Math 9 – Coordinate Geometry Name: Mr. Hagen

Lesson #3: Slope of a Line - Notes Date: March 1, 2018

In this lesson, we will explore the most significant property of a linear relationship: the slope! The slope of a line tells us how the relationship is changing and can be thought of as how slanted/steep the line is. It has many important applications such as engineering the initial climb of a roller coaster to making safe ramps, but today we will focus on the algebra and understanding how to calculate the slope of a line.

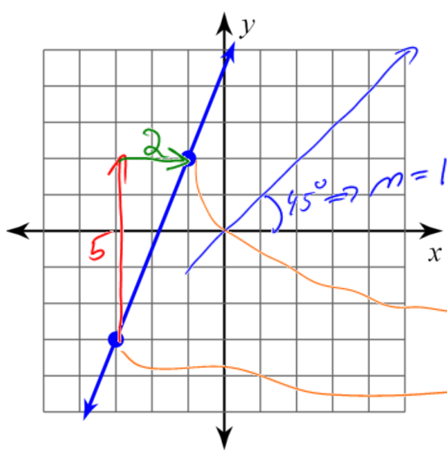
First, let's look at the slope from a geometric perspective. The slope, defined by the letter *m* for no apparent reason, is: $m = \frac{Rise}{Run}$

Rise → how far are you increasing or decreasing vertically

Run → how far are you increasing or decreasing horizontally

m → move

Example 1: Given the line with two points, calculate the slope.



$m = \frac{Rise}{Run}$

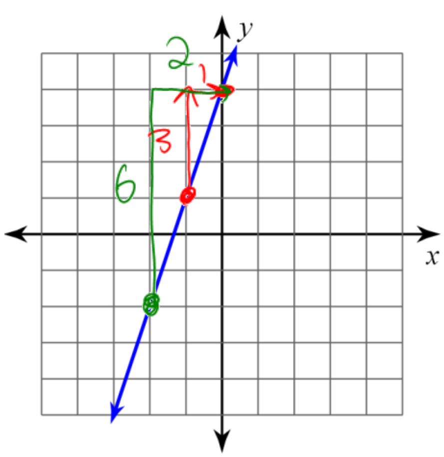
$m = \frac{5}{2}$

$(-1, \boxed{2})$ =5

$(-3, \boxed{-3})$

$\overset{y}{2} - \overset{y}{(-3)} = 5$

Example 2: Given the line, locate two points, then calculate the slope.

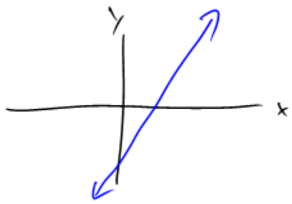


$m = \frac{3}{1} = 3$

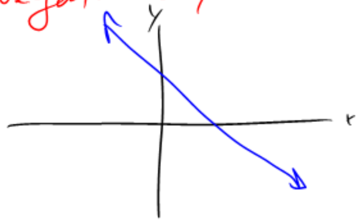
$m = \frac{6}{2} = 3$

Are slopes always positive? There are 4 possible slopes:

① Positive Slope

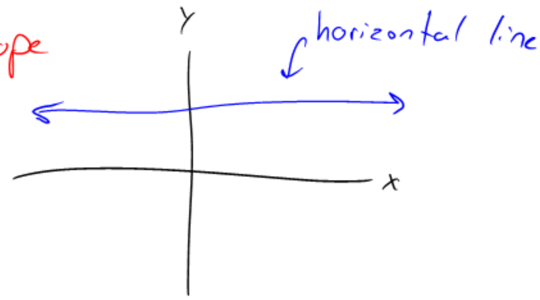


② Negative Slope



③ Zero Slope

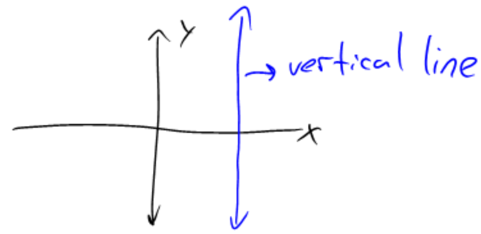
$m = 0$



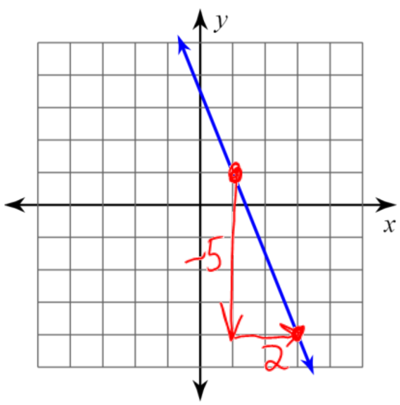
④ undefined slope

$m = \infty$

$m = \frac{\#}{0}$

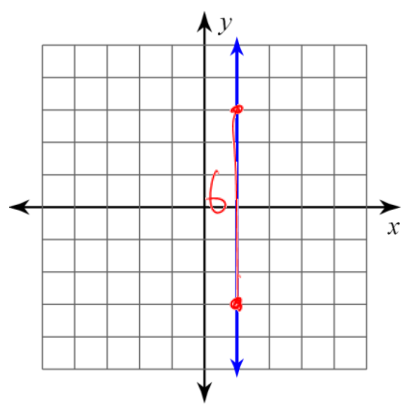


Example 3 and 4: Calculate the slopes of each line.



