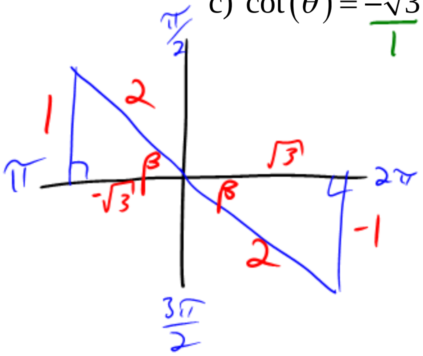
$\beta = \frac{\pi}{4}$

$\theta_1 = \frac{4\pi}{4} - \frac{\pi}{4} = \frac{3\pi}{4}$

$\theta_2 = \frac{4\pi}{4} + \frac{\pi}{4} = \frac{5\pi}{4}$

$\therefore \theta = \frac{3\pi}{4}$ and $\frac{5\pi}{4}$

c) $\cot(\theta) = \frac{-\sqrt{3}}{1}$ $\frac{a}{o}$



$\beta = \frac{\pi}{6}$

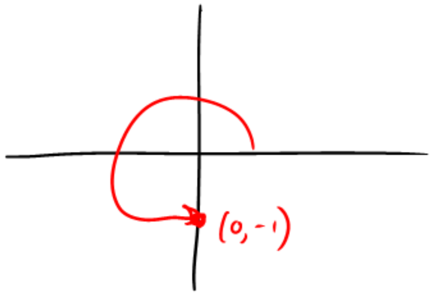
$\theta_1 = \frac{6\pi}{6} - \frac{\pi}{6} = \frac{5\pi}{6}$

$\theta_2 = \frac{12\pi}{6} - \frac{\pi}{6} = \frac{11\pi}{6}$

$\therefore \theta = \frac{5\pi}{6}$ and $\frac{11\pi}{6}$

d) $\sin(\theta) = -1 = y$

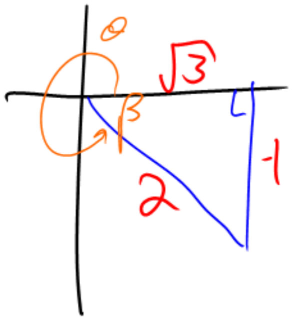
$\boxed{\text{recall: } \sin\theta = y, \ \cos\theta = x}$



$\theta = \frac{3\pi}{2}$

Example 5.4.3

Determine θ where $\underbrace{\frac{3\pi}{2} \le \theta \le 2\pi}_{Q4}$ for $\csc(\theta) = \frac{-2}{1}$ $\frac{h}{o}$



$\beta = \frac{\pi}{6}$

$\theta = \frac{12\pi}{6} - \frac{\pi}{6}$

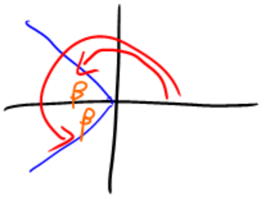
$\theta = \frac{11\pi}{6}$

Practice Problems

Determine the angles of rotation, θ, for $0 \le \theta \le 2\pi$:

a) $\sin(\theta) = -\frac{\sqrt{3}}{2}$	b) $\sec(\theta) = \sqrt{2}$
c) $\tan(\theta) = \frac{1}{\sqrt{3}}$	d) $\cot(\theta) = -1$
e) $\csc(\theta) = \frac{2}{\sqrt{3}}$	f) $\cos(\theta) = 0$
g) $\sin(\theta) = 1$	h) $\sqrt{3}\cos(\theta) - 2\cos(\theta)\cdot\sin(\theta) = 0$

$\cos\theta = -0.8213$



$\cos\beta = 0.8213$

$\beta = \cos^{-1}(0.8213)$

$\beta = 0.61$

$\theta_1 = \pi - 0.61 = 3.14 - 0.61 = \boxed{2.53}$

$\theta_2 = 3.14 + 0.61 = \boxed{3.75}$

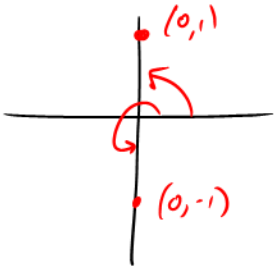
h) $\sqrt{3}\,\underline{\cos\theta} - 2\underline{\cos\theta}\sin\theta = 0$

$(\cos\theta)(\sqrt{3} - 2\sin\theta) = 0$

Solve:

