Advanced Functions

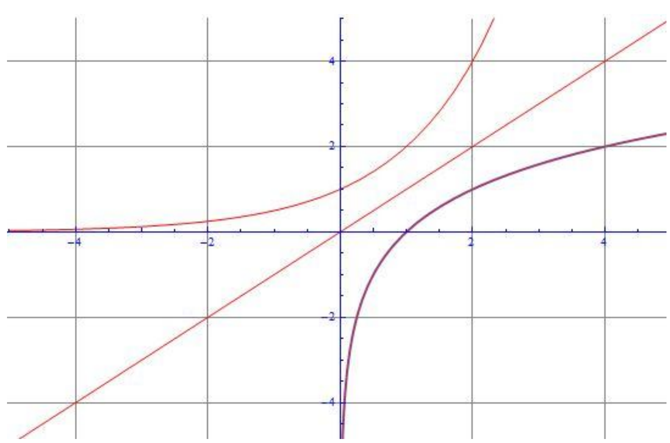
Fall 2017

Course Notes

Unit 7 – Exponential and Logarithmic Functions

We will learn

- *how Exponential and Logarithmic Functions are related*
- *how to describe the characteristics of Logarithmic Functions and their graphs*
- *techniques for simplifying and evaluating logarithmic expressions and equations*
- *how to solve exponential and logarithmic equations*
- *how to apply concepts and techniques to solve real world problems involving exponential and logarithmic functions*



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Chapter 7 – Exponential and Logarithmic Functions

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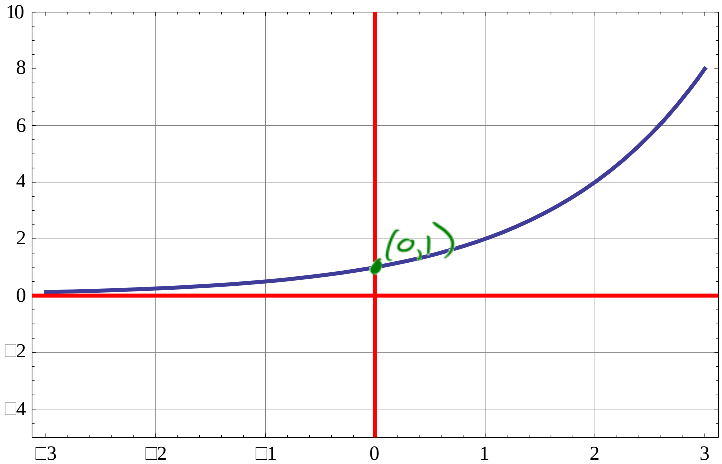
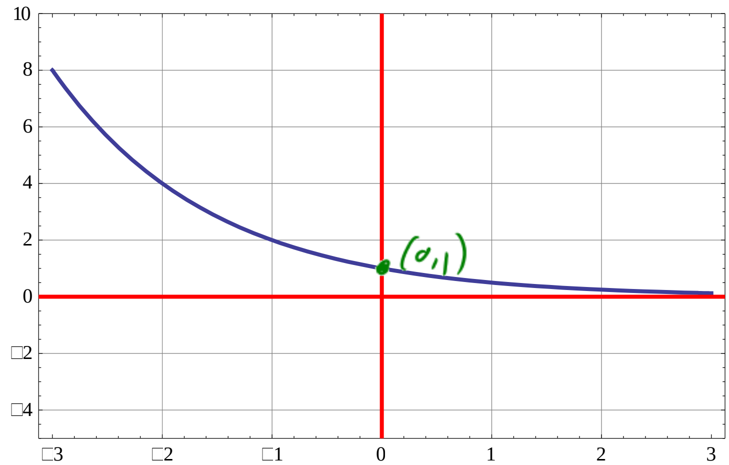
Test on Tuesday, December 5th

7.1 Exploring the Logarithmic Function

In grade 11 Functions, you spent a bunch of time considering Exponential Functions, and it seems like a good idea to spend a little time reviewing that type of function.

Consider the sketches of the graphs of the functions $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^x$:





Features of $f(x) = \boxed{2}^x$: Exponential Growth

$b > 0$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Axis Intercepts:

x-int: None

y-int: (0,1)

Asymptotes:

V.A: none

H.A: y = 0

Intervals of inc/decrease:

inc: $(-\infty, \infty)$ dec: never!

Features of $g(x) = \left(\frac{1}{2}\right)^x$: Exponential Decay.

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Axis Intercepts:

x-int: None

y-int: (0,1)

Asymptotes:

V.A: none

H.A: y = 0

Intervals of inc/decrease:

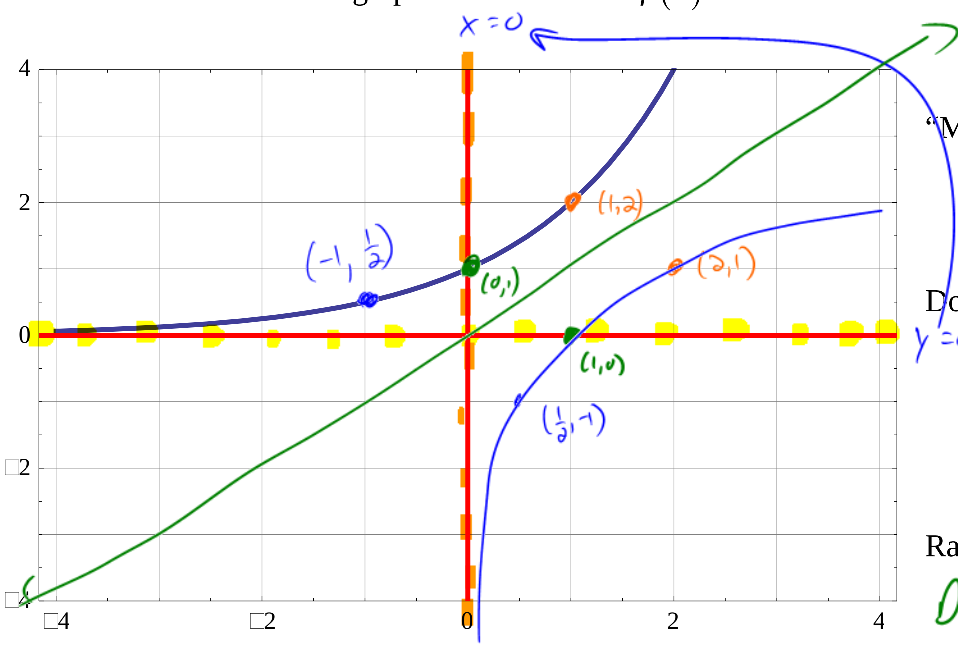
inc: never dec: $(-\infty, \infty)$

One important aspect of functions is the concept of **inverse**. We have in fact spent a good chunk of time talking about the inverses of functions. In case you've forgotten, let's take a look at how to find the inverse of a function.

There are **two methods**. One is a **geometric approach**, and the other (more useful) method is an **algebraic approach**.

Inverse of a function geometrically

Consider the sketch of the graph of the function $f(x) = 2^x$:



"Mirror" $y = x$

Domain of $f(x)$ is the $y = 0$ Range of $f^{-1}(x)$

Range of $f(x)$ is the Domain of $f^{-1}(x)$

Why isn't this technique very useful for the mathematician? graphs are not always accurate.

Inverse of a function algebraically

Recall that the basic technique for finding the algebraic inverse of a function is to switch x and $f(x)$ and then solve for $f^{-1}(x)$.

e.g. Determine the inverse of $f(x) = 2(x-1)^2 + 2$

$x = 2(y-1)^2 + 2$

$x - 2 = 2(y-1)^2$

$\frac{x-2}{2} = (y-1)^2$

$\pm\sqrt{\frac{x-2}{2}} + 1 = y$

$f^{-1}(x) = \pm\sqrt{\frac{x-2}{2}} + 1$

O.K., now let's try this same technique for an exponential function.

Example 7.1.1

Determine the inverse of $f(x) = 2^x$ (First we switch x and $f(x)$)

$x = 2^{(y)}$

$x = 2^{f^{-1}(x)}$ (this is called the exponential form of the inverse)

O.K. now we just need to isolate for $f^{-1}(x)$ …

how do we invert the operation of exponentiation?

Definition 7.1.1

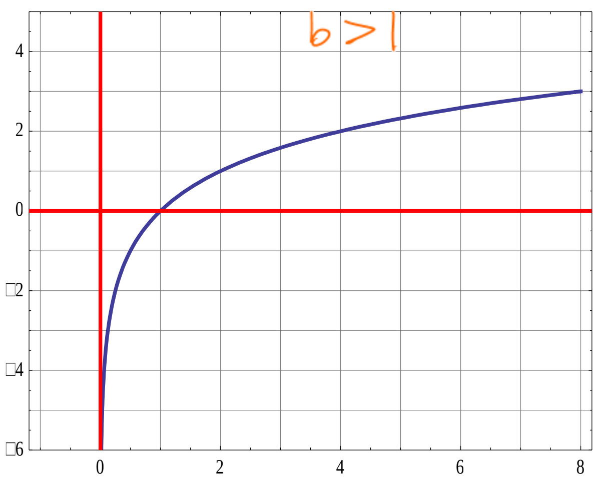
Given $y = a^x$

$x = \boxed{\log_a}(y)$

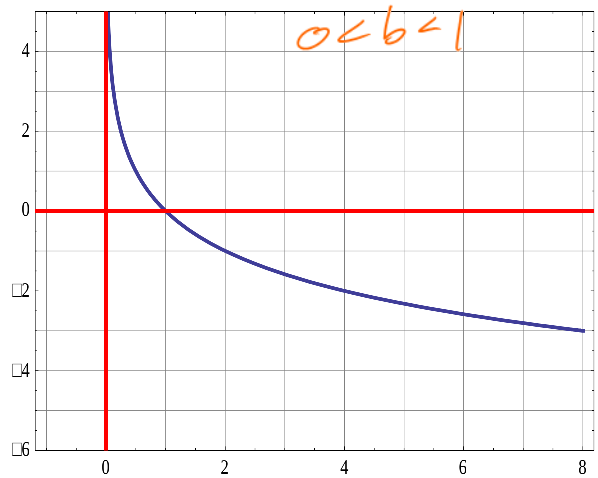
Note: A **Logarithm** is an exponent.

We will use the following form for Logarithmic Functions:

Basic Sketches of the General Logarithmic Function $f(x) = \log_b(x)$



$f(x) = \log_2(x)$



$g(x) = \log_{\frac{1}{2}}(x)$

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Axis Int's:

x-int: $(1,0)$

y-int: none

Asymptotes:

V.A: $x=0$

H.A: none

Intervals of inc/dec

inc: $(0, \infty)$

dec: never

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Axis Int's:

x-int: $(1,0)$

y-int: none

Asymptotes

V.A: $x=0$

H.A: none

Intervals of inc/dec

inc: never

dec: $(0, \infty)$

Example 7.1.2

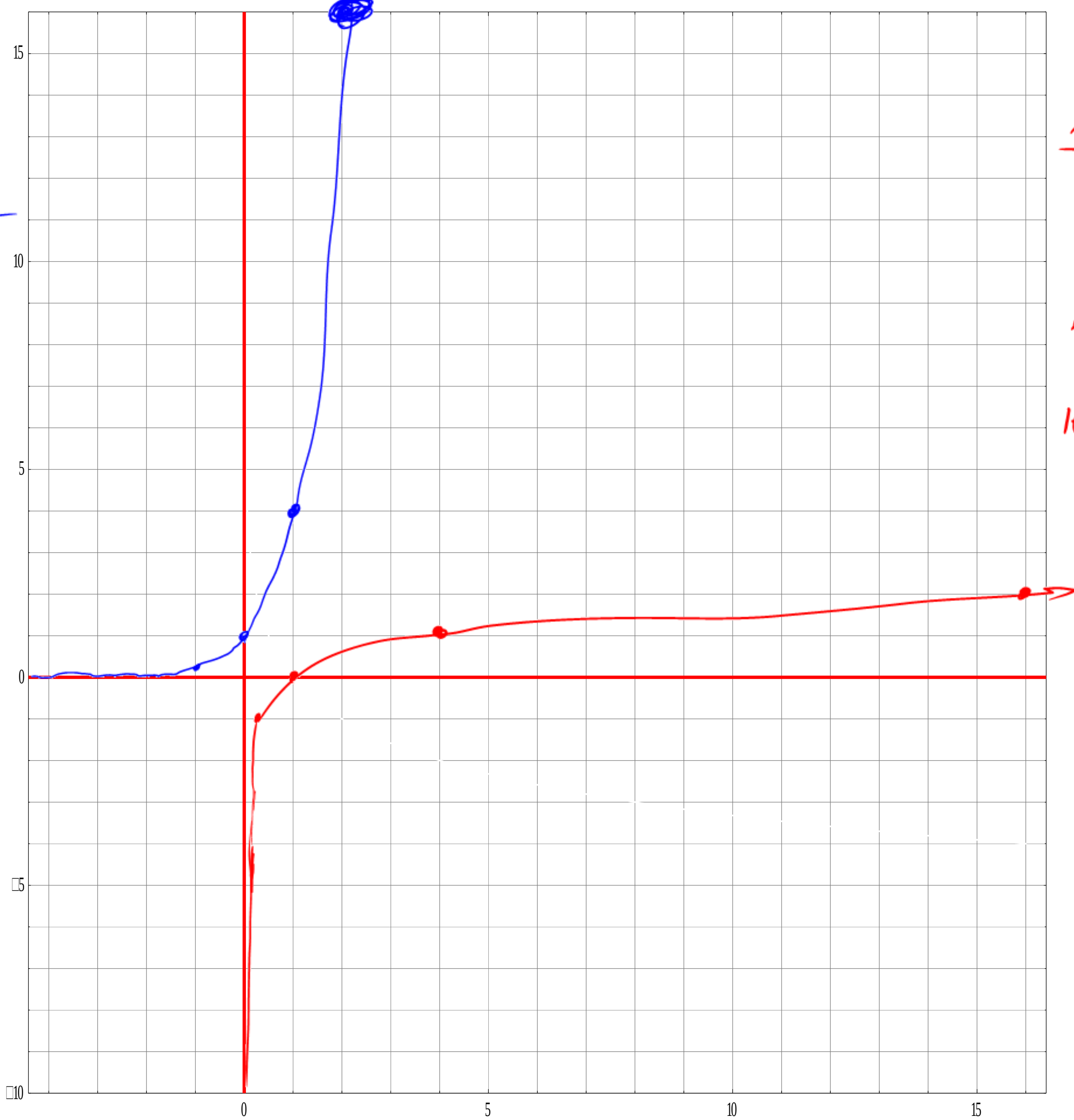
From your text: Pg. 451 #1a)

Sketch the inverse of $f(x) = 4^x$

$f^{-1}(x) = \log_4(x)$

$f(x) = 4^x$

x	4^x
-1	$1/4$
0	1
1	4
2	16



x	$\log_4(x)$
$\frac{1}{4}$	-1
1	0
4	1
16	2

Example 7.1.3

From your text: Pg. 451 #2

Write the equation of the above inverse function in i) exponential and ii) logarithmic form:

$f(x) = 4^x$ i) $x = 4^{f^{-1}(x)}$ ii) $f^{-1}(x) = \log_4(x)$

Example 7.1.4

From your text: Pg. 451

a) #5, 6cd Write the inverse of the given exponential function in i) exponential and ii) logarithmic form:

1) $y = \left(\frac{1}{4}\right)^x$

i) $x = \left(\frac{1}{4}\right)^{y^{-1}}$ ii) $y^{-1} = \log_{\frac{1}{4}}(x)$

2) $y = m^x$

i) $x = m^{y^{-1}}$

ii) $y^{-1} = \log_m(x)$

b) #7c) Write the equation of the given logarithmic function in exponential form:

$\boxed{y} = \log_3(x)$ answer

exponent

base

$x = 3^y$

c) #8c) is a question which can be very confusing if you don't carefully consider the context of the problem.

Write the equation of the INVERSE **FUNCTION** of given logarithmic function (in exponential form…well duh).

$f(x) = \log_3(x)$ $f^{-1}(x) = 3^x$

Know: $\sqrt{x} = x^{\frac{1}{2}}$

d) #9adf) Evaluate (remember that **evaluate** means to **calculate** a number)

1) $\log_2 4 = x$

$2^x = 4 \therefore x = 2$

2) $\log_5 1 = x$

$5^x = 1 \therefore x = 0$

3) $\log_3 \sqrt{3} = x$

$3^x = \sqrt{3} = 3^{\frac{1}{2}} \therefore x = \frac{1}{2}$

e) #11 For $y = \log_2 x$ determine the coordinates of the points on the graph with functional (y) values of $-2, -1, 0, 1, 2$

Class/Homework for Section 7.1

Pg. 451 #1c, 4, Finish 5 – 7, 9, 11

7.2 Evaluating Logarithms (Part 1)

We will learn to "evaluate" through many examples, keeping in mind **the fact that logarithms and exponentials are inverses of each other.**

Example 7.2.1

Evaluate

base

exponent

a) $\log_2(32) = x$

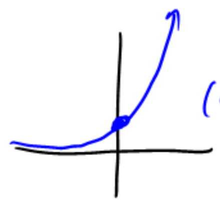
$2^x = 32$

$2^x = 2^5$

$x = 5$

b) $\log_3(-9) = x$

$3^x = -9$



$(0, \infty)$

no solutions! $x > 0$

also, $f(x) = \log(x)$ has domain $(0, \infty)$

c) $\log_4\left(\frac{1}{64}\right) = x$

$4^x = \frac{1}{64}$

$4^x = \frac{1}{4^3}$

$4^x = 4^{-3}$

$x = -3$

d) $\log_{254}(1) = x$

$254^x = 1$

$x = 0$

e) $\log_4(32) = x$

$4^x = 32$

$(2^2)^x = 2^5$

$2^{2x} = 2^5$

$2x = 5$

$x = \frac{5}{2}$

f) $\log_3\left(\frac{1}{81}\right) = x$

$3^x = \frac{1}{81}$

$3^x = \frac{1}{3^4}$

$3^x = 3^{-4}$

$x = -4$

g) $\log_5\left(\sqrt[4]{125}\right) = x$

Recall: $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

$5^x = \sqrt[4]{125}$

$5^x = (125)^{1/4}$

$5^x = (5^3)^{1/4}$ multiply

$5^x = 5^{3/4}$

$x = \frac{3}{4}$

h) $\log_9\left(\sqrt{27}\right) = x$

$\sqrt{x} = x^{\frac{1}{2}}$

$9^x = \sqrt{27}$

$(3^2)^x = (3^3)^{\frac{1}{2}}$

$3^{2x} = 3^{3/2}$

$2x = \frac{3}{2}$

$x = \frac{3}{4}$

Practice Problem: Pg. 466 #3

7.3 Evaluating Logarithms (Part 2)

Here we take this idea of evaluation up a notch. We will continue learning "by example".

Example 7.3.1

Evaluate

a) $\log_3\left(\sqrt[5]{27}\right) = x$

$3^x = \sqrt[5]{27}$

$3^x = \left(3^3\right)^{1/5}$

$3^x = 3^{3/5}$

$x = \frac{3}{5}$

b) $\log_5(32) = x$

$5^x = 32$

use graphing tech.

→ graph $y = 5^x$

and $y = 32$

$x = 2.153$

c) $\log_3(19) = x$

$3^x = 19$

graph $y = 3^x$

graph $y = 19$

$x = 2.68$

d) $\log(1000) = x$

10

$10^x = 1000$

$10^x = 10^3$

$x = 3$

e) $\log\left(10^6\right) = x$

$10^x = 10^6$

$x = 6$

f) $\boxed{\log_3}\left(3^5\right) = x$

$3^x = 3^5$

$x = 5$

g) $\log(27)$ use the $\boxed{\log}$ button

$= 1.431$

Example 7.3.2

From your text: Pg. 467 #12

Half-life is the time it takes for half of a sample of a radioactive element to decay. The function $M(t) = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$ can be used to calculate the mass remaining if the half-life is h and the initial mass is P. The half-life of radium is 1620 years.

a) If a laboratory has 5 g of radium, how much will there be in 150 years?

b) How many years will it take until the laboratory has only $\boxed{4\text{ g}} = M(t)$ of radium?

