Test #3 – Trigonometric Ratios and Sinusoidal Functions

Marks:

K: ___/20 T: ___/28 C: ___/5 A: ___/12

Name: SOLUTIONS

1. Determine the trig ratio correct to 4 decimal places: [K:2]

a) $\cos(130°) = -0.6428$

b) $csc(239°) = -1.1666$

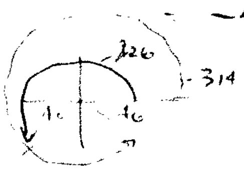
$\hookrightarrow \frac{1}{\sin(239)}$

2. Find the possible values for θ, (between 0° and 360°) given the trig ratios: [K:4]

a) $\sin\theta = -0.7231$

$\theta = \sin^{-1}(-0.7231)$

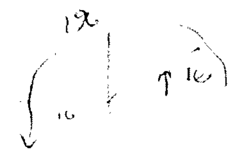
$= -46°$



$\theta = 226°$

or $\theta = 314°$

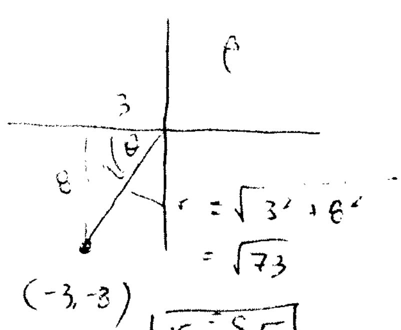
b) $\cot\theta = 3.4231$ $\Rightarrow$ $\tan\theta = \frac{1}{3.4231}$



$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{3.4231}\right) = 16°$

or $\theta = 196°$

3. (-3,-8) lies on the terminal arm of an angle in standard position. [K:7]
   i) Draw a sketch of the angle [1]
   ii) Determine the exact value of r to the nearest tenth. [1]
   iii) Determine the primary trigonometric ratios for angle θ [3]
   iv) Calculate the value of the related acute (θ) and principal (β) to the nearest degree. [2]



$r = \sqrt{3^2 + 8^2}$

$= \sqrt{73}$

$\boxed{r \doteq 8.5}$

$\sin\beta = -\frac{8}{8.5}$

$\cos\beta = -\frac{3}{8.5}$

$\tan\beta = \frac{8}{3}$

$\tan\theta = \frac{8}{3}$

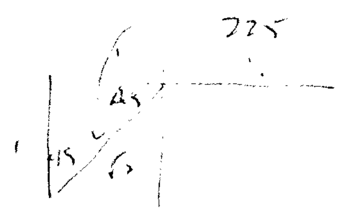
$\Rightarrow \theta = \tan^{-1}\left(\frac{8}{3}\right)$

$= 69°$

$\Rightarrow \beta = 180 + 69 = 249°$

4. Determine the exact value of the following (draw a diagram for each): [T:6]

a) $\cos(225°) = -\frac{1}{\sqrt{2}}$

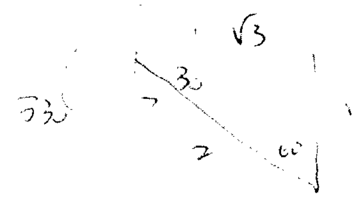


b) $csc(330°)$

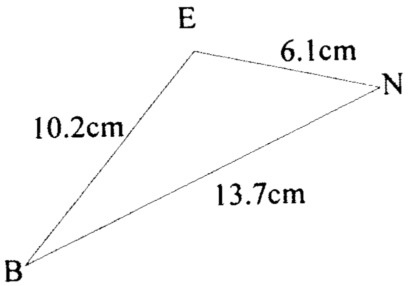
$= \frac{1}{\sin(330)}$

$= -\frac{2}{1}$

$= -2$



5. Solve ΔBEN. Properly label your solutions. [T:6]



$$\cos E = \frac{b^2 + n^2 - e^2}{2bn}$$

$$\Rightarrow E = \cos^{-1}\left(\frac{6.1^2 + 10.2^2 - 13.7^2}{2(6.1)(10.2)}\right)$$

$$= 112°$$

$$\frac{\sin N}{n} = \frac{\sin E}{e}$$

$$\Rightarrow N = \sin^{-1}\left(\frac{(10.2)(\sin(112))}{13.7}\right)$$

$$= 44°$$

$B = 24°$ (+) $\quad b = 6.1$ cm ✓

$E = 112°$ (cc) $\quad e = 13.7$ cm ✓

$N = 44°$ (sc) $\quad n = 10.2$ cm ✓

[T: 4]

6. Prove the following Identities, using proper form:

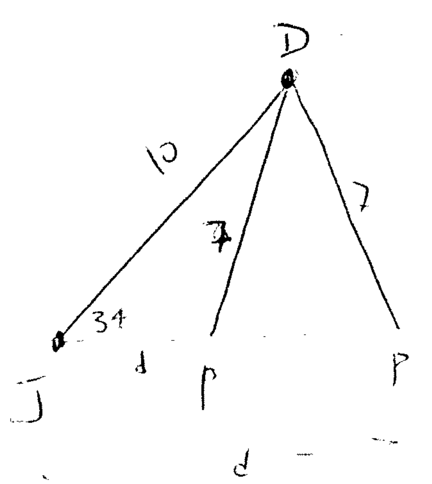
$$\frac{\sin^3\theta + \cos^2\theta \sin\theta}{\cos^2\theta} = \tan\theta \sec\theta$$

$$LHS = \frac{\sin(\theta)\left(\sin^2\theta + \cos^2\theta\right)}{\cos^2\theta}$$

$$= \frac{\sin\theta}{\cos^2\theta}$$

$$= \frac{\sin\theta}{\cos(\theta)\cdot\cos\theta} = \tan\theta \cdot \sec\theta = RHS \quad \square$$