$\cos\theta = 0$

$\boxed{\cos\theta = x}$



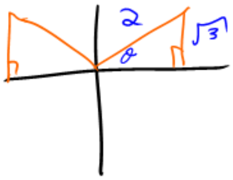
$\theta = \frac{\pi}{2}$

$\theta = \frac{3\pi}{2}$

Solve: $\sqrt{3} - 2\sin\theta = 0$

$\sqrt{3} = 2\sin\theta$

$\frac{{}^{o}\sqrt{3}}{{}_{h}2} = \sin\theta$



$\beta = \frac{\pi}{3}$

$\theta_1 = \frac{\pi}{3}$

$\theta_2 = \frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3}$

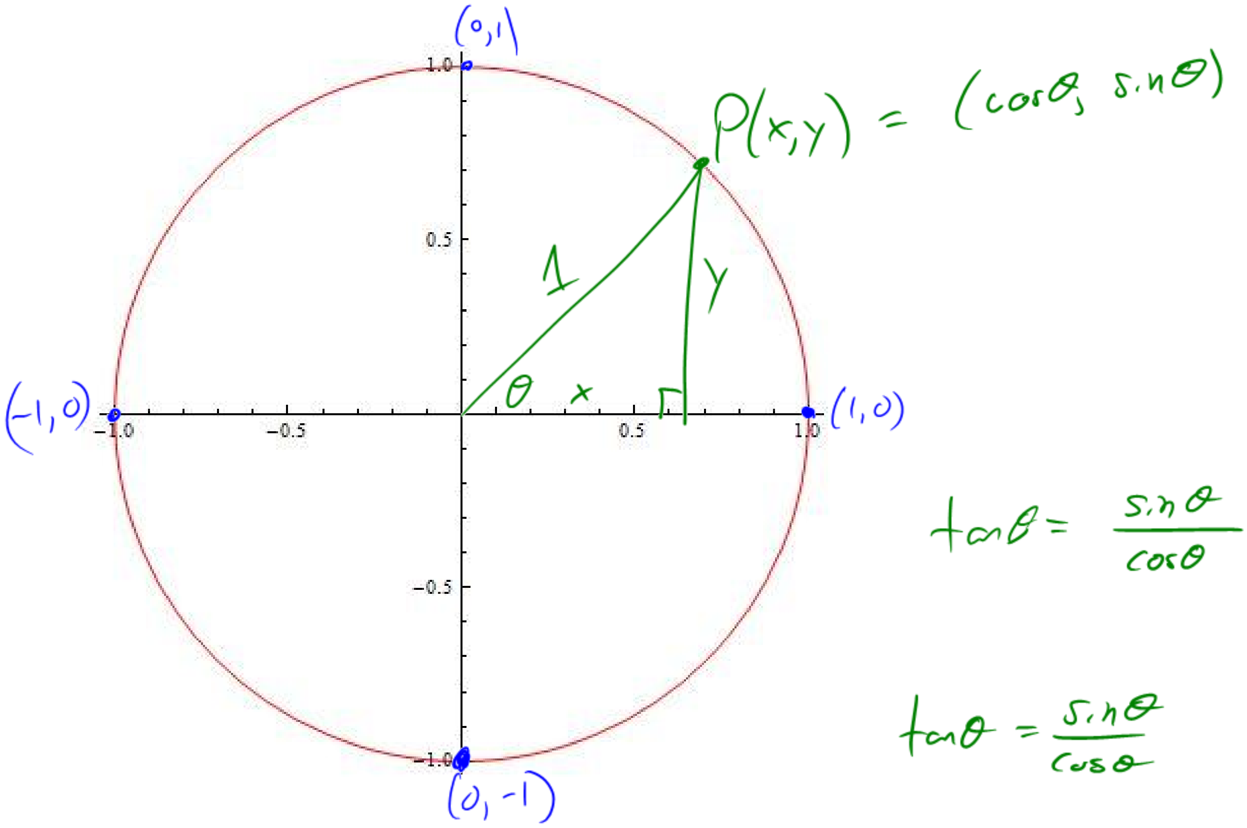
$\therefore \theta = \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{2}$

Class/Homework for Section 5.4

Pg. 331 #6, 11, 16

5.5 Sketching the Trigonometric Functions

Before beginning the sketches, recall the diagram of the unit circle that we have been using to explore the basic ideas in trigonometry:



$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(0) = 0$

$\sin(\frac{\pi}{2}) = 1$

$\sin(\pi) = 0$

$\sin(\frac{3\pi}{2}) = -1$

$\sin(2\pi) = 0$

$\cos(0) = 1$

$\cos(\frac{\pi}{2}) = 0$

$\cos(\pi) = -1$

$\cos(\frac{3\pi}{2}) = 0$

$\cos(2\pi) = 1$

$\tan(0) = 0$

$\tan(\frac{\pi}{2}) =$ undefined

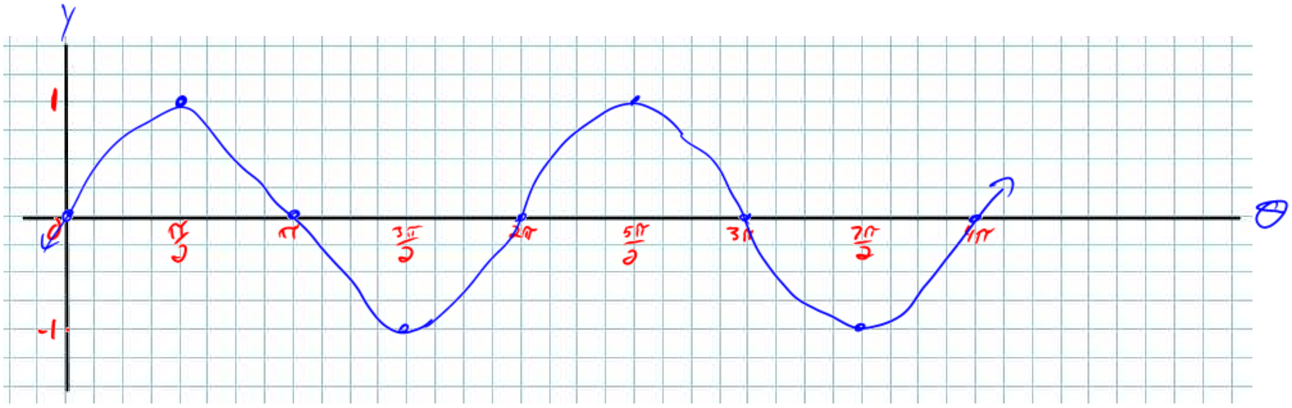
$\tan(\pi) = 0$

$\tan(\frac{3\pi}{2}) =$ undefined

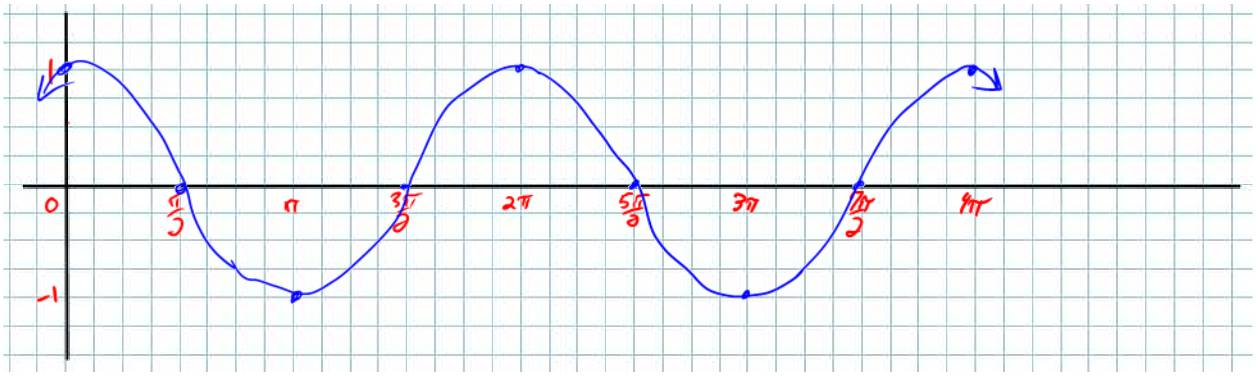
$\tan(2\pi) = 0$

The Primary Trigonometric Functions

$f(\theta) = \sin(\theta), \quad \theta \in [0, 4\pi]$



$g(\theta) = \cos(\theta)$

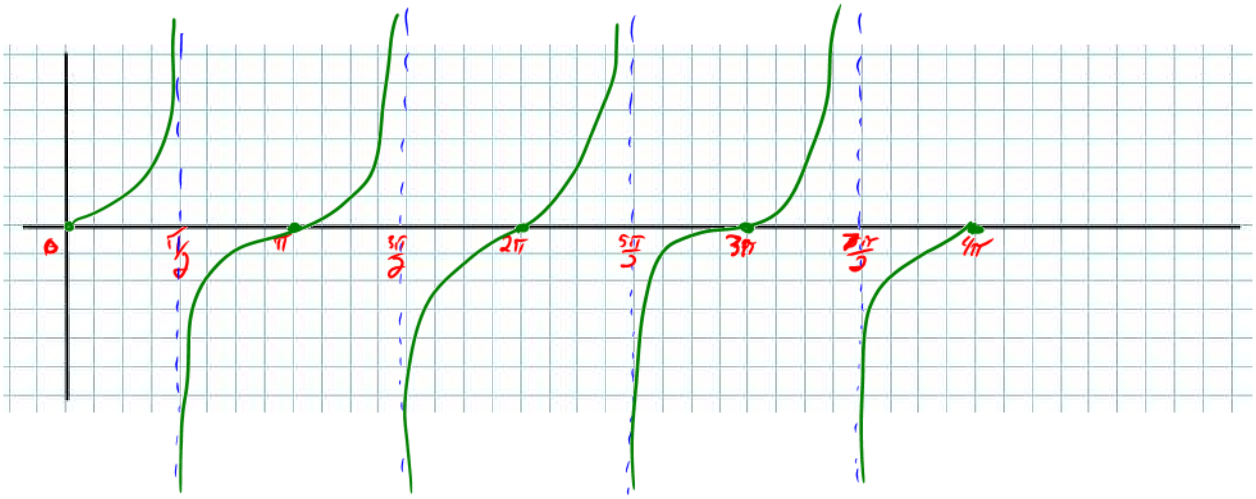


Note: $\cos\theta$ is just $\sin\theta$ shifted to the left by $\frac{\pi}{2}$

Since $\tan\theta = \frac{\sin\theta}{\cos\theta}$, we have

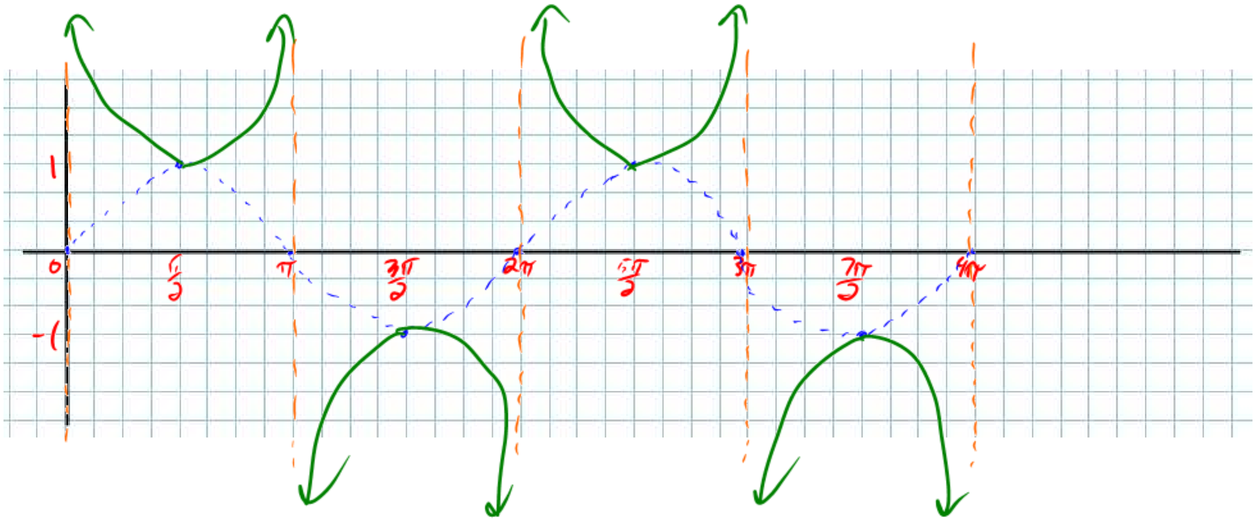
$h(\theta) = \tan(\theta)$

V.A.'s when $\cos\theta = 0$

