

Lesson #1: Collecting Like Terms

In this unit, you will be introduced to one of the most important component to Mathematics: Algebra. Algebra comes from the Arabic word “*al-jabr*”, meaning “the coming together of broken parts”, and math is about bringing together ideas to solve problems. In Algebra, we will look at how to use Mathematical symbols and the rules for manipulating them. Typically, the symbols are letters.

Learning Goal: We are learning common math terminology, and using those terms to simplify algebraic expressions.

To begin, let's define some terminology that is important in Algebra.

Expression: is a mathematical sentence, made up of terms and $+/-$ signs.
ex: $2x^2 - 3xy + 4$

Variable: is the letters used as placeholders for numbers.
↳ the unknown.

Coefficient: is the sign ($+$ / $-$) and number in front of a variable

ex: $\boxed{2}xy \boxed{-} 8w \boxed{+} 5$

Constant:

- a number without a variable

- never changing

Like terms:

↳ a term is one item from an expression
↳ it is made up of coefficients and variables
↳ separated by $+/-$ signs

ex: $\boxed{2}x \boxed{+} \boxed{3}y \boxed{-} 8z$

Unlike terms: like terms are terms with the exact same variable and exponent combination.

$4xy$ and $3yx$
are like terms

ex: $3x^2y$ and $4x^2y$ are like terms
 $4xw$ and $3y$ are unlike terms
 $2x^2y$ and $3xy^2$ are unlike terms

Example: Given the following expressions, state the number of terms, the coefficients, and the constant term.

a) $3x^2 - 5x + 7$

3 terms

3 and -5 are coefficients

7 is the constant

b) $-5y + 10x + 8 - 12y$

4 terms

-5, 10 and -12 are coefficients

8 is the constant.

In the above example, the second expression has 4 terms, but two of them had the same variable. This means that we can combine them together. All you need to do is add, or subtract, their coefficients. This process is called **collecting like terms**.

Collect the like terms in the above example:

$$\begin{array}{r} -5y + 10x + 8 - 12y \\ \hline -5 - 12 = -17 \end{array}$$

$$-17y + 10x + 8$$

More examples:

a) $\underline{-6} - 3r^2 - 4r + 2 + 6r$

$$= -4 - 3r^2 + 2r$$

$$= -3r^2 + 2r - 4$$

c) $\underline{7a^2b^2} + \underline{2a^4} - \underline{8a^3b^3} - \underline{4a^2b} - \underline{2a^4} - \underline{2a^3b^3} + \underline{8a^2b^2}$

$$= 15a^2b^2 - 10a^3b^3 - 4a^2b$$

b) $\underline{-4k^3} - \underline{8k^2} + \underline{4} + \underline{7k^4} - \underline{k^3} - \underline{8k^2} - \underline{1}$

$$= -5k^3 - 16k^2 + 3 + 7k^4$$

$$= 7k^4 - 5k^3 - 16k^2 + 3$$

Now for a super duper big example:

$$\begin{aligned} \text{d) } & -8x - \underline{x^2y^2} - \underline{8x^3y^5} + \underline{3x^3y} + \underline{2x^3y} + 6x + \underline{2x^2y^2} + \underline{2xy} - \underline{2x^2y^2} + \underline{5x^3y^4} + \underline{3xy} + \underline{5x} \\ & = 3x - x^2y^2 - 8x^3y^5 + 5x^3y + 5xy + 5x^3y^4 \end{aligned}$$

There's more! Did you ask, "what term should I write first?" If you did, good thinking! There is a definite order to writing out expressions. It is called **descending order**.

Descending order is: writing out the expression starting with the highest exponent, then work your way down to the constant term.

* only do this on expressions with one variable.



Now go back to the above examples and put them in descending order.

When an expression is simplified, we call that a polynomial, meaning many terms. However, we also have special names for expressions with one, two, or three terms.

$4x^2$ is called a monomial.

$3x^5 - 2xy$ is called a binomial.

$7y^2 + 5y - 1$ is called a trinomial.

Anything over that we just call a polynomial.

Finally, all polynomials have a **degree**, which is just the term largest exponent (5).

Examples: For each expression, collect the like terms, state what type of polynomial, and the degree.

a) $\underline{-2v} - \underline{2v^5} - 8 + \underline{2v^5} + \underline{7v}$

$$= 5v^1 - 8$$

Binomial

Degree: 1

b) $\underline{3xy} - \underline{4x^2y} + \underline{8x^4y} + \underline{6xy} - \underline{7x^2y} - \underline{7x^4y}$

$$= 9xy^1 - 11x^2y^1 + x^4y^1$$

Trinomial with degree 5.
(4+1)

c) $\frac{7}{4}x^5 - \frac{2}{3}x^4 - \frac{4}{3}x^4 + \frac{6}{7}x^5$

$$= \frac{7}{4}x^5 + \frac{6}{7}x^5 - \left[\frac{2}{3}x^4 + \frac{4}{3}x^4 \right]$$

$$= \left[\frac{49}{28}x^5 + \frac{24}{28}x^5 \right] - \left[\frac{6}{3}x^4 \right]$$

$$= \frac{73}{28}x^5 - 2x^4$$

Binomial with degree 5

Success Criteria:

- I can correctly define the following terms: expression, variable, coefficient, constant, like term, unlike term, monomial, binomial, trinomial, polynomial, and degree
- I can group like terms within algebraic expressions
- I can identify the degree and type of various polynomials