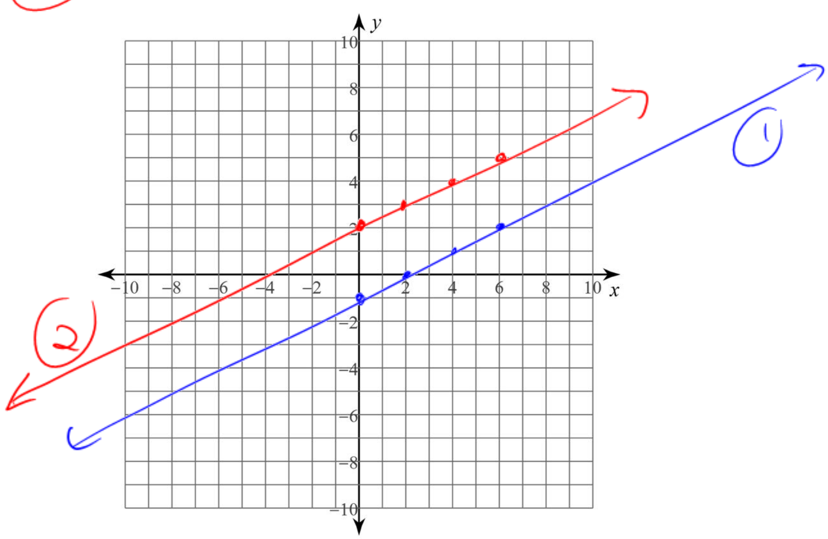
Math 9 – Analytic Geometry Name: Mr. Hagen

Lesson #5: Parallel and Perpendicular Slopes – Notes Date: March 27, 2018.

Graph the following two lines on the same grid.

① $y=\frac{1}{2}x\boxed{-1}$ b

② $y=\frac{1}{2}x+2$



These lines are _parallel_, meaning that their _slope_ are _equal._. In fact, if you have two equations and you want to know if they are parallel, just find their slopes.

Example: Determine the slopes of each line to determine if they are parallel or not.

a) ① $y=\frac{2}{3}x-6$ $m_1=\frac{2}{3}$

② $4x\boxed{-3y}+9=0$ $+3y$

need $y=$

$\frac{4x}{3}+\frac{9}{3}=\frac{3y}{3}$

$y=\frac{4}{3}x+3$ $m_2=\frac{4}{3}$

$\frac{2}{3}\neq\frac{4}{3}$ ∴ not parallel

b) ① $8x-2y=7$ $+2y$ -7 $\Longrightarrow$ $\frac{2y}{2}=\frac{8x}{2}-\frac{7}{2}$

$y=4x-\frac{7}{2}$

$m_1=4$

② $\frac{4y}{4}=\frac{16x}{4}+\frac{3}{4}$

$y=4x+\frac{3}{4}$

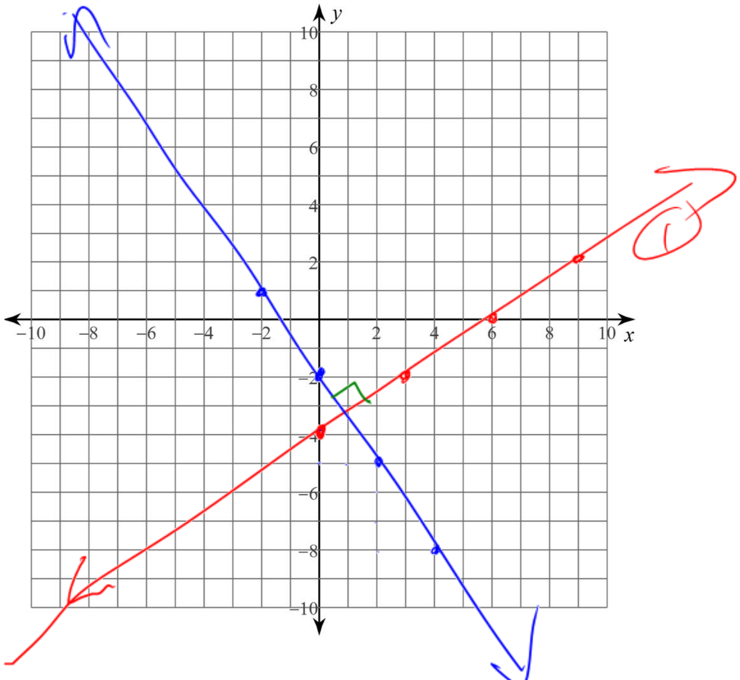
$m_2=4$

$4=4$ ∴ parallel!

Now, graph these two lines on the same grid.

① $y = \frac{2}{3}x - 4$ ↑2 → 3

② $y = -\frac{3}{2}x - 2$ ↓3 → 2



This time, the lines do intersect. However, it is not the point of intersection that is important, it is the angle at which these lines are intersecting each other which is important. These lines are crossing at a 90 degree angle. We call these lines perpendicular. Just with parallel lines, it is the slopes that help us determine whether lines are perpendicular.

The slope of the first line is: $m_1 = \frac{2}{3}$

The slope of the second line is: $m_2 = \frac{-3}{2}$

These slopes are called the negative reciprocal of each other. This means that one slope is negative and one slope is positive. Reciprocal means that the fraction is flipped around.