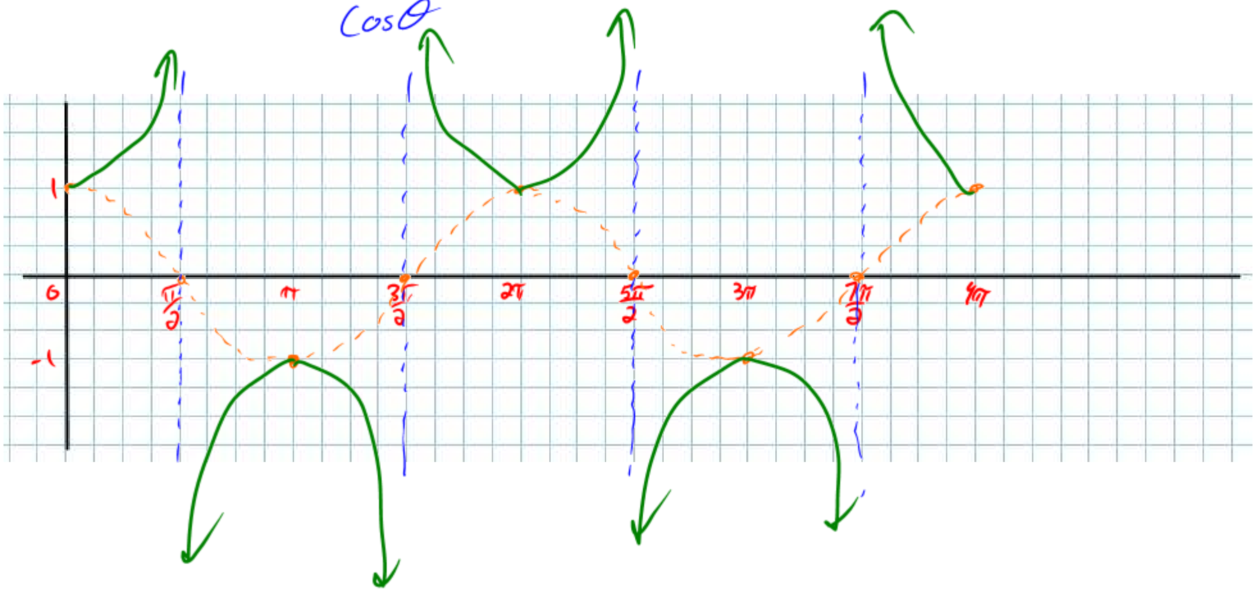


The Reciprocal Trig Functions

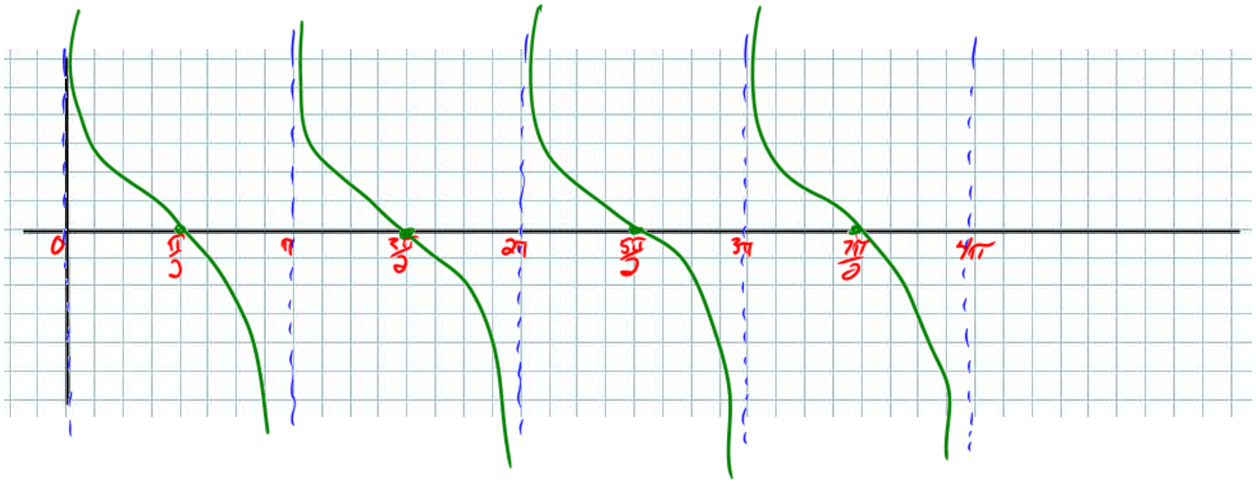
$f(\theta) = \csc(\theta) = \frac{1}{\sin\theta}$, ∴ V.A's whenever $sn\theta = 0$



$g(\theta) = \sec(\theta) = \frac{1}{\cos\theta}$, $\therefore$ VA's whenever $\cos\theta = 0$!



$h(\theta) = \cot(\theta) = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$ $\therefore$ V.A's when $\sin\theta = 0$



5.6 Transformations of Trigonometric Functions

By this point in your illustrious High School careers, you have a solid understanding of Transformations of Functions in general. In terms of the trig functions Sine and Cosine in particular, the concepts are as you expect, but the transformations have specific meanings relating to nature of the sinusoidal "wave".

General Form of the Sine and Cosine Functions

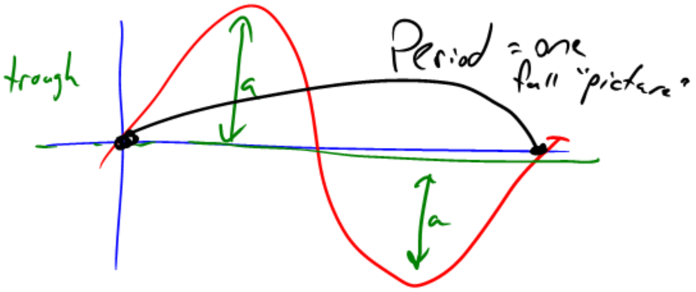
$f(\theta) = a\sin(k(\theta - d)) + c$ $\qquad$ $g(\theta) = a\cos(k(\theta - d)) + c$

$|a| =$ Amplitude

$a = \dfrac{\overset{\text{Peak}}{\max} - \min}{2} \rightarrow \text{trough}$

$a = \max - c$

$a = c - \min$



$k =$ Period factor

$\text{Period} = \dfrac{2\pi}{|k|} \Rightarrow k = \dfrac{2\pi}{\text{period}}$

$d =$ Phase Shift

"starting point" when graphing

Note: To determine d you MUST isolate the θ or x

$c =$ central axis

→ the middle

$c = \dfrac{\max + \min}{2}$

Example 5.6.1

Determine the amplitude, period, phase shift and the equation of the central axis for:

a) $f(\theta) = 2\sin\left(\theta + \frac{\pi}{3}\right) + 1$

amp = 2

$k = 1 \therefore$ Period $= \frac{2\pi}{1} = 2\pi$

Phase Shift: $-\frac{\pi}{3}$ (left $\frac{\pi}{3}$)

central axis: $y = 1$

b) $g(\theta) = 3\cos\left(\boxed{2\theta - \frac{\pi}{2}}\right) + 0 \rightarrow 2\left(\theta - \frac{\pi}{4}\right)$

