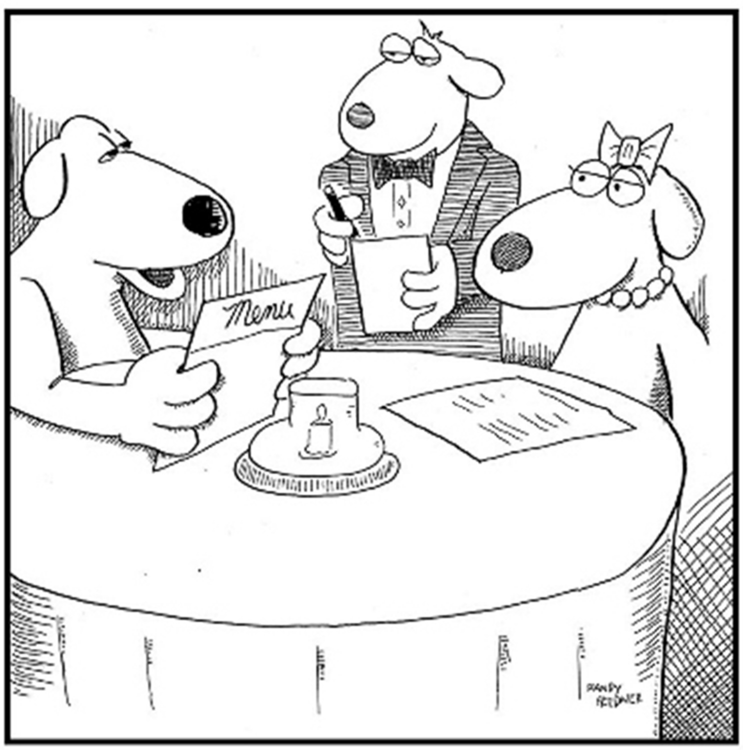
Math 10P, 11C, 11UC

Course Notes

The Algebra of Quadratics



"I'll have the math homework."

2.1 Working with Quadratic Expressions

Learning Goal: We are learning to expand and simplify quadratic expressions

This chapter is mostly a review. We will work with algebra to multiply binomials (FOIL) and then factoring. In order to work with Quadratics and convert from one form to another, which we will do in later chapters, we need to understand and be comfortable with this algebra.

Adding/Subtracting Polynomials:

$-7-(-12) = -7+12$

a) $(-7x^2 + 9x - 8) + (2x^2 - 10x - 8)$

$= -5x^2 - 1x - 16$

b) $(4x^2 - 7x + 3) - (x^2 - 12x)$

$= 3x^2 + 5x + 3$

First
Outer
Inside
Last

Expand:

a) $2x(3x-8)$

$= 6x^2 - 16x$

b) $(2x-3)(5x+7)$

$= 10x^2 + 14x - 15x - 21$

$= 10x^2 - x - 21$

c) $-2(x-5)(3x-2)$

$= -2(3x^2 \boxed{-2x - 15x} + 10)$

$= -2(3x^2 - 17x + 10)$

$= -6x^2 + 34x - 20$

d) $4n(n-3) + (5n+1)(3n+2)$

$= 4n^2 - 12n + 15n^2 + 10n + 3n + 2$

$= 19n^2 + 1n + 2$

e) $(4x-5)(4x+5)$

$= 16x^2 + 20x - 20x - 25$

$= 16x^2 - 25$

f) $(4x-5)^2$

$= (4x-5)(4x-5)$

$= 16x^2 - 20x - 20x + 25$

$= 16x^2 - 40x + 25$

Special Cases: There are two special cases which, when encountered in through expanding, can be done just by foiling (as we did with *e* and *f* above). However, if you understand their respective patterns, you will be able to understand how to factor them with ease.

Case 1: Difference of Squares – Notice what happened in *e*. After we expanded, the middle terms of *20x* cancelled out. This happens in ALL Difference of Squares questions. All we need to do, then, is square the first term, and subtract the square of the second term. Let's try it out and see how quickly it goes!

a) $(3x+2)(3x-2)$

$= 9x^2 - 4$

b) $(8x-5)(8x+5)$

$= 64x^2 - 25$

c) $(12x-9)(12x+9)$

$= 144x^2 - 81$

Case 2: Perfect Square – Again, notice what happened in *f*. The first and last terms are squared, but what about the middle terms? They were two 20x's, making a total of 40x. Instead of cancelling out like in Difference of Squares, they double up. How can we use this to our benefit? Because this will happen with all perfect squares, the process is this: square the first term, multiply the two terms together and double them, then add the square of the second term. Let's try it out!

a) $(3x+2)^2$ $(3x)(2) = 6x$

$= 9x^2 + 12x + 4$

b) $(5x-8)^2$ $(5x)(-8) = -40x$

$= 25x^2 - 80x + 64$

c) $(9x+1)^2$ $(9x)(1) = 9x$ double

$= 81x^2 + 18x + 1$

d) $(10x-3)^2$ $-30x$ double it

$= 100x^2 - 60x + 9$

Success Criteria:

- I can use the distributive property or FOIL to expand quadratic expressions
- I can recognize the "Difference of Squares" and "Perfect Squares" patterns

2.2 Factoring Polynomials: Common Factors

Learning Goal: We are learning to factor polynomials by dividing out the greatest common factor.