a) $M(t) = 5\left(\frac{1}{2}\right)^{\frac{t}{1620}}$

$M(150) = 5\left(\frac{1}{2}\right)^{\frac{150}{1620}}$

$M(150) = 4.69$ grams

b) $4 = 5\left(\frac{1}{2}\right)^{t/1620}$

$y = \uparrow$ $\quad$ $y = \uparrow$

$t = 521.5$ years

Example 7.3.4

From your text: Pg. 467 #13

The function $s(d) = 0.159 + 0.118 \log d$ relates the slope, s, of a beach to the average diameter, d, in millimetres, of the sand particles on the beach. Which beach has a steeper slope: beach A, which has very fine sand with $d = 0.0625$, or beach B, which has very coarse sand with $d = 1$? Justify your decision.

Beach A: $s(0.0625) = 0.159 + 0.118\log(0.0625)$

$= 0.0169$

Beach B: $s(1) = 0.159 + 0.118\boxed{\log(1)} = 0$

$= 0.159$

$\therefore$ Beach B has the steeper slope.

Class/Homework for Section 7.3

Pg. 466 – 468 #4 – 7, 9, 10, 11, 14 – 17, 19

7.4 The Laws of Logarithms

In terms of human understanding, new knowledge can be built from previous knowledge. If this weren't the case, then humans would either have no knowledge at all, or we would prepossess all knowledge without any learning. Now, I know that I personally don't have all knowledge. I'm fairly certain that you do not have all knowledge either. At the same time, I know that I know more today than I did in the past. That is to say, I know that my knowledge has increased. I can only hope that yours has too, at least in terms of mathematics. If it hasn't, then there is only one thing that can be done. Work. Amen and amen.

At this point in our mathematical discussions, we have the knowledge that:

- There are exponent laws
- Logarithms are the inverse of exponentials

We expect, then, that there will be some **Laws of Logarithms** which will **allow us to manipulate logarithms algebraically**. To find (build?) these elusive Log Laws, we will begin by assuming the following to be true:

axioms

1) The Exponent Rules
2) Logarithms are the inverses of exponentials
3) If $M = N$, then $\log_a(M) = \log_a(N)$

$$a^m \cdot a^n = a^{m+n}$$
$$\frac{a^m}{a^n} = a^{m-n}$$
$$(a^m)^n = a^{mn}$$
$$a^0 = 1$$
$$a^{-1} = \frac{1}{a}$$

The Logarithm Laws

1. Evaluate $\log_a(a)$

Let $x = \log_a(a)$

exponent, answer, base

$$a^x = a^1$$
$$x = 1$$

2. Evaluate $\log_a(a^b)$

undoes itself

Let $x = \log_a(a^b)$

$$a^x = a^b$$
$$x = b$$

Rule: $\log_a(a^{\boxed{b}}) = b\log_a(a)$

3. The Product Law of Logarithms

Prove $\log_a(m \cdot n) = \log_a(m) + \log_a(n) \quad a, m, n > 0$

Consider $\log_a(m \cdot n)$ and let $\underbrace{m = a^x}_{\log_a(m) = x}$ and $\underbrace{n = a^y}_{\log_a(n) = y}$

$= \log_a(a^x \cdot a^y)$

$= \boxed{\log_a(a}^{x+y})$ by rule #2

$= x + y$

$= \log_a(m) + \log_a(n)$ $\square$

Note: Logarithm Laws go backwards too!!!

4. The Quotient Law of Logarithms

Prove $\log_a\left(\dfrac{m}{n}\right) = \log_a(m) - \log_a(n) \quad a, m, n > 0$

Let $\boxed{m = a^x}$ and $\boxed{n = a^y}$

$\log_a(m) = x$

$\log_a(n) = y$

$\log_a\left(\dfrac{m}{n}\right) = \log_a\left(\dfrac{a^x}{a^y}\right)$

$= \boxed{\log_a(a}^{x-y})$ undoes itself

$= x - y$

$= \log_a(m) - \log_a(n)$ $\square$

5. The Power Law of Logarithms

$2^3 = 2 \times 2 \times 2$

Prove $\log_a\left(m^n\right) = n \cdot \log_a\left(m\right) \quad a, m > 0$

Consider $\log_a(m^n)$

$= \log_a(\underbrace{m \cdot m \cdot m \cdot m \cdot \ldots \cdot m}_{n \text{ of these}})$

Product Rule of Logs

$= \underbrace{\log_a(m) + \log_a(m) + \ldots + \log_a(m)}_{n \text{ of these}}$

$= n \log_a(m)$

Change of Base

Changing base allows one to use a calculator to evaluate numbers like $\log_3 7$ without graphing, or using "guess and check" (unless you happen to possess a cheater calculator like Patrick Clark). Our goal will be to change the "base 3 log" to a "base 10 log".