amp = 3

$k = 2 \therefore$ period $= \frac{2\pi}{2} = \pi$

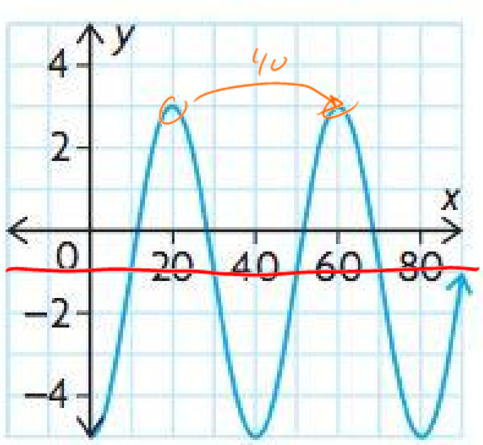
Phase Shift: $d = \frac{\pi}{4}$

Central Axis: $y = 0$

Example 5.6.2

From your text: Pg. 346 #14c

Determine a ***sinusoidal*** function for the given sketch of a graph



Amplitude: $a = 4$ (by counting...) | $a = \frac{3 - -5}{2} = \frac{8}{2} = 4$

Period: 40 $\therefore k = \frac{2\pi}{40} = \frac{\pi}{20}$

Phase Shift: As Cosine → Peak or trough

Peak: $d = 20$ Trough: $d = 0$

As Sine → middle

$d = 10$ ↗ or $d = 30$ ↘

Equation of Central Axis: $y = -1$

Equation as a Cosine Wave

using Peak: $f(\theta) = 4\cos\left(\frac{\pi}{20}(\theta - 20)\right) - 1$

using Trough: $f(\theta) = -4\cos\left(\frac{\pi}{20}\theta\right) - 1$

Equation as a Sine Wave

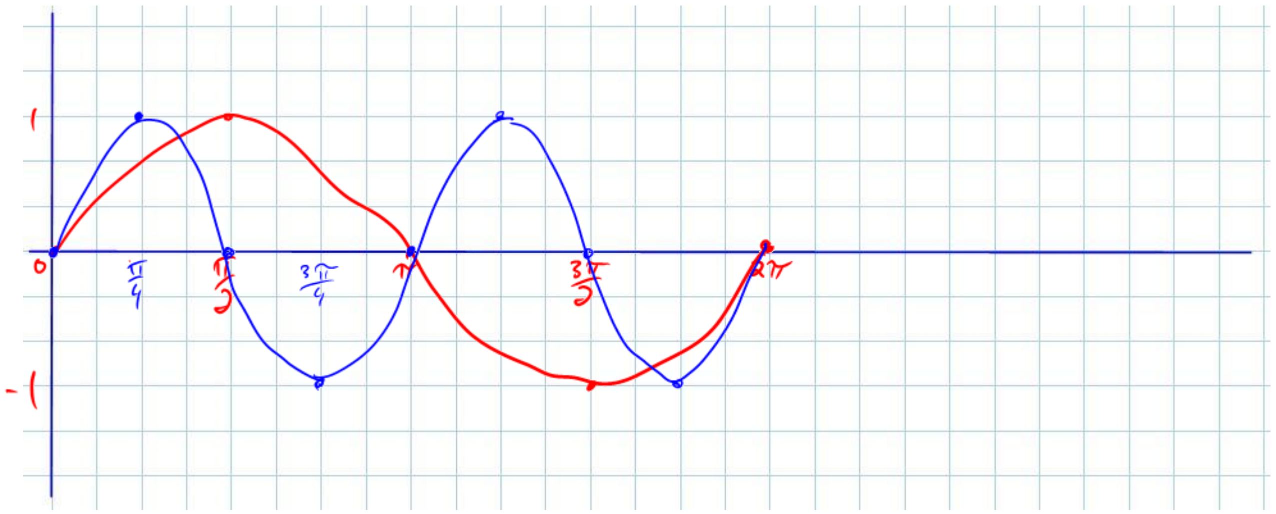
↗ $f(\theta) = 4\sin\left(\frac{\pi}{20}(\theta - 10)\right) - 1$

↘ $f(\theta) = -4\sin\left(\frac{\pi}{20}(\theta - 30)\right) - 1$

k=2 ∴ period = $\frac{2\pi}{2} = \pi$

Example 5.6.3

Sketch $f(x)=\sin(x)$ and $g(x)=\sin(2x)$ for $0 \le x \le 2\pi$ on the same set of axes.



peak or trough

Start with dotted line.