Okay, I feel rather silly teaching this concept. A common factor is something that can divided into each term. You then divide every term by that common factor, leaving the common factor in front of the expression. Remember, you are creating equivalent expressions.

Example:

GCF: 4

a) $4x^2 + 8x - 20$

$\frac{4x^2}{4} \quad \frac{8x}{4} \quad \frac{20}{4}$

$= 4(x^2 + 2x - 5)$

GCF: 5xy

b) $15x^3y + 40x^2y - 35xy$

$\frac{15x^3y}{5xy} \quad \frac{40x^2y}{5xy} \quad \frac{35xy}{5xy}$

$= 5xy(3x^2 + 8x - 7)$

Not too shabby. As we move into other factoring, it is ***essential*** that you ***always*** ask yourself if there is a common factor. If there is, your factoring will be much easier as you will be dealing with small numbers. So, while this is an easy concept, it is something that cannot be overlooked. Okay, now for something trickier.

a) $7x(2x-5) + 4(2x-5)$

Identical brackets ∴ it is the common factor.

$= (2x-5)(7x+4)$

b) $\underbrace{12n^3 - 16n^2}_{4n^2} + \underbrace{15n - 20}_{5}$

At first, nothing. But what if we look at groups?

$= 4n^2(3n-4) + 5(3n-4)$

$= (3n-4)(4n^2+5)$

This called: Factoring by Grouping

Success Criteria:

- I can recognize that factoring is the opposite of expanding
- I can identify the GCF in an algebraic expression and factor it out

2.3 Factoring Quadratic Expressions: $x^2 + bx + c$

Learning Goal: We are learning to factor simple quadratic trinomials.

Factoring is the opposite of expanding. When you expand $(x+4)(x+7)$ give us $x^2 + \overset{A}{11x} + \underline{28}$.
Therefore, when we factor a trinomial of the form x^2+bx+c, we should get $(x+r)(x+s)$, where r and s are integers (whole numbers, positive or negative).

There are four different trinomials. Take a look at the chart.

Trinomial	Factors
$x^2 + bx + c$	$(x + r)(x + s)$
$x^2 - bx + c$	$(x - r)(x - s)$
$x^2 - bx - c$	$(x - r)(x + s)$, where $r > s$
$x^2 + bx - c$	$(x + r)(x - s)$, where $r > s$

Shortcut Rules
1) x^2 must be positive
2) x^2 must have a coefficient of 1

Example time!!

a) $x^2 + 12x + 32$

M: 32
A: 12

1 32
2 16
$\boxed{+4\ +8}$

$=(x+4)(x+8)$

b) $x^2 + 5x - 36$

M: −36 ✓
A: +5 ✓

1 36
2 18
3 12
$\boxed{-4\ +9}$
6 6

$=(x-4)(x+9)$

c) $2x^2 - 20x + 48$

M: 24
A: −10

$=2(x^2-10x+24)$

$=2(x-4)(x-6)$

1 24
~~−2 −12~~
3 8
$\boxed{-4\ -6}$

d) $x^2 - 12x - 1485$

M: −1485
A: −12

$=(x+33)(x-45)$

1 1485
3 495
5 299
9 165
11 135
15 −99
27 −55
$\boxed{+33\ -45}$

Success Criteria
- I can factor a quadratic trinomial, that has an "*a*" value of 1, into two binomials

2.4 Factoring Quadratic Expressions: $ax^2 + bx + c$

Learning Goal: We are learning to factor quadratic trinomials where a ≠ 1.

When there is a number in front of the x^2 term that can't be factored out, we need to use the method of decomposition.

