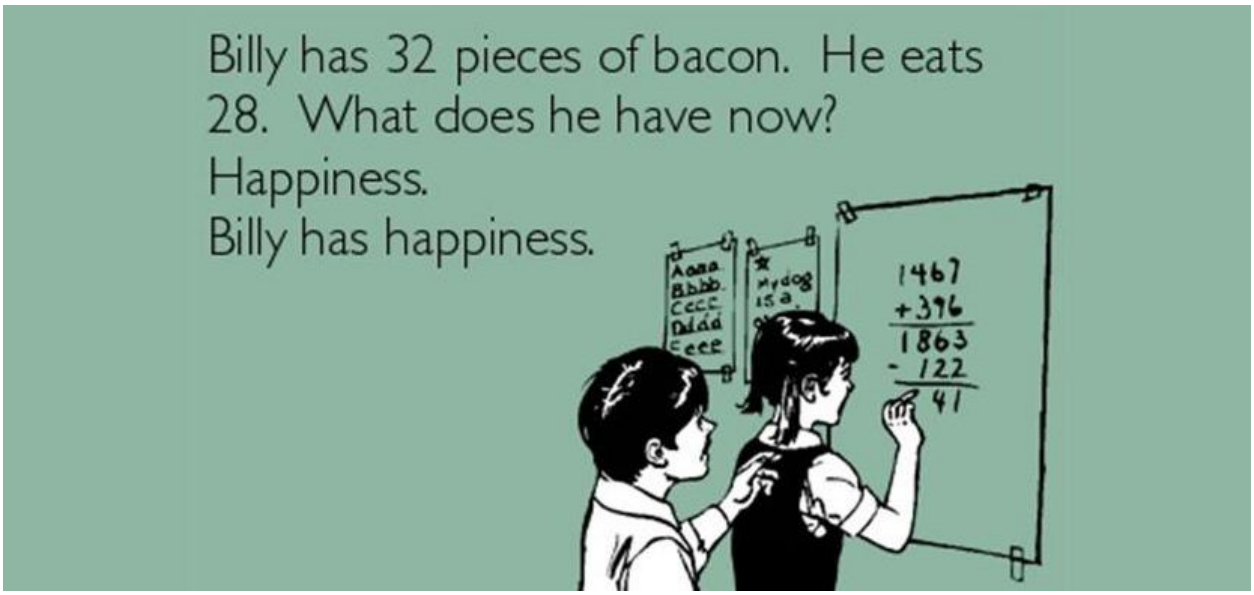
Math 10P, 11C, 11UC

Course Notes

Problems with Quadratics



4.1 Solving Quadratics Graphically

Learning Goal: We are learning to use technology to help us solve problems that involve quadratics.

There is fantastic software out there that can graph quadratics (or lines) instantly. They can also tell us important pieces of information about our quadratics such as the vertex or the zeros. The software is also flexible and can recognize equations in standard, factored, and vertex forms

To use the technology effectively you must:

1) Carefully type your equation into the software
    a. Often the software depends on the variables "y" and "x", so if your question uses different variables, you will have to switch them to y and x.
2) Draw a rough sketch of the parabola in your notes/homework
3) Identify what you are trying to find on your sketch (the vertex, the zeros, some sort of intersection)
4) State your solution

Example 1: A computer software company models the profit on its latest video game using the equation $y = -2x^2 + 32x - 110$, where x is the number of games, in thousands, that the company produces and y is the profit, in millions of dollars.
What is the maximum profit the company can earn?

Example 2: Sally is standing on the top of a river slope and throws a ball. The height of the ball at a given time is modelled by the equation $h = -5t^2 - 10t + 250$, where *h* is the height in metres and *t* is the time in seconds.
Rewrite the equation using "y" and "x"

When will the ball be 10m above the ground?

Example 3: The equation $d = 0.0056s^2 + 0.14s$ models the stopping distance of a car, d, in metres and its speed, s, in kilometres per hour.
What is the car's speed when the stopping distance is 7m?

Example 4: A ball is thrown vertically upward from the top of a cliff. The height of the ball is modelled by the equation $y = 65 + 10x - 5x^2$, where y is the height in metres and x is the time in seconds.
When does the ball reach its maximum height?

When does the ball hit the ground?

Success Criteria:

- I can rewrite an equation using the variables "y" and "x"
- I can type an equation into an online graphing calculator
- I can use technology to help me solve problems involving quadratics

4.3a Solving Quadratics Algebraically

Learning Goal: We are learning to solve problems involving quadratics algebraically.