Procedure

1) Invert the non-base 10 log to an exponential equation.
2) Take the log $(\log_{10})$ of "both sides"
3) Solve using your calculator.

e.g. Evaluate $\log_3 7 = x$

$3^x = 7$

$\log_{10}(3^{\boxed{x}}) = \log_{10}(7)$

$\dfrac{x \log_{10}(3)}{\log_{10}(3)} = \dfrac{\log_{10}(7)}{\log_{10}(3)}$

$x = \dfrac{\log_{10}(7)}{\log_{10}(3)}$

$x = 1.77\text{ish}$

This idea leads to another Log Law: **The Change of Base Law**

If $x = \log_a(b)$, then $x = \dfrac{\log(b)}{\log(a)}$

$$\boxed{\sqrt{x} = x^{\frac{1}{2}}}$$

Example 7.4.1

From your text: Pg. 475 #6a)

Evaluate $\log_{\underset{a}{25}}\left(\underset{b}{5}^3\right)$

Change the base:

$$x = \frac{\log(5^3)}{\log(25)}$$

$$x = 1.5$$

$$= 3\log_{25}(5)$$

$$= 3\,\boxed{\log_{25}\left(25^{\frac{1}{2}}\right)}$$

undoes itself

$$= 3\left(\frac{1}{2}\right)$$

$$= \frac{3}{2}$$

Example 7.4.2

Evaluate $\log_3(54) - \boxed{\frac{1}{2}}\log_3(36)^{1/2}$

$$= \log_3(54) - \log_3\left(36^{\frac{1}{2}}\right)$$

$$= \log_3(54) - \log_3(6)$$

$$= \log_3\left(\frac{54}{6}\right)$$

$$= \log_3(9) \implies 3^x = 9$$

$$= 2$$

Example 7.4.3

Evaluate $\log_8(2) + 3\log_8(2)^3 + \frac{1}{2}\log_8(16)$

$= \log_8(2) + \log_8(2^3) + \log_8(16^{\frac{1}{2}})$

$= \log_8(2) + \log_8(8) + \log_8(4)$

$= \log_8(2 \cdot 8 \cdot 4)$

$= \log_8(64)$ $\qquad 8^x = 64$

$= 2$

Example 7.4.4

Express as a single logarithm:

a) $\log(3) + \log(9)$

$= \log(27)$

b) $\log_5(6) + \log_5(4) - \boxed{\log_7(7)}$

$= 1 \Rightarrow \log_5(5)$

$= \log_5\left(\frac{6 \cdot 4}{5}\right)$

$= \log_5\left(\frac{24}{5}\right)$

Example 7.4.5

Simplify and evaluate:

a) $\log_4(432) - \log_4(27)$

$= \log_4\left(\frac{432}{27}\right)$

$= \log_4(16)$

$4^x = 16$

$= 2$

b) $\log\left(\sqrt[7]{1000}\right)$

$= \log\left(1000^{\frac{1}{7}}\right)$

$= \frac{1}{7}\log(1000)$

$= \frac{1}{7}(3)$

$= \frac{3}{7}$

Example 7.4.6

Express **IN** **TERMS** **OF** $\log_b(x)$, $\log_b(y)$, and $\log_b(z)$

$$\log_b\left(\frac{z^3y^2}{x^5}\right)$$

$= \log_b(z^3) + \log_b(y^2) - \log_b(x^5)$

$= 3\log_b(z) + 2\log_b(y) - 5\log_b(x)$

Example 7.4.7

Solve the equation:

$$\log_3(x)+\log_3(10)=5\log_3(2)^{5}+\log_3(5)\ -\log_3(10)$$

$$\log_3(x)=\log_3\left(\frac{32\cdot 5}{10}\right)$$

$$\underbrace{\log_3}(x)=\underbrace{\log_3}(16)$$

$$\therefore\ x=16$$

Class/Homework for Section 7.4

Pg. 475 – 476 #2, 4 – 8, 10, 11

7.5 Solving Exponential Equations

Recall that solving an equation means (in usual terms) "getting *x* by itself". That's generally pretty easy to do, for a student in grade 12.

e.g. Solve for *x*: $2x^3 - 8x = 0$

$$2x(x^2-4)=0$$

$2x = 0$

$x = 0$

$x^2 - 4 = 0$

$x^2 = 4$

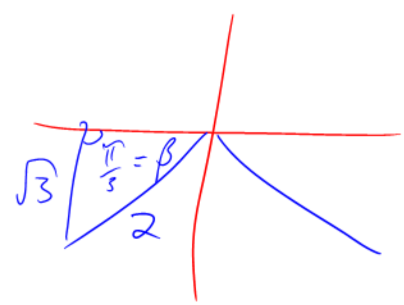
$x = \pm 2$

Easy as pi! However, "getting *x* by itself" **isn't always** a matter of standard **algebra**.

e.g. Solve for *x*: $\sin(x) = -\frac{\sqrt{3}}{2}$

$\therefore x = \frac{4\pi}{3}$

and $x = \frac{5\pi}{3}$



We now turn our attention to **Solving Exponential Equations** ("algebraically").

