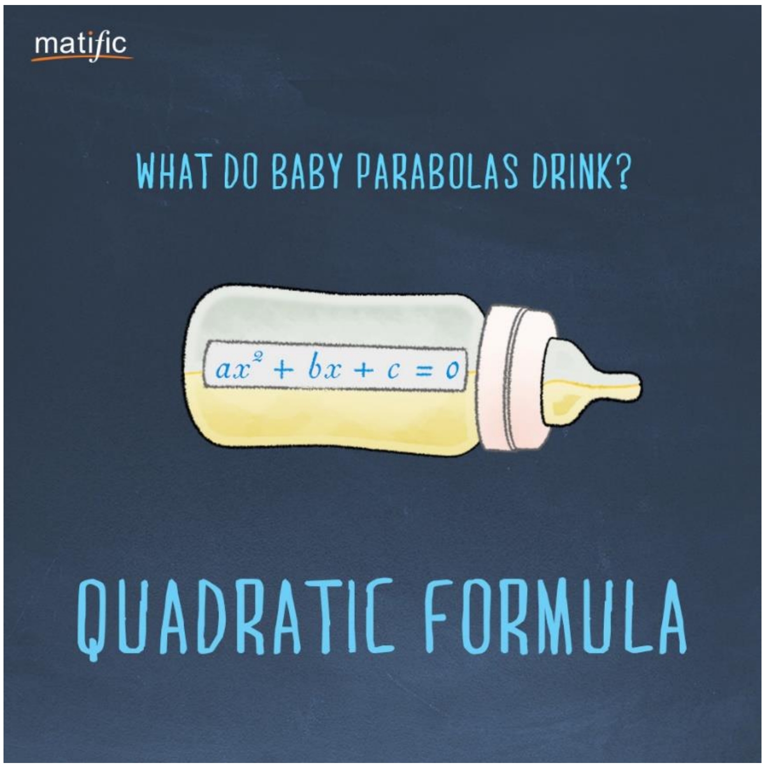
Functions & Applications 11

MCF3M

Course Notes

Chapter 4: Standard & Vertex Form



Homework

Contents with suggested problems from the Nelson Textbook. You are welcome to ask for help, from myself or your peers, with any of the following problems. They will be handed in on the day of the Unit Test as a homework check. Alternatively, you may be handing your homework in more regularly for me to correct.

Section 4.1 – Page 203
#1, 3, 4, 6, 8, 9, 10

Section 4.2 – Page 214
#6, 7 (do not graph), 8-11

Section 4.3 – Page 22
#3, 6, 8, 9

4.1 The Vertex Form of a Quadratic Function

Learning Goal: We are learning the properties of the standard and vertex forms of a quadratic equation, and how they interrelate.

In this Chapter, we will be exploring the relationship between the Standard Form and the Vertex Form of quadratic functions. The Standard Form is $f(x) = ax^2 + bx + c$ and the Factored Form is $f(x) = a(x-h)^2 + k$. These two forms are equivalent, meaning that they generate the exact same information or graph. In the Vertex Form, the *h* and *k* form together to give the coordinate of the vertex (h,k). The value *k* is also known as the maximum or minimum value, which is handy when writing out the range of the function.

Question 1: Convert $f(x) = 3(x+4)^2 - 18$ to the Standard Form.

Question 2: Given $g(x) = -4(x+5)^2 + 3$, state the vertex, axis of symmetry, direction of opening and range.

Question 3: Given the vertex (-3, -8) and the coordinate (-6, 37), determine the equation of the parabola.

Question 4: The height above the ground of a bungee jumper is modelled by the quadratic function $h(t) = -5(t-0.3)^2 + 110$, where height, *h(t)*, is in metres and time, *t*, is in seconds.

a) When does the bungee jumper reach a maximum height? Why is it a maximum?

b) What is the maximum height reached by the jumper?

c) Determine the height of the platform from which the bungee jumper jumps.

