Math 9 – Unit 2: Algebra One

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Lesson #1: Collecting Like Terms

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In this unit, you will be introduced to one of the most important component to Mathematics: Algebra. Algebra comes from the Arabic word *"al-jabr"*, meaning "the coming together of broken parts", and math is about bringing together ideas to solve problems. In Algebra, we will look at how to use Mathematical symbols and the rules for manipulating them. Typically, the symbols are letters.

To begin, let's define some terminology that is important in Algebra.

Expression: is like a math sentence, made up of terms and +/- signs ex: $2x^2 - 3xy + 4$

Variable: is the letters used as placeholders for numbers.
- "unknown" → the value varies

Coefficient: is the sign (+/-) and number infront of a variable

ex: $2xy$ $-8w$ $+5$

Constant:
- is a number without a variable
- never changing

Like terms:
- a term is one item from an expression
- it is made up of coeffients and variable.
- terms are separated by + and − signs.

ex: $2x + 3y - 5z$

→ like terms are terms with the exact same variable and exponent combination.

Unlike terms:

ex: $3xy$ and $4yx$ are like terms.

ex: $3x^2y$ and $4x^2y$ are like terms

ex: $4xw$ and $3y$ are unlike terms

ex: $2x^2y$ and $5xy^2$ are unlike terms

Example: Given the following expressions, state the number of terms, the coefficients, and the constant term.

a) $3x^2 - 5x + 7$

3 terms

3 and -5 are coefficients

7 is the constant term

b) $-5y + 10x + 8 - 12y$

4 terms

-5, 10, -12 are coefficients

8 is the constant term.

In the above example, the second expression has 4 terms, but two of them had the same variable. This means that we can combine them together. All you need to do is add, or subtract, their coefficients. This process is called collecting like terms.

Collect the like terms in the above example: $-5y + 10x + 8 - 12y$

put these together: $-5 - 12$

$= -17y + 10x + 8$

More examples:

a) $-6 - 3r^2 - 4r + 2 + 6r$

$= -4 - 3r^2 + 2r$

$= -3r^2 + 2r - 4$

b) $-4k^3 - 8k^2 + 4 + 7k^4 - k^3 - 8k^2 - 1$

$= -5k^3 - 16k^2 + 3 + 7k^4$

$= 7k^4 - 5k^3 - 16k^2 + 3$

c) $7a^2b^2 + 2a^4 - 8a^3b^3 - 4a^2b - 2a^4 - 2a^3b^3 + 8a^2b^2$

$= 15a^2b^2 - 10a^3b^3 - 4a^2b$

$= -10a^3b^3 + 15a^2b^2 - 4a^2b$

Now for a super duper big example:

d) $-8x - x^2y^2 - 8x^3y^5 + 3x^3y + 2x^3y + 6x + 2x^2y^2 + 2xy - 2x^2y^2 + 5x^3y^4 + 3xy + 5x$

$= 3x - x^2y^2 - 8x^3y^5 + 5x^3y + 5xy + 5x^3y^4$

$= -8x^3y^5 + 5x^3y^4 + 5x^3y - x^2y^2 + 5xy + 3x$

There's more! Did you ask, "what term should I write first?" If you did, good thinking! There is a definite order to writing out expressions. It is called descending order.

Descending order is: writing the expression starting with the highest exponent on the variable which comes first alphabetically, then you work your way down, ending with the constant term.

Now go back to the above examples and put them in descending order.

When an expression is simplified, we call that a polynomial, meaning many terms. However, we also have special names for expressions with one, two, or three terms.

$4x^2$ is called a monomial.

$3x^5 - 2xy$ is called a binomial.

$7y^2 + 5y - 1$ is called a trinomial.

Anything over that we just call a polynomial.

Finally, all polynomials have a degree, which is just the largest exponent.

Examples: For each expression, collect the like terms, state what type of polynomial, and the degree.

a) $-2v - 2v^5 - 8 + 2v^5 + 7v$

$= 5v^1 - 8$

Binomial

Degree: 1.

b) $3xy - 4x^2y + 8x^4y + 6xy - 7x^2y - 7x^4y$

$= 9xy - 11x^2y + x^4y$

$= x^4y^1 - 11x^2y^1 + 9x^1y^1$

Trinomial, with degree 5

because $x^4y^1 \Rightarrow 4+1=5$

c) $\frac{7}{4}x^5 - \frac{2}{3}x^4 - \frac{4}{3}x^4 + \frac{6}{7}x^5$

$= \frac{7(7)}{4(7)}x^5 + \frac{6(4)}{7(4)}x^5 - \frac{2}{3}x^4 - \frac{4}{3}x^4$

$= \frac{49}{28}x^5 + \frac{24}{28}x^5 - \frac{6}{3}x^4$

$= \frac{73}{28}x^5 - 2x^4$

Binomial with degree 5