When solving quadratics algebraically, it is important to consider what you are trying to find. Does the question require you to find the vertex? Or are you looking for one of the zeros? These two are the most common things you are trying to find. You may also be asked to find the y-intercept, or when something starts.

But first, a quick review of factoring. Many word problems require you to factor either by pulling out common terms (if there are only two terms), or by factoring by decomposition (if you have a proper trinomial).

$y = 7x^2 - 14x$ Pull out common factor

$7x$ $7x$

$y = (7x)(x-2)$

If factoring, the solution is the value of x, when $y=0$.

$0 = (7x)(x-2)$ *

$7x = 0$ $x-2 = 0$

$\frac{7x}{7} = \frac{0}{7}$ $+2 \quad +2$

$x = 0$ $x = 2$

Zeros:

The solutions $\begin{cases} x=0 \\ \& \\ x=2 \end{cases}$

$y = -3x^2 + 3x + 126$

Find the zeros!

$\frac{0}{-3} = \frac{-3x^2}{-3} + \frac{3x}{-3} + \frac{126}{-3}$

$0 = 1x^2 - x - 42$

$0 = (x+6)(x-7)$

$x+6 = 0$ $x-7 = 0$

$-6 \quad -6$ $+7 \quad +7$

$x = -6$ $x = 7$

Zeros: $x = -6$ and $x = 7$

M: 42

1 42

2 21

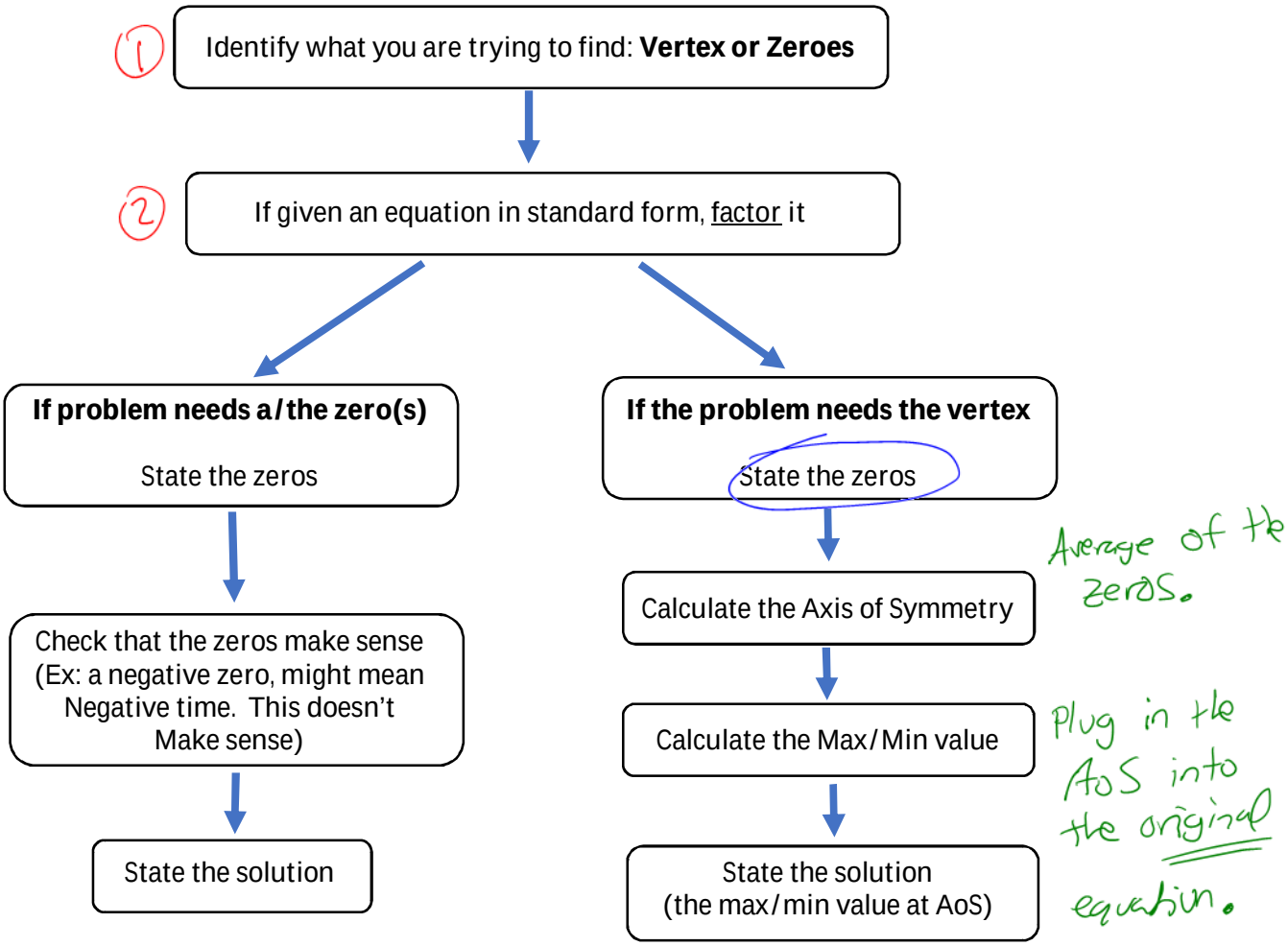
3 14

+6 -7

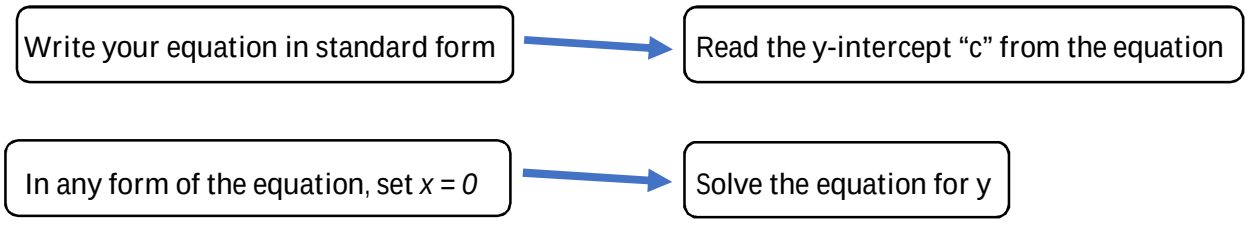
Some Examples of Common Word Problems

Find the zeros	**Find the vertex**	**Find the y-intercept**
- A ball is thrown up. When does the ball hit the ground? - A person dives off a cliff. When do they splash into the water? - A company is trying to find how many products they need to sell to reach the break-even point(s) (Profit = $0)	- A company is trying to find out how many toys to make in order to have the maximum profit? - A ball is thrown up. When does it reach its maximum height?	- A ball is thrown off a cliff. What was its initial height? - How much money did a company start with before selling a product?

