MTH1W Analytic Geometry

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Lesson 8.3: Unique Slopes and Lines

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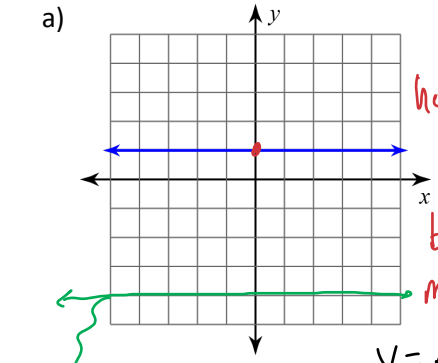
Learning Goal: We are learning the properties of horizontal, vertical, parallel and perpendicular lines.

Horizontal and Vertical Lines: Given the graph, determine the equation of the line:

a)



horizontal

b = 1

m = 0

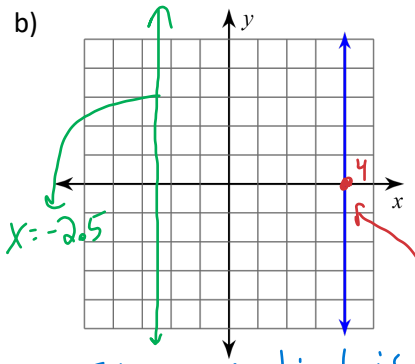
$y = mx + b$

$y = 0x + 1$

$y = 1$

$y = -4$

b)



m = undefined

b = none

$y = mx + b$ fails ∴

since vertical is opposite of horizontal

↳ x = x-int ∴ it is x = 4

must memorize!

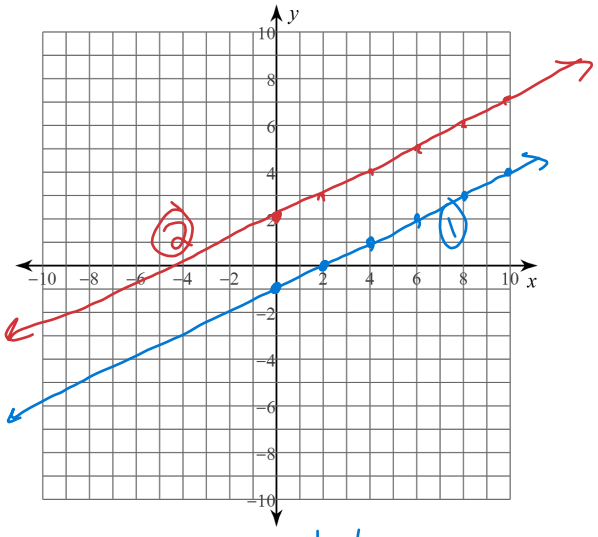
→ horizontal equation are $y = b$

y-int.

Graph the following two lines on the same grid.

① $y = \frac{1}{2}x - 1$ $m = \frac{1}{2}$ $b = -1$

② $y = \frac{1}{2}x + 2$ $m = \frac{1}{2}$ $b = 2$

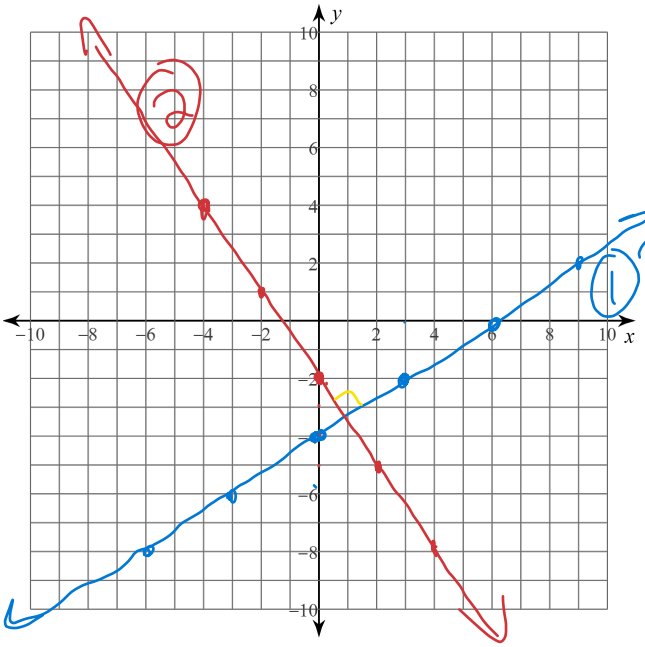


These lines are parallel, meaning that their slopes are equal. In fact, if you have two equations and you want to know if they are parallel, just find their slope

Now, graph these two lines on the same grid.

(1) $y = \frac{2}{3}x - 4$ $m = \frac{2}{3}$ $b = -4$

(2) $y = -\frac{3}{2}x - 2$ $m = -\frac{3}{2}$ $b = -2$



This time, the lines do intersect. However, it is not the point of intersection that is important, it is the angle at which these lines are intersecting each other which is important. These lines are crossing at a 90 degree angle. We call these lines perpendicular. Just with parallel lines, it is the slopes that help us determine whether lines are perpendicular.

The slope of the first line is:

$m = \frac{2}{3}$

The slope of the second line is:

$m = -\frac{3}{2}$

$\frac{2}{3} \times \frac{-3}{2}$

$= \frac{-6}{6}$

$= -1$

These slopes are called the negative reciprocal of each other. This means that one slope is negative and one slope is positive. Reciprocal means that the fraction is flipped around.

In general terms, we write:

$m = \frac{a}{b}$ then $m_{\perp} = \frac{-b}{a}$

perpendicular.

$m \times m_{\perp} = -1$

Example: Determine the slope perpendicular to the given slope:

a) $m = \frac{-3}{4}$

$m_{\perp} = \frac{4}{3}$

b) $m = \frac{8}{1}$

$m_{\perp} = \frac{-1}{8}$

c) $m = \frac{12}{23}$

$m_{\perp} = -\frac{23}{12}$

d) $m = 0$ horizontal

$m_{\perp}$ = undefined Vertical

Example: Determine the slopes of each line to determine if they are parallel, perpendicular, or neither.

a) ① $y = \frac{2}{3}x - 6$ → $m_1 = \frac{2}{3}$

② $2x + 3y + 8 = 0$

$\frac{3y}{3} = \frac{-2x}{3} - \frac{8}{3}$

$y = -\frac{2}{3}x - \frac{8}{3}$ → $m_2 = -\frac{2}{3}$

> neither.

b) ① $8x - 2y = 7$ ⟹ $\frac{8x}{2} - \frac{7}{2} = \frac{2y}{2}$

$4x - 3.5 = y$

② $\frac{4y}{4} = \frac{16x}{4} + \frac{3}{4}$

$y = 4x + 0.75$

$m_1 = 4$

$m_2 = 4$

> parallel

c) ① $5x + 3y - 18 = 0$

$\frac{3y}{3} = \frac{-5x}{3} + \frac{18}{3}$

$y = -\frac{5}{3}x + 6$

② $y = \frac{3}{5}x + 2$

$m_1 = \frac{-5}{3}$

$m_2 = \frac{3}{5}$

> perpendicular!

Always reduce the fractions to lowest terms.

$m = \frac{10}{2}$ $m = 5$

Success Criteria:

- I can determine the equation of a vertical and horizontal line
- I can determine if two lines are parallel by seeing if they have the same slope
- I can determine if two lines are perpendicular if their slopes are negative reciprocals of each other