In general terms, we write:

$$m = \frac{a}{b} \qquad m_\perp = \frac{-b}{a}$$

↑ symbol for perpendicular

Example: Determine the slope perpendicular to the given slope:

a) $m = \frac{-3}{4}$

$m_\perp = \frac{4}{3}$

b) $m = 8$

$m_\perp = \frac{-1}{8}$

c) $m = \frac{12}{23}$

$m_\perp = \frac{-23}{12}$

d) $m = 0$ → horizontal

$\cancel{m_\perp = \frac{-1}{0}}$

$m_\perp$ = undefined → vertical

Now for the big questions. The goal of these questions is to create an equation with properties taken from other equations. Remember, to create an equation of a line, we need a slope and a point.

1. Create a line in Standard Form which is parallel to $y=\frac{4}{5}x-8$ and has the same x-intercept as $2x-3y+8=0$.

① Slope → parallel to $y=\frac{4}{5}x-8$

$m=\frac{4}{5}$

∴ our slope is also $\frac{4}{5}=m$

② Point → x-int of $2x-3y+8=0$

x-int, $y=0$

$2x-\cancel{3(0)}+8=0$; -8 from both sides

$\frac{2x}{2}=\frac{-8}{2}$

$x=-4$

(x_1, y_1)

∴ $(-4,0)$ is our point

③ Equation Time

$y-y_1=m(x-x_1)$

$(y-0)=\frac{4}{5}(x+4)$

$5y = 4x+16$; $-5y$ from both sides

$$\boxed{4x-5y+16=0}$$

2. Create a line in slope-intercept form which is perpendicular to $3x+5y+2=0$ and goes through the point (6,1).

$y=mx+b$

① Slope: ⊥ to $3x+5y+2=0$

$3x+5y+2=0$; $-3x$, -2

$\frac{5y}{5}=\frac{-3x}{5}-\frac{2}{5}$

$y=\frac{-3}{5}x-\frac{2}{5}$

$m=\frac{-3}{5}$

$$\boxed{m_{\perp}=\frac{5}{3}}$$

② Point → $(6,1)$

③ Equation Time

$y-y_1=m(x-x_1)$

$y-1=\frac{5}{3}(x-6)$

$3y-3=5x-30$; $+3$, $+3$

$\frac{3y}{3}=\frac{5x}{3}-\frac{27}{3}$

$$\boxed{y=\frac{5}{3}x-9}$$

3. Create a line in Slope-Intercept Form that has the same y-intercept as $4x-7y=35$ and is parallel to $5x-9y+27=0$.

$y=mx+b$

Slope thing

① Slope → parallel to $5x-9y+27=0$

$$5x-9y+27=0 \quad +9y$$

$$\frac{5x}{9}+\frac{27}{9}=\frac{9y}{9}$$

$$y=\frac{5}{9}x+3$$

$$\boxed{m=\frac{5}{9}}$$

② Point → y-int from

$$4x-7y=35$$

y-int, $x=0$

$$\frac{4(0)-7y}{-7}=\frac{35}{-7}$$

$$y=-5$$

$\therefore (0,-5)$

③ Equation Time

$$y=mx+\boxed{b}$$

$$y=\frac{5}{9}x-5$$

$$y-y_1=m(x-x_1)$$

$$y+5=\frac{5}{9}(x-0)$$

$$y+5=\frac{5}{9}x$$

$$y=\frac{5}{9}x-5$$

4. Create a line in Standard Form that is perpendicular to the slope formed by the points (5,2) and (-1,8) and goes through the origin.

① Slope: $\perp$ to $(5,2),(-1,8)$

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$m=\frac{8-2}{-1-5}$$

$$m=\frac{6}{-6}$$

$$m=-\frac{1}{1}$$

$$\boxed{m_{\perp}=1}$$

② Point → origin is $(0,0)$

③ Equation Time

$$y-y_1=m(x-x_1)$$

$$y-0=1(x-0)$$

$$y=1(x)$$

$$y-y=x-y$$

$$0=x-y$$

or

$$\boxed{x-y=0}$$

Thursday, April 5 is the Analytic Geometry Test

Wednesday, April 18 is the Exam, covering only the 2nd half.

What we have learned.

① Slope $= \underline{m} = \dfrac{\text{Rise}}{\text{Run}} = \dfrac{y_2 - y_1}{x_2 - x_1}$

② Slope-intercept Form: $y = mx + b$

↳ slope ↳ y-intercept.

③ Standard Form: $Ax + By + C = 0$

→ A, B, and C are integers

→ A is positive

④ x-int, meaning $y = 0$

$(x, 0)$

⑤ y-int, meaning $x = 0$

$(0, y)$

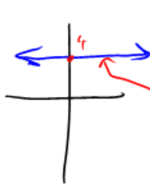
⑥ Point-slope form $y - y_1 = m(x - x_1)$

→ we use it to create the other forms.

⑦ Parallel slopes → slopes are equal.

⑧ Perpendicular slopes $m = \dfrac{a}{b}$ $m_{\perp} = \dfrac{-b}{a}$

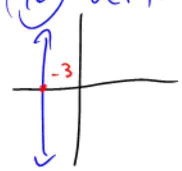
⑨ Horizontal Lines



$m = 0$

$y = 4$ is the equation

⑩ Vertical Lines



$m =$ undefined

$x = -3$