Here are the steps you need to take to solve a quadratic problem for the vertex or zeros:



If you are trying to find the y-intercept, you have two options:



Example 1: A rocket is launched from the ground. Its path through the air is modelled by the equation $y = -5t^2 + 45t$, where t is the time in seconds. How long does it take before the rocket hits the ground?

① Find the zeros.

② Factor it!

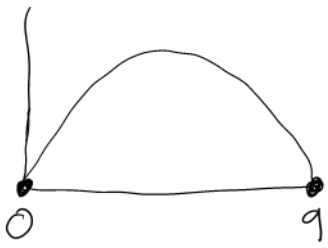
$y = \frac{-5t^2}{-5t} + \frac{45t}{-5t}$

$y = (-5t)(t - 9)$

$0 = (-5t)(t - 9)$

$\frac{-5t}{-5} = \frac{0}{-5}$ $\qquad$ $t - 9 = 0$ $\quad +9 \quad +9$

$t = 0$ $\qquad$ $t = 9$



The rocket was in the air for 9 seconds.

Example 2: A computer software company models the profit on its latest video game with the equation $y = -2x^2 + 32x - 110$, where x is the number of games the company produces in thousands and y is the profit in thousands of dollars.

How many games should the company make in order to earn the maximum profit?

① Find the vertex

② Factor the equation

$y = -2x^2 + 32x - 110$

$\frac{0}{-2} = \frac{-2x^2}{-2} + \frac{32x}{-2} - \frac{110}{-2}$

$0 = 1x^2 - 16x + 55$

$0 = (x-5)(x-11)$

1 55

$-5 \quad -11$ ✓

③ State the Zeros

$x - 5 = 0$

$+5 \quad +5$

$x = +5$

$x - 11 = 0$

$+11 \quad +11$

$x = 11$

④ Find the AoS

$\frac{5+11}{2} = \frac{16}{2} = 8$

$x = 8$

⑤ Find max/min value

$y = -2(8)^2 + 32(8) - 110$

$y = -128 + 256 - 110$

$y = 18$

The maximum profit is \$18,000 if the company sells 8000 games.