$m = \frac{Rise}{Run}$

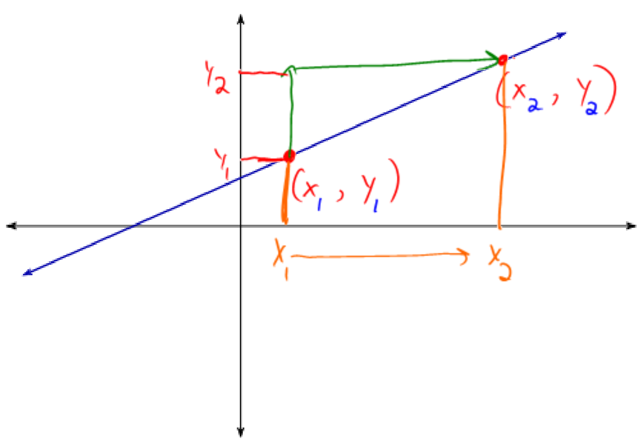
$m = \frac{-5}{2}$



$m = \frac{6}{0}$

$m =$ undefined

Now that we know about slope, we can derive a formula so that we do not need a graph.



$m = \frac{Rise}{Run}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ ✱ memorize

$m = \frac{\Delta y}{\Delta x}$ → delta, meaning "the change in"

"Rise rhymes with y's"

Examples 5-8: Given the points, calculate the slope.

5. $\boxed{(7,-10)}, \boxed{(9,-7)}$

$x_1 \quad y_1 \quad x_2 \quad y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-7 - (-10)}{9 - 7}$$

$$m = \frac{3}{2}$$

6. $\boxed{(-6,-17)}, \boxed{(-20,11)}$

$x_1 \quad y_1 \quad x_2 \quad y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{11 - (-17)}{-20 - (-6)} = \frac{28}{-14} = -2$$

7. $\boxed{(6,-12)}, \boxed{(6,1)}$

$x_1 \quad y_1 \quad x_2 \quad y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{1 - (-12)}{6 - 6} = \frac{13}{0}$$: undefined!

8. $(-3,9), (3,9)$

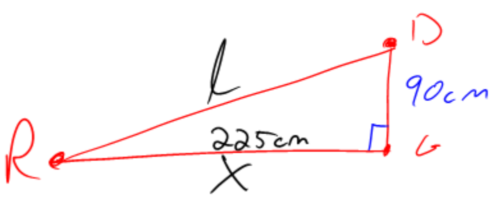
$1 \quad 2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{9 - 9}{3 - (-3)} = \frac{0}{6} = 0$$

Example 9: A ramp needs to be constructed to go from the ground to a doorway. The doorway is 90 cm from the ground and the ramp needs a slope of $\frac{2}{5}$.

a) Calculate how far the ramp will start from the edge of the house.



$$m = \frac{2}{5} = \frac{\text{Rise}}{\text{Run}}$$

cross multiply

$$\frac{2}{5} = \frac{90}{x}$$

$$\frac{2x}{2} = \frac{450}{2}$$

$$x = 225\text{cm}$$

b) Calculate the length of the ramp.

$$a^2 + b^2 = c^2$$

$$(225)^2 + (90)^2 = c^2$$

$$50625 + 8100 = c^2$$

$$\sqrt{58725} = \sqrt{c^2}$$

$$c = 242.33\text{cm}$$

Example 10 and 11: Calculate the missing coordinate.

10. $\underset{x_2\ \ y_2}{(2, y)}$ and $\underset{x_1\ \ y_1}{(-3, -2)}$; slope: $\frac{3}{5} = m$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{3}{5} = \frac{y - (-2)}{2 - (-3)}$$

$$\cancel{5}\left(\frac{3}{\cancel{5}}\right) = \cancel{5}\left(\frac{y + 2}{\cancel{5}}\right)$$

$$3 \ {}^{-2} = y + 2 \ {}^{-2}$$

$$1 = y$$

$\therefore$ the missing coordinate is $y = 1$.

11. $\underset{x_2,\ y_2}{(x, 4)}$ and $\underset{x_1\ \ y_1}{(-5, 10)}$; slope: $\frac{3}{2} = m$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{3}{2} = \frac{4 - 10}{x - (-5)}$$

$$\frac{3}{2} = \frac{-6}{(x + 5)}$$

$$3(x + 5) = -12$$

$$3x + 15 \ {}^{-15} = -12 \ {}^{-15}$$

$$\frac{3x}{3} = \frac{-27}{3}$$

$$x = -9$$

$\therefore$ the missing coordinate is $x = -9$.