Here are the steps to set it up:

1) Multiply $a \times c$
2) List ALL the factors of that new number
3) Find a pair of factors that add to *b*. Keeping in mind that their signs must match *ac*.
4) Factor by Decomposition
    a. It's easier to teach with examples, so let's just dive in:

a) $5x^2 - 7x + 2$

M: 10
A: −7

−1, −10
−2, −5

$= \underbrace{5x^2 - 5x}_{5x} \underbrace{-2x + 2}_{-2}$

$= 5x(x-1) - 2(x-1)$

$= (x-1)(5x-2)$

b) $6x^2 + 13x + 5$

M: 30
A: +13

1, 30
2, 15
3, 10 =13
5, 6

$= \underbrace{6x^2 + 3x}_{3x} \underbrace{+10x + 5}_{5}$

$= 3x(2x+1) + 5(2x+1)$

$= (2x+1)(3x+5)$

Let's try to cut out a step (you don't have to if you don't want to!!)

c) $2x^2 - 7x - 30$

M: −60
A: −7

1, −60
2, −30
3, −20
4, −15
5, −12 = −7
6, −10

$= \underbrace{2x^2 - 12x}_{2x} \underbrace{+5x - 30}_{+5}$

$= (2x+5)(x-6)$

d) $10x^2 + x - 24$

M: −240
A: +1

−1, 240
−2, 120
−3, 80
−4, 60
−5, 48
−6, 40
−8, 30
−10, 24
−12, 20
−15, 16
= 1

$= \underbrace{10x^2 - 15x}_{5x} \underbrace{+16x - 24}_{+8}$

$= (5x+8)(2x-3)$

e) $\underline{35x^2 - 190x + 75}_{5}$

First, take out a GCF if it exists.

$= 5(7x^2 - 38x + 15)$

M: 105
A: −38

−1, −105
$\boxed{-3, -35}$ = −38 ☺

$= 5(\underbrace{7x^2 - 35x}_{7x} \underbrace{-3x + 15}_{-3})$

$= 5[7x(x - 5) - 3(x - 5)]$

$= 5(x - 5)(7x - 3)$

f) $60x^2 - 222x - 216$

M: −360
A: −37

1, −360
2, −180
3, −120
4, −90
5, −72
6, −60
$\boxed{8, -45}$
= −37

$= 6(10x^2 - 37x - 36)$

$= 6(\underbrace{10x^2 + 8x}_{2x} \underbrace{-45x - 36}_{-9})$

$= 6(2x - 9)(5x + 4)$

Success Criteria:

- I can test that a trinomial can be factored if two integers can be found whose product is $a \times c$ and whose sum is b
- I can use “Factoring by Decomposition” to write a quadratic as the product of two binomials

2.5 Factoring Special Cases

Learning Goal: We are learning factoring shortcuts for two special cases of quadratics.

Case 1: Difference of Squares

Remember expanding $(3x-5)(3x+5)$? We got: $9x^2 \cancel{+15x} \cancel{-15x} - 25 = 9x^2 - 25$

Hence, when we factor difference of squares, recognize that it is a binomial, it has two square terms, and they are being subtracted. If it all fits, it is a difference of squares.

Factor:

a) $16x^2-49$

$\sqrt{16}=4 \quad \sqrt{49}=7$

$(4x+7)(4x-7)$

b) $64x^2-81$

$\sqrt{64}=8 \quad \sqrt{81}=9$

$(8x+9)(8x-9)$

c) $5x^2-45$

$=5(1x^2-9)$

$=5(x-3)(x+3)$

d) $25x^2+36$

$\sqrt{25}=5 \quad \sqrt{36}=6$

= No minus in the middle. So not a difference of squares.

Case 2: Perfect Squares

Similarly, what happens when we expand $(3x+5)^2$? $(3x+5)(3x+5) = 9x^2+15x+15x+25 = 9x^2+30x+25$ Not Factorable

When we come up a trinomial that looks like a perfect square, we just need to do a quick mental check: **1.** Is the first term a perfect square? **2.** Is the last term a perfect square? **3.** Take the square roots of each term, multiply them and double them. If you get the middle term, you have a perfect square!

Factor:

a) $9x^2-24x+16$

$\sqrt{9}=3 \quad \sqrt{16}=4$

$3\times4=12$ double it $=24$

$(3x-4)(3x-4)$

$=(3x-4)^2$

b) $49x^2+84x+36$

$\sqrt{49}=7 \quad \sqrt{36}=6$

$7\times6=42$ double it $=84$

$(7x+6)(7x+6)$

$=(7x+6)^2$

c) $200x^2+360x+162$

$=2(100x^2+180x+81)$

$\sqrt{100}=10 \quad \sqrt{81}=9$

$9\times10\times2=180$ ✓

$=2(10x+9)(10x+9)$

$=2(10x+9)^2$

Success Criteria:

- I can factor a Difference of Squares as $(ax+b)(ax-b)$
- I can factor Perfect Squares as $(ax \pm b)^2$