7. Jim and Pam, who are on the ground, are holding ropes which are attached at the same spot on Dwight's hot air balloon, which is in the air. Jim's 10m rope is at an angle of elevation of 34° and Pam's rope is 7m. How far apart are Jim and Pam? Give one answer to represent if they are on the same side of the balloon and one for if they are on the opposite sides. [A:6]



$$\frac{\sin P}{10} = \frac{\sin 34}{7}$$

$$\Rightarrow P = \sin^{-1}\left(\frac{10 \cdot \sin(34)}{7}\right)$$

$$= 53°$$

If $P = 53°$

$\Rightarrow D = 180 - 34 - 53$

$= 93$

$$\frac{d}{\sin(93)} = \frac{7}{\sin(34)} \Rightarrow \boxed{d = 12.5\text{m}}$$

OR

$P = 180 - 53$

$= 127°$

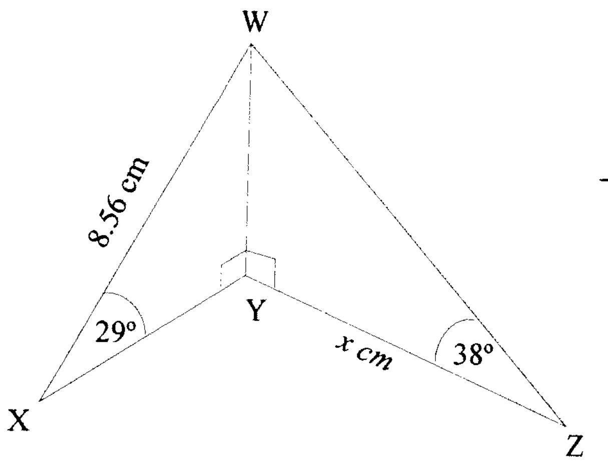
$\Rightarrow D = 180 - 34 - 127$

$= 19$

$$\frac{d}{\sin(19)} = \frac{7}{\sin(34)}$$

$$\Rightarrow \boxed{d = 4.1\text{m}}$$

9. In the given diagram, determine the unknown x correct to two decimal places. [T:4]



$\sin 29 = \frac{WY}{8.56}$

$\Rightarrow WY = (8.56)(\sin 29)$

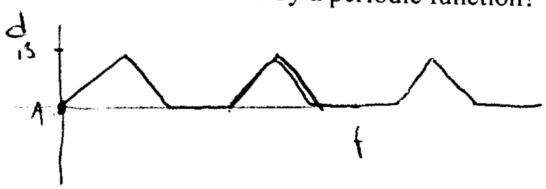
$= 4.15$

$\tan(38) = \frac{WY}{x}$

$\Rightarrow x = \frac{4.15}{\tan(38)}$

$\boxed{x = 5.31 cm}$

10. A man delivers packages from location A to location B and then returns to location A to get more packages. He does this in equal amounts of time each trip, but takes a 10 minute break when he gets to location B. Can the man's distance from location A be modelled by a periodic function? Explain. [A:2]

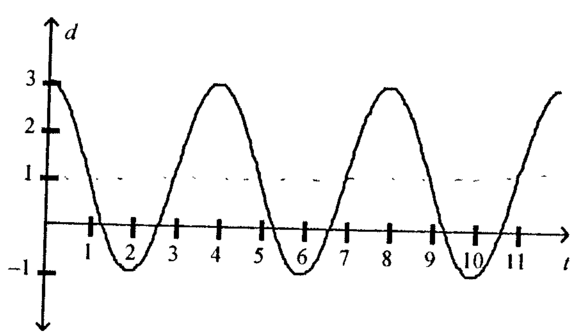


yes, as a fn of time.

12. A scientist records the motion of a particle. A graph of that motion is shown below. [K:7]

a) What is the period of one complete cycle?

$4 \Rightarrow k = \frac{360}{4} = 90$



b) What is the range of this function?

$-1 \leq d \leq 3$

c) Determine the equation of the axis.

$y = 1$

d) Determine the amplitude.

$a = 2$

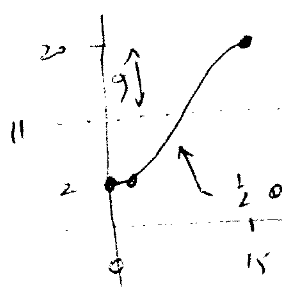
e) Give an equation for this sketch.

$d = 2\cos(90t) + 1$

f) If the particle can survive for twenty complete cycles, determine the domain of the function.

$0 \leq t \leq 80$

13. Determine a sinusiodal function that satisfies the information below:
A certain town has a windmill with a tip of one of the blades painted red. Over 15 seconds, that red tip moves from 2m above the ground up to 20m from the ground. Write the equation that models the red tip's distance from the ground in terms of time, assuming that the red tip starts nearest to the ground. [A:4]



$\boxed{y = -9\cos(12(t)) + 11}$

$\frac{1}{2}$ a cycle ⇒ 1 period is 30 seconds

⇒ $k = \frac{360}{30} = 12$

14. State the peak, trough, equation of axis, amplitude and period of *f(x)*, then graph it for 3 cycles. [T:8]

$f(x) = -2\sin(3(x-30)) + 4$

$a = 2$

$k = 3 \Rightarrow p = \frac{360}{3} = 120$

$d = 30$ right

$c = 4$