Success Criteria:

- I can convert the vertex form into the standard form by expanding and simplifying it
- If a quadratic equation is in vertex form, I can
  - Find the vertex at (h,k)
  - Find the axis of symmetry at x = h
  - Determine the direction of opening by seeing if “a” is positive or negative

4.2 Completing the Square

Learning Goal: We are learning to convert from the standard form equation into the vertex form equation by "completing the square"

As we learned in 4.1, the Vertex Form gives the vertex, which in turn tells us the maximum or minimum value of the parabola. Given the Standard Form, it would be handy to have a way to convert it to the Vertex Form so we could determine the maximum or minimum. Thankfully, we have a method to do just that!

First, we need to explore perfect squares, and given missing pieces, how it can be completed.

Expand the following: (notice the pattern!)

$f(x) = a(x-h)^2 + k$

$(x+4)^2 = (x+4)(x+4) = x^2+4x+4x+16 = x^2+8x+16$

$(x-1)^2 = x^2 - 2x + 1$

$(x \underline{-7})^2 = x^2 - 14x + \underline{49}$

$\div 2$ $\quad ()^2$

$(x \underline{+5})^2 = x^2 + 10x + \underline{25}$

$\div 2$ $\quad ()^2$

$(x \underline{-174})^2 = x^2 - 348x + \underline{30276}$

$\div 2$ $\quad ()^2$

$(x \underline{-16.5})^2 = x^2 - 33x + \underline{272.25}$

$(x \underline{-1.42})^2 = x^2 - 2.84x + \underline{2.016}$

$\div 2$

Important: Vertex Form has $(x-h)^2$ which is a perfect square. We will need to create this when completing the square.

Example 1: Complete the Square to convert to Vertex Form, then state the Vertex.

$f(x) = 2x^2 - 12x + 13$

$f(x) = 2(x^2 - 6x + \_\_\_) + 13$

$f(x) = 2(\underbrace{x^2 - 6x + 9}_{\text{A perfect square}} - 9) + 13$

$f(x) = 2(x^2 - 6x + 9) + 13 - 18$

$f(x) = 2(x - 3)^2 - 5$

Vertex: $(3, -5)$

1. Factor "a" from the first two terms
2. Complete the square. "b" ÷ 2 then square it.
   $\frac{-6}{2} = -3 \rightarrow (-3)^2 = 9$
   *we are finding what # is needed to make a perfect square.
3. Remove the "−9" from the bracket. Remember to multiply it by "a though".
4. Rewrite as a perfect square factored. $(x-h)^2$

Example 2:

$g(x) = -4x^2 - 40x - 7$

$g(x) = -4(x^2 + 10x + \_\_\_) - 7$

$\left(\frac{b}{2}\right)^2 \rightarrow \left(\frac{10}{2}\right)^2 = \boxed{25}$

$g(x) = -4(\underbrace{x^2 + 10x + 25}_{\text{perfect square}} - 25) - 7$

remove from bracket

$g(x) = -4(x^2 + 10x + 25) - 7 + 100$

$g(x) = -4(x + 5)^2 + 93$

∴ Vertex $(-5, 93)$

Example 3:

$$f(x) = \underbrace{\frac{1}{2}x^2 + 7x}_{\text{Factor } a} + 6$$

$$f(x) = \frac{1}{2}(x^2 + 14x + \_\_\_) + 6$$

$$\left(\frac{14}{2}\right) \rightarrow 7, \quad 7^2 = \boxed{49}$$

$$f(x) = \frac{1}{2}(\underbrace{x^2 + 14x + 49} - 49) + 6$$

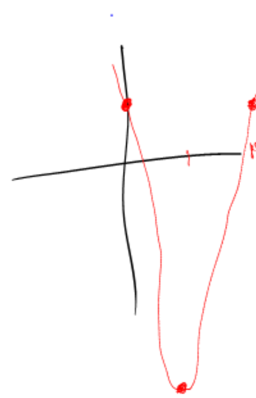
$$f(x) = \frac{1}{2}(x^2 + 14x + 49) + 6 - 24.5$$

$$f(x) = \frac{1}{2}(x+7)^2 - 18.5$$

Vertex: $(-7, -18.5)$

Example 4:

$$g(x) = 4.5x^2 - 102.1x + 23.4$$



$$g(x) = 4.5(x^2 - 22.69x + \_\_\_) + 23.4$$

$$\frac{-22.69}{2} = -11.345, \quad (-11.345)^2 = \boxed{128.71}$$

$$g(x) = 4.5(\underbrace{x^2 - 22.69x + 128.71} - 128.71) + 23.4$$

$$g(x) = 4.5(x^2 - 22.69x + 128.71) + 23.4 - 579.2$$

$$g(x) = 4.5(x - 11.345)^2 - 555.8$$

Vertex $(11.35, -555.8)$

Success Criteria

- I can convert a quadratic function from standard form into vertex form by completing the square
- I can use both standard and vertex forms to gather useful information about the quadratic function, and to help me graph the parabola

4.3 Quadratic Formula

Learning Goal: We are learning to use the quadratic formula to solve quadratic equations, especially those that can't be factored.

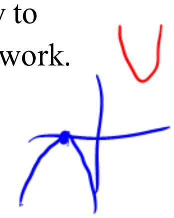
The quadratic formula is the one formula to solve them all. With great power, comes great responsibility. It is a tricky formula, but once you learn how to properly use it, you will be that much happier. The key is to write and communicate your math carefully.

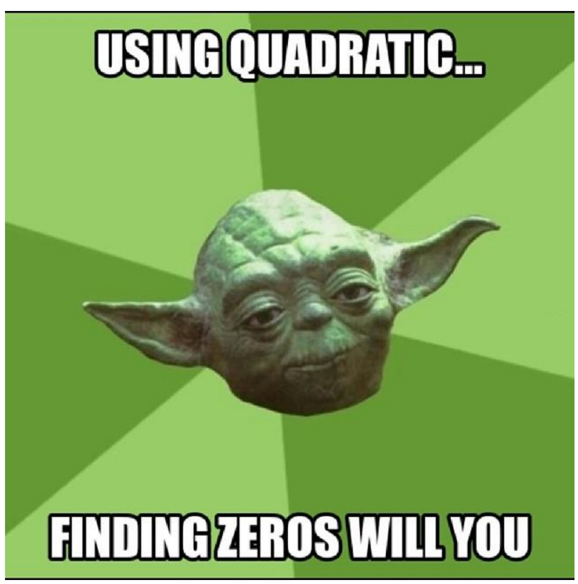
Given $ax^2 + bx + c = 0$, then $\boxed{x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

Two notes:

1. Inside the square root, you start with b^2. No matter what you plug in, you get a positive number. If $b = 9, \quad b^2 = 81$. If $b = -5, \quad b^2 = 25$. Why do I make note of this? I have seen many people do it wrong, so don't be one of those people.

2. Since we are calculating a square root, we have three options. If the number inside the square root is positive, there are two solutions. If the number is zero, there is only one solution. If the number is negative, there are no solutions since you cannot square root a negative. This is a fact. Don't try to square root a negative as it absolutely 100% cannot be done. Please don't try to do it. It doesn't work. No solution is an answer, so do not fret. If you don't like the negative, double check your work.





Example 1:

$$3x^2 - 24x + 45 = 0$$

Simplify First!

$$x^2 - 8x + 15 = 0$$

$a = 1$ $b = -8$ $c = 15$

Could have factored normally.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{64 - 60}}{2}$$

$$x = \frac{8 \pm \sqrt{4}}{2}$$

$$x_1 = \frac{8 + \sqrt{4}}{2} \qquad x_2 = \frac{8 - \sqrt{4}}{2}$$

$$x_1 = \frac{8 + 2}{2} \qquad x_2 = \frac{8 - 2}{2}$$

$$\boxed{x_1 = 5} \qquad \boxed{x_2 = 3}$$

Example 2:

$$3x^2 + 2x + 15 = 0$$

$a = 3$

$b = 2$

$c = 15$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(3)(15)}}{2(3)}$$

$$x = \frac{-2 \pm \sqrt{4 - 180}}{6}$$

$$x = \frac{-2 \pm \sqrt{-176}}{6}$$

No solutions / No zeros (never crosses the x-axis)

Example 3:

$4x^2 - 8x + 10 = 2x + 7$ Get "stuