

Lesson #4: More Distributive Property and Powers of Monomials**Learning Goal:** We are learning to expand and simplify more complicated expressions.

Let's start off by continuing our lesson on the Distributive Property. Take a look at the following questions:

Expand AND simplify (put your answers in descending order):

a) $3x(4x^2 - 7x + 2) + 4x^2(2x - 3)$

$$= \underline{12x^3} - \underline{21x^2} + 6x + \underline{8x^3} - \underline{12x^2}$$

$$= 20x^3 - 33x^2 + 6x$$

b) $-4y^2(3y^2 - 5) - 5y^3(6 + y)$

$$= \underline{-12y^4} + 20y^2 - 30y^3 - \underline{5y^4}$$

$$= -17y^4 - 30y^3 + 20y^2$$

c) $3mn(2m - 7n) - 5m^2(4n + 8) + 6n^2(3m - n)$

$$= \underline{6m^2n} - \underline{21mn^2} - \underline{20m^2n} - 40m^2 + \underline{18mn^2} - 6n^3$$

$$= -14m^2n - 3mn^2 - 40m^2 - 6n^3$$

Now we are going to go back to discussing monomials. How do we simplify $(3x^2y^5)^3$? This is called a monomial raised to a power. How does the outside exponent affect the question? First, how does it work with just a number?

$$\text{Simplify } (4^3)^2 = (4^3)(4^3) = (4 \times 4 \times 4)(4 \times 4 \times 4) = 4^6$$

The initial exponents were 3 and 2, with the final exponent a 6. So, $3 \times 2 = 6$! This leads to our second exponent law. When raising a power to a power, multiply the exponents. Try it out!

$$\begin{aligned} \text{a) } (x^4)^5 \\ = x^{20} \end{aligned}$$

$$\begin{aligned} \text{b) } (y^2)^8 \\ = y^{16} \end{aligned}$$

$$\begin{aligned} \text{c) } (m^3n^6)^4 \\ = m^{12}n^{24} \end{aligned}$$

That's all well and good (hopefully), but how do you handle a question with a coefficient?

Consider the expression from before, $(3x^2y^5)^3$. Expand it without using the laws.

$$\begin{aligned} &= (3x^2y^5)(3x^2y^5)(3x^2y^5) \\ &= 27x^6y^{15} \end{aligned}$$

The coefficient was just raised to the power of 3! Awesome. Try out some more, this time following the laws.

$$\begin{aligned} \text{a) } (2x^4y^2)^5 \\ = 32x^{20}y^{10} \end{aligned}$$

$$\begin{aligned} \text{b) } (-3m^7n)^2 \\ = 9m^{14}n^2 \end{aligned}$$

$$\begin{aligned} \text{c) } (5a^2b^3c^4d^5)^6 \\ = 15625a^{12}b^{18}c^{24}d^{30} \end{aligned}$$

$$\begin{aligned} \text{d) } (3x^2y^5)^2(2xy^3) \\ = (9x^4y^{10})(2xy^3) \\ = 18x^5y^{13} \end{aligned}$$

$$\begin{aligned} \text{e) } (-4m^3n^2)^3(3m^4n^3)^2 \\ = (-64m^9n^6)(9m^8n^6) \\ = -576m^{17}n^{12} \end{aligned}$$

multiply exponents...
add exponents

$$\text{f) } (5x^2y^4z^6)^0 \quad \text{Whoah!! Exponent of zero? How does that work?}$$

There are multiple explanations. We will look at a pattern, starting with 4^1 then moving up the ladder.

$$\begin{aligned} 4^4 &= 256 \quad \uparrow \times 4 \quad 2 \div 4 \\ 4^3 &= 64 \quad \uparrow \times 4 \quad 2 \div 4 \\ 4^2 &= 16 \quad \uparrow \times 4 \quad 2 \div 4 \\ 4^1 &= 4 \quad \uparrow \times 4 \quad 2 \div 4 \\ 4^0 &= 1 \quad 2 \div 4 \\ 4^{-1} &= \frac{1}{4} \end{aligned}$$

As you move up the ladder, you keep multiplying by 4. If you were to go down the ladder, you would divide by 4. Follow the pattern to determine what four to the power of zero is.

This leads to another exponent law: Anything to the power of zero is equal to 1.

$$(5x^2y^4z^6)^0 = 1$$

$$\frac{1}{4} \div 4 = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

$$4^{-2} = \frac{1}{16} = \frac{1}{4^2}$$

$$4^{-3} = \frac{1}{64} = \frac{1}{4^3}$$

$$\begin{aligned} (2x^3y^4)^0(3x^2y^5)^2 \\ = 9x^{10}y^{10} \end{aligned}$$

$$(3x^2)^4 = 81x^8$$

Success Criteria:

- I can use the distributive property to multiply a polynomial with a monomial ✓
- I can use the distributive property to combine multiple variables into a single term ✓
- I can simplify a monomial raised to a power by multiplying the exponents of each variable ✓
- I can recognize that when a coefficient is raised to a power, it is NOT NOT NOT multiplied ✓
- I can understand that raising to the power of zero equals one. ✓