Example 7.5.1

Solve $5^{x-2} = 1.7$

$$\log(5^{\boxed{x-2}}) = \log(1.7)$$

$$\frac{(x-2)\log(5)}{\log(5)} = \frac{\log(1.7)}{\log(5)}$$

$$x - 2^{+2} = \frac{\log(1.7)}{\log(5)} + 2$$

$$x = 2.33$$

No matter what type of equation, "**getting *x* by itself**" requires using "**inverse operations**". So, for **Exponential Equations**, it's **Logarithms to the rescue**!

$$\log_a b^m = m\log_a b$$

Example 7.5.2

From your text: Pg. 485 #3a

Solve $x = \log_3(243)$

There are two methods for this problem, but one method is very rare…and, yes, I know this is a logarithmic equation in a lesson on exponential equations…but logs and exponentials are related!!!!!!!!!!

$$3^x = 243$$

$$\log(3^{\boxed{x}}) = \log(243)$$

$$x\log(3) = \log(243)$$

$$x = \frac{\log(243)}{\log(3)}$$

change of base

$$x = 5$$

Example 7.5.3

Solve $32^{3x-5} = \left(4^{\frac{5}{2}}\right)^x = 4^{\frac{5x}{2}}$

$$(\quad)\log(32^{3x-5}) = (\quad)\log(4^{\frac{5x}{2}})$$

$$(3x-5)\log(32) = \frac{5x}{2}\log(4)$$

$$3x\log(32) - \boxed{5\log(32)} = \boxed{\frac{5x}{2}\log(4)}$$

Factor

$$3\boxed{x}\log(32) - \frac{5\boxed{x}}{2}\log(4) = 5\log(32)$$

$$x\left(3\log(32) - \frac{5}{2}\log(4)\right) = 5\log(32)$$

$$x = \frac{5\log(32)}{3\log(32) - \frac{5}{2}\log(4)} = \frac{7.5257}{3.01} = 2.5$$

Example 7.5.4

Solve $3^{x-4} = 17^{2x+1}$

$$(x-4)\log(3) = (2x+1)\log(17)$$

$$x\log(3) - 4\log(3) = 2x\log(17) + \log(17)$$

$$x\log(3) - 2x\log(17) = \log(17) + 4\log(3)$$

$$x\left(\log(3) - 2\log(17)\right) = \log(17) + 4\log(3)$$

$$x = \frac{\log(17) + 4\log(3)}{\log(3) - 2\log(17)}$$

$$x = \frac{3.1389}{-1.9838} = -1.58$$

Example 7.5.5

From your text: Pg. 485 #6b

A \$1000 investment is made in a trust fund that pays 12%/a, **compounded monthly**. How long will it take the investment to grow to \$5000?

A_0 $i = \frac{0.12}{12} = 0.01$ $A(n)$

$A(n)$ = Final Amount

A_0 = initial

$1 = 1$

$i = \frac{\text{interest rate}}{\text{compounding periods}}$

n = # of c.p.

$$A(n) = A_0(1+i)^n$$

$$\frac{5000}{1000} = \frac{1000(1.01)^n}{1000}$$

$$5 = (1.01)^n$$

$$\frac{\log(5)}{\log(1.01)} = \frac{n\log(1.01)}{\log(1.01)}$$

$$n = \frac{\log(5)}{\log(1.01)}$$

$n = 162$ months

$n = 162/12 = 13.5$ years.

Example 7.5.6

From your text: Pg. 485 #7

doubling period

A bacteria culture doubles every 15 minutes. How long will it take for a culture of 20 $= P_0$ bacteria to grow to a population of $\underbrace{163\,840}_{P(t)}$ bacteria?

Doubling Formula:

$$P(t) = P_0(2)^{\frac{t}{D}}$$

Half-life Formula:

$$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$P(t) = P_0(2)^{\frac{t}{D}}$$

$$163\,840 = 20(2)^{\frac{t}{15}}$$

$t = 195$ minutes

or $t = 3.25$ hours

Class/Homework for Section 7.5

Pg. 485 – 486 #2, 3cdf, 4, 5, 6acd, 8adf (see Ex 2, Pg. 483), 10

Solve: A) $4^{2x+3} = 9^x$ $x = -7.23$

B) $5^{x-2} = 3^{x+1}$ $x = 8.45$

C) $(0.312)^{2x+1} = 4^{3x-1}$ $x = 0.0341$

7.6 Solving Logarithmic Equations

This is so much fun that it's ridiculous, but there is a

CAUTION

Some solutions may have to be discarded as **INADMISSIBLE**

Remember: Given $f(x) = \log_a(x)$, then the domain is $(0, \infty)$

must be bigger than zero.

And now, for the lesson:

But First…

We will be making use of the following facts:

1) $\log_a(x) \Rightarrow$ [illegible], $x > 0$
2) If $\log_a(M) = \log_a(N)$, then $M = N$
3) $\log_a(m) + \log_a(n) = \log_a(m \cdot n)$
4) $\log_a(m) - \log_a(n) = \log_a\left(\frac{m}{n}\right)$
5) $\log_a(m^n) = n \cdot \log_a(m)$
6) Given $x = \log_a(b)$, then $a^x = b$ (or vice-versa)

$\text{base}^{\text{exponent}} = \text{answer} \quad \| \quad \log_{\text{base}}(\text{answer}) = \text{exponent}$

Example 7.6.1

Solve $\log_2(x) = 2\log_2(5)$

Admissible domain

$x > 0$

$\log_2(x) = \log_2(5^2)$

$x = 25$

Example 7.6.2

Solve $\log(5x-2) = 3$

$10^{3} \; {}^{+2} = 5x - 2 \; {}^{+2}$

$1002 = 5x$

$\frac{1002}{5} = x = 200.4$

Admissible domain

$5x - 2 > 0$

$5x > 2$

$\boxed{x > \frac{2}{5}}$

Example 7.6.3

$\log_x(0.04) = -2$

$x^{-2} = \boxed{0.04} = \frac{4}{100} = \frac{1}{25}$

$\frac{1}{x^2} = \frac{0.04}{1}$

$x^2 = \frac{1}{0.04}$

$\sqrt{x^2} = \sqrt{25}$

$x = \pm 5$

Admissible domain

$x > 0$

$\log_{-3}(5) \Rightarrow \frac{\log(5)}{\log(-3)}$

But, $x = -5$ is a no go.

$\therefore x = 5$

Example 7.6.4

Solve $\log_2(x)+\log_2(3)=3$

Admissible domain

$x>0$

$\log_2(3x)=3$

$\frac{2^3}{3}=\frac{3x}{3}$

$\frac{8}{3}=x$

Example 7.6.5

Solve $3\log(x)-\log(3)=2\log(3)$

Admissible donain

$x>0$

$3\log(x^3)=2\log(3^2)+\log(3)$

$\log(x^3)=\log(9)+\log(3)$

$\log(x^3)=\log(27)$

$x^3=27$

$\therefore x=3$

Example 7.6.6

Solve $\log_6(x)+\log_6(x-5)=2$

Admissible domain

$\left.\begin{array}{l} x>0 \\ x>5 \end{array}\right\}$ must be true at all times.

$\therefore x>5$

$\log_6(x(x-5))=2$

$6^2=x(x-5)$

$36=x^2-5x$

$0=x^2-5x-36$

$0=(x-9)(x+4)$

$\therefore x=9$ or $\cancel{x=-4}$ inadmissible.

Example 7.6.7 **Admissible domain**

Solve $\log_7(x+1)+\log_7(x-5)=1$

$x > -1$

$x > 5$

$\log_7((x+1)(x-5)) = 1$

$7^1 = (x+1)(x-5)$

$7 = x^2 - 4x - 5$

$0 = x^2 - 4x - 12$

$0 = (x-6)(x+2)$

$\therefore x = 6$ and $\cancel{x = -2}$ inadmissible.

Class/Homework for Section 7.6

Pg. 491 – 492 #1bef, 2bcf, 4ade, 5, 7, 9, 10, 11d, 12, 14

7.7 Problem Solving with Exponentials and Logarithms – *An Independent Study*

In this lesson you will have two class periods to **read some examples** and **think about some problems** and **write some solutions**. Your solutions will be collected at the end of the second period.

Exponential Functions and Logarithmic Functions both have a number of real-world applications. In this lesson you will consider the following applications:

A) pH in Chemistry

pH is used as a measure of the acidity or alkalinity of a solution. pH is calculated using the formula $\mathrm{pH} = -\log\left[\mathrm{H}^{+}\right]$, where $\left[\mathrm{H}^{+}\right]$ is the concentration of Hydrogen ions in mol/L in the solution in question.

Read Example 1 on Pgs. 494 – 495

B) The Richter Scale and Earthquake Intensity

Why is an earthquake with a measure of 7.6 on the Richter Scale so much more damaging than a quake with a measure of 4.6 on the same scale? It's only a difference of 3, after-all. The following paragraph is from the website "How Stuff Works:"

The most common standard of measurement for an earthquake is the ***Richter scale****, developed in 1935 by Charles F. Richter of the California Institute of Technology. The Richter scale is used to rate the* ***magnitude*** *of an earthquake -- the amount of energy it released. This is calculated using information gathered by a* ***seismograph****. The Richter scale is* ***logarithmic****, meaning that whole-number jumps indicate a tenfold increase. In this case, the increase is in wave amplitude. That is, the wave amplitude in a level 6 earthquake is 10 times greater than in a level 5 earthquake, and the amplitude increases 100 times between a level 7 earthquake and a level 9 earthquake.*

Read Example 2 on Pg. 496

C) Growth or Decay Rate problems

There are many sort of applications here. Financial applications, population growth and radioactive decay are the sort we see most often in High School. The **basic growth/decay formulas** are:

Growth – $A(t) = A_0 (1+r)^t$

Decay – $A(t) = A_0 (1-r)^t$

Where (in both formulas) $A(t)$ is the 'amount of stuff' at time *t*, A_0 is the initial amount of stuff, and *r* is the growth/decay rate. **Note that the formulas are exponential, and so one may need logarithms to solve if looking for *t*.**
Of course, these formulas can be modified to better describe the situation being modelled as growth or decay.
Read Example 3 on Pgs. 496 – 497

D) Decibels and Sound Intensity

You've all probably heard that people listening to music using earbuds are in danger of **permanent hearing loss**. If you are interested, read the article found at: http://www.nidcd.nih.gov/health/hearing/pages/noise.aspx
Long-time exposure to sound over 85 dB can cause hearing loss.
Read Example 4 on Pgs. 497 – 498

You are expected to submit solutions to the following problems at the end of class on Friday.

Pg. 499 – 490 #1, 2, 3, 4, 5bd, 6ac, 8, 10, 13, 14, 15, 17, 18