K=1 ∴ period = 2π

d = $\frac{\pi}{3}$

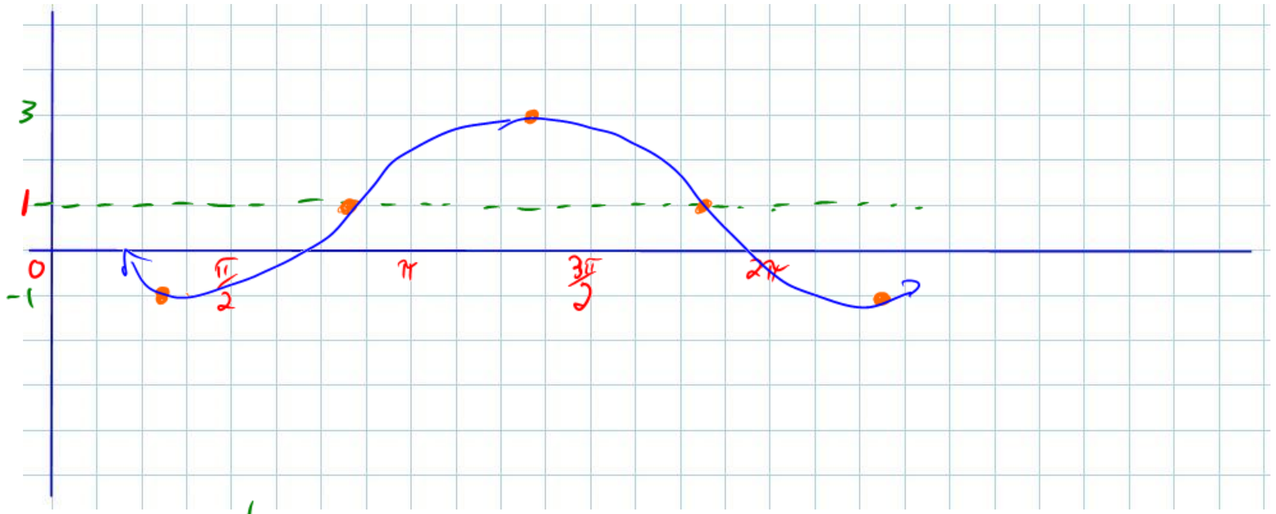
Example 5.6.4

Sketch $f(\theta)=-2\cos\left(\theta-\frac{\pi}{3}\right)+1$ on $0 \le \theta \le 2\pi$

trough

amp = 2 → put the max/min on scale

Start at $\left(\frac{\pi}{3}, -1\right)$



End at $\left(\frac{\pi}{3} + \overset{\text{period}}{2\pi}, -1\right)$

$\frac{7\pi}{3}$

Example 5.6.5

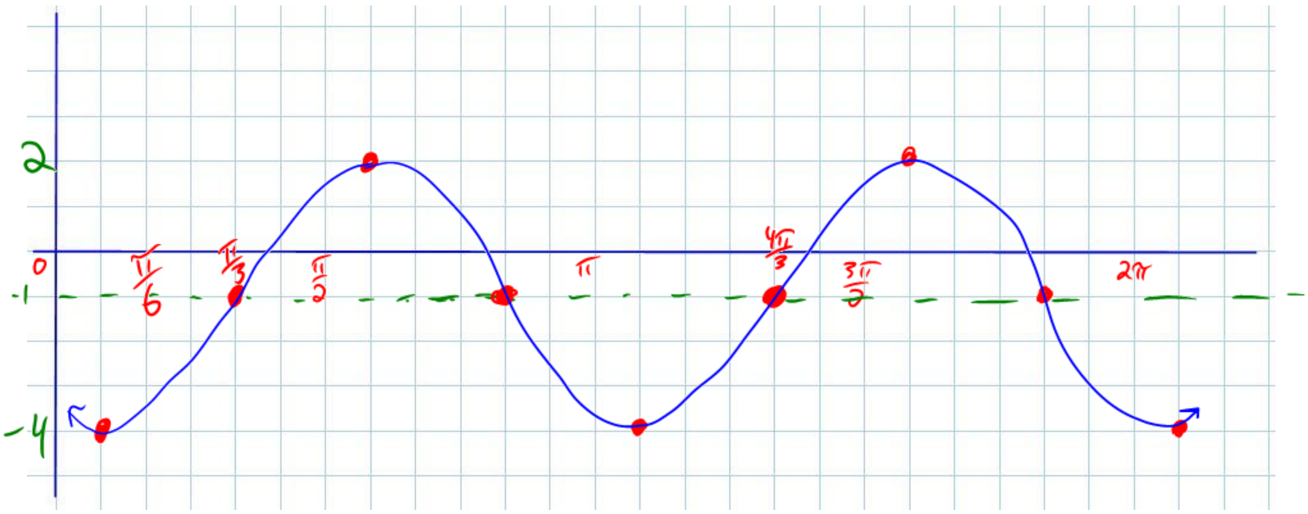
begins in middle ∴ $\left(\frac{\pi}{3}, -1\right) \rightarrow$ end at $\left(\frac{\pi}{3} + \pi, -1\right)$

