Math 9 – Analytic Geometry Name: Mr. Hagen

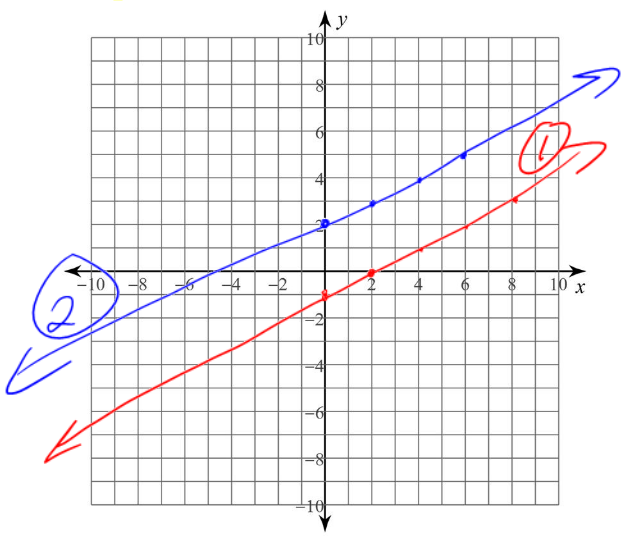
Lesson #5: Parallel and Perpendicular Slopes – Notes Date: March 25, 2019

Learning Goal: We are learning the properties of parallel and perpendicular lines.

Graph the following two lines on the same grid.

(1) $y = \frac{1}{2}x - 1$ m, b

(2) $y = \frac{1}{2}x + 2$ b



These lines are parallel, meaning that their slopes are equal or the same. In fact, if you have two equations and you want to know if they are parallel, just find their slopes.

Example: Determine the slopes of each line to determine if they are parallel or not.

a) $y = \frac{2}{3}x - 6$ $m_1 = \frac{2}{3}$

$4x - 3y + 9 = 0$

$\frac{4x}{3} + \frac{9}{3} = \frac{3y}{3}$

$y = \frac{4}{3}x + 3$ $m_2 = \frac{4}{3}$

$m_1 \neq m_2 \therefore$ these lines are not parallel

b) $8x - 2y = 7$

$\frac{8x}{2} - \frac{7}{2} = \frac{2y}{2}$

$y = 4x - \frac{7}{2}$

$m_1 = 4$

$\frac{4y}{4} = \frac{16x}{4} + \frac{3}{4}$

$y = 4x + \frac{3}{4}$

$m_2 = 4$

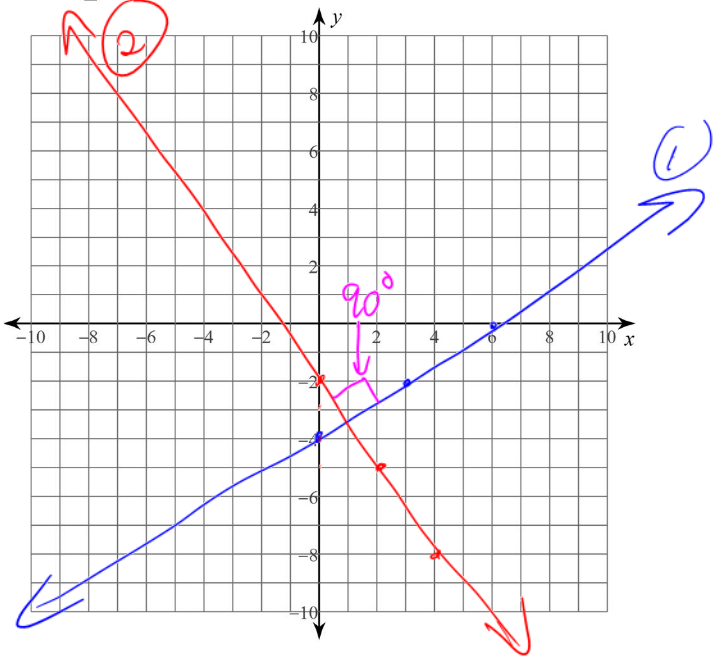
$m_1 = m_2$

$\therefore$ these lines are parallel

Now, graph these two lines on the same grid.