What would be the company's profit if they didn't make or sell any games at all?

↳ Find y-intercept. $x = 0$

$y = -2(0)^2 + 32(0) - 110$

$y = -110$

The company loses \$110,000 if they make 0 games.

Example 3: A helicopter drops an aid package. The height of the package above the ground at any time is modelled by the equation $y = -5t^2 - 30t + 675$, where t is the time in seconds and y is the height in meters.

How long will it take the package to hit the ground?

① Find the zeros

② $0 = -5t^2 - 30t + 675$

$\frac{0}{-5} = \frac{-5t^2}{-5} - \frac{30t}{-5} + \frac{675}{-5}$

$0 = t^2 + 6t - 135$

$0 = (t - 9)(t + 15)$

1	135
3	45
5	27
−9	+15

$t - 9 = 0$

$+9 \quad +9$

$t = 9$

$t + 15 = 0$

$-15 \quad -15$

$t = -15$

Does not make sense. Negative time?

It takes 9 seconds for the package to hit the ground.

4.3b Solving Quadratics Algebraically

Learning Goal: We are learning to solve problems involving quadratics algebraically.

Example 4: A ball is tossed upward from a cliff that is 40m above water. The height of the ball is modelled by the equation $y = -5t^2 + 10t + 40$, where *t* is the time in seconds and *y* is the height of the ball.

a. When will the ball hit the water?

① Find the zeros

$y = 0$

$\frac{0}{-5} = \frac{-5t^2}{-5} + \frac{10t}{-5} + \frac{40}{-5}$

$0 = t^2 - 2t - 8$

1 8
+2 -4

$0 = (t+2)(t-4)$

$t+2 = 0$ $\quad$ $t-4 = 0$

$-2 \quad -2$ $\quad$ $+4 \quad +4$

$t = -2$ $\quad$ $t = 4$

~~-2 seconds~~ and 4 seconds.

The ball hits the water after 4 seconds.

b. What is the maximum height reached by the ball?

Find the vertex!

AoS : average the zeros.

$\frac{-2+4}{2} = \frac{2}{2} = 1$

$x = 1$

Plug $x = 1$ into original equation.

$y = -5(1)^2 + 10(1) + 40$

$y = -5 + 10 + 40$

$y = 45$

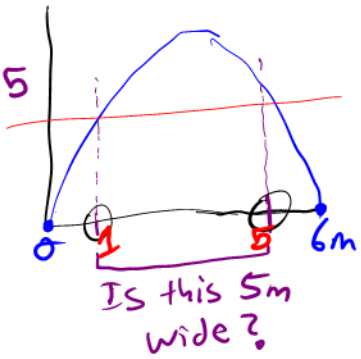
Vertex: $(1, 45)$

The max height is 45m.

Example 5: A big rig is carrying an entire house through a narrow tunnel. The tunnel is in the shape of a parabola and can be modelled by the equation $h = -x^2 + 6x$, where *h* is the height of the tunnel in meters, and *x* is the width of the tunnel in meters.

The big rig and house together are 5m wide, and just under 5 m tall. Is the tunnel big enough for the big rig and house to get through?

Start w/ a sketch



Let's factor to sketch

$h = -x^2 + 6x$

$0 = (-x)(x - 6)$

So, $-x = 0$ $\quad x = 0$

$x - 6 = 0$ $\quad x = 6$

Here, we are trying to find the x values when $h = 5$.

So our equation is:

$5 = -x^2 + 6x$

$-5 \quad\quad -5$

we solve by getting "stuff $= 0$" and factoring.

Then find the zeros!

$\frac{0}{-1} = \frac{-x^2}{-1} + \frac{6x}{-1} - \frac{5}{-1}$

$-1 \quad -5$

$0 = x^2 - 6x + 5$

$0 = (x - 1)(x - 5)$

So our zeros are $x = 1$ and $x = 5$.

what is the distance between them? $5 - 1 = 4\text{m}$

No, the truck will not fit.

Success Criteria

- I can determine whether a word problem is requiring me to find the zeroes, vertex, or y-intercept of a quadratic equation.
- I can solve for the zeros or vertex of a quadratic equation by factoring
- I can solve for the y-intercept by setting x = 0 in the quadratic equation.
- I can sketch a parabola that illustrates the word problem.

4.1 Solving with Quadratic formula

Only for 11UC's. Standard -> Vertex form