$\left(\frac{4\pi}{3}, -1\right)$

Sketch $f(\theta) = \boxed{3}\sin\left(2\theta - \frac{2\pi}{3}\right) - 1$

$\left(2\left(\theta - \frac{\pi}{3}\right)\right)$

$\rightarrow K = 2 \therefore \text{Period} = \frac{2\pi}{2} = \pi$



Class/Homework for Section 5.6

Pg. 343 - 345 #1, 4, 6 – 8, 13, 14ab

5.7 Applications of Trigonometric Functions

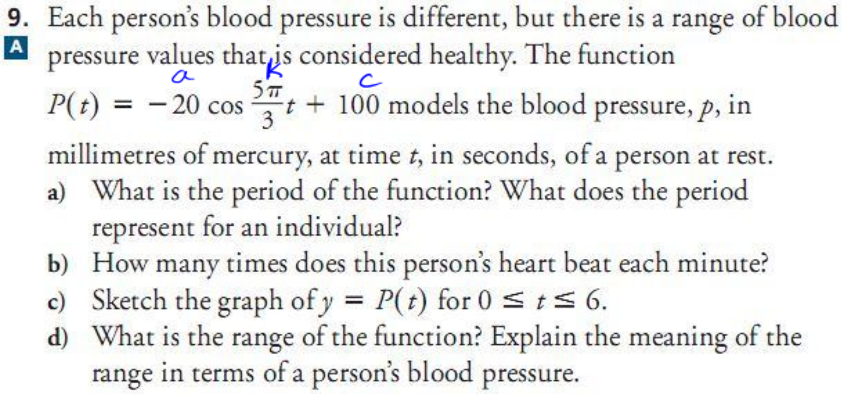
For any phenomenon in the real world which has a periodically repeating behaviour, Trigonometric Functions can be used to describe and analyze that behaviour. There are a myriad of such phenomena. From the rise and fall of tides to computer gaming habits, Trigonometric Functions have a say.

We will look at a few real world applications of Trigonometric Functions here.

Figure 5.7.1 A periodic rise and fall in online gamers

Example 5.7.1

From your text: Pg. 345 #9



9. Each person's blood pressure is different, but there is a range of blood pressure values that is considered healthy. The function

$P(t) = \overset{a}{-20} \cos \overset{k}{\frac{5\pi}{3}} t + \overset{c}{100}$ models the blood pressure, p, in millimetres of mercury, at time t, in seconds, of a person at rest.

- a) What is the period of the function? What does the period represent for an individual?
- b) How many times does this person's heart beat each minute?
- c) Sketch the graph of $y = P(t)$ for $0 \leq t \leq 6$.
- d) What is the range of the function? Explain the meaning of the range in terms of a person's blood pressure.

a) $k = \frac{5\pi}{3}$

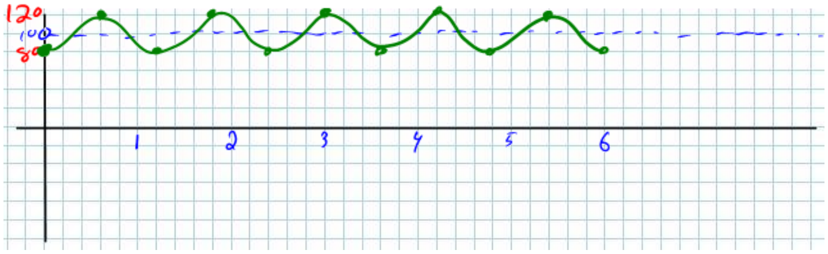
$\text{Period} = \frac{2\pi}{\frac{5\pi}{3}}$ ÷

$= \frac{2\cancel{\pi}}{1} \times \frac{3}{5\cancel{\pi}}$

$\boxed{\text{Period} = \frac{6}{5} \text{ seconds for every beat.}}$

↳ 1.2 sec/beat

b) $\overset{10}{\cancel{60}} \frac{\cancel{\text{sec}}}{\text{min}} \cdot \frac{5 \text{ beats}}{\cancel{6} \cancel{\text{ seconds}}} = 50 \text{ beats/min}$



d) $R = P(t) \in [80, 120]$

Example 5.7.2

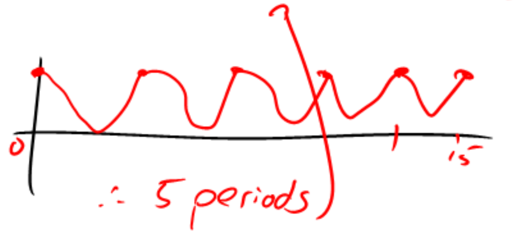
From your text Pg. 361 #7

7. A person who was listening to a siren reported that the frequency of the sound fluctuated with time, measured in seconds. The minimum frequency that the person heard was 500 Hz, and the maximum frequency was 1000 Hz. The maximum frequency occurred at $t = 0$ and $t = 15$. The person also reported that, in 15, she heard the maximum frequency 6 times (including the times at $t = 0$ and $t = 15$). What is the equation of the cosine function that describes the frequency of this siren?

Min = 500

Max = 1000

Period = $\frac{15}{5} = 3$ seconds



$a = \frac{max - min}{2}$

$a = \frac{1000 - 500}{2}$

$a = 250$

$c = \frac{max + min}{2}$

$c = \frac{1000 + 500}{2}$

$c = 750$

$d = 0$

t=0, at Max!

∴ + cosine

$k = \frac{2\pi}{3 \text{ (period)}}$

$$\therefore f(t) = 250\cos\left(\frac{2\pi}{3}t\right) + 750$$

The end. of this unit.

Class/Homework for Section 5.7

Pg. 360 – 362 #4, 6, 9, 10