① $y = \frac{2}{3}x - 4$

② $y = -\frac{3}{2}x - 2$



This time, the lines do intersect. However, it is not the point of intersection that is important, it is the angle at which these lines are intersecting each other which is important. These lines are crossing at a 90 degree angle. We call these lines perpendicular. Just with parallel lines, it is the slopes that help us determine whether lines are perpendicular.

The slope of the first line is:

$m_1 = \frac{2}{3}$

The slope of the second line is:

$m_2 = \frac{-3}{2}$

These slopes are called the negative reciprocal of each other. This means that one slope is negative and one slope is positive. Reciprocal means that the fraction is flipped around.

In general terms, we write:

$m = \frac{a}{b}$ $\quad m_\perp = \frac{-b}{a}$

Example: Determine the slope perpendicular to the given slope:

a) $m = \frac{-3}{4}$

$m_\perp = \frac{4}{3}$

b) $m = \frac{8}{1}$

$m_\perp = \frac{-1}{8}$

c) $m = \frac{12}{23}$

$m_\perp = \frac{-23}{12}$

d) $m = 0$

$m_\perp =$ undefined

Now for the big questions. The goal of these questions is to create an equation with properties taken from other equations. Remember, to create an equation of a line, we need a slope and a point.

$Ax + By + C = 0$

1. Create a line in Standard Form which is parallel to $y = \frac{4}{5}x - 8$ and has the same x-intercept as $2x - 3y + 8 = 0$.

① Slope: parallel to:

$y = \frac{4}{5}x - 8$

$\boxed{m = \frac{4}{5}}$ ✓

② Point: x-int from: $y = 0$

$2x - 3y + 8 = 0$

$2x - \cancel{3(0)} + 8 = 0$

$2x = -8$

$x = -4$

$\boxed{(-4, 0)}$ ✓

③ Equation:

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{4}{5}(x + 4)$ (-4)

$5y = 4x + 16$

$0 = 4x - 5y + 16$

$y = mx + b$

2. Create a line in slope-intercept form which is perpendicular to $3x + 5y + 2 = 0$ and goes through the point (6,1).

① Slope: ⊥ to:

$3x + 5y + 2 = 0$

$\frac{5y}{5} = \frac{-3x}{5} - \frac{2}{5}$

$y = \frac{-3}{5}x - \frac{2}{5}$

$m = \frac{-3}{5}$

$\boxed{m_\perp = \frac{5}{3}}$ ✓

② Point

$\boxed{(6, 1)}$ ✓

③ Equation

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{5}{3}(x - 6)$

$3y - 3 = 5x - 30$ $+3$

$\frac{3y}{3} = \frac{5x}{3} - \frac{27}{3}$

$y = \frac{5}{3}x - 9$

$y = mx + b$

3. Create a line in Slope-Intercept Form that has the same y-intercept as $4x - 7y = 35$ and is parallel to $5x - 9y + 27 = 0$.

① Slope: parallel to:

$5x - 9y + 27 = 0$

$\frac{5x}{9} + \frac{27}{9} = \frac{9y}{9}$

$y = \frac{5}{9}x + 3$

$\boxed{m = \frac{5}{9}}$ ✓

② Point: y-int

$\cancel{4x}^{0} - 7y = 35$

$-7y = 35$

$y = -5$

$\boxed{(0, \underset{b}{-5})}$ ✓

③ Equation.

$y = \frac{5}{9}x - 5$

4. Create a line in Standard Form that is perpendicular to the slope formed by the points (5,2) and (-1,8) and goes through the origin.

① Slope: $\perp$ to:

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{8 - 2}{-1 - 5}$

$m = \frac{6}{-6}$

$m = -1$

$m_\perp = 1$

② Point:

Origin $= (0, 0)$

③ Equation

$y - y_1 = m(x - x_1)$

$y - 0 = 1(x - 0)$

$y = x$

$\boxed{x - y = 0}$ ✓

Success Criteria:

- I can determine if two lines are parallel by seeing if they have the same slope
- I can recognize that parallel lines have different y-intercepts
- I can determine if two lines are perpendicular by seeing if their slopes are negative reciprocals of each other
- I can create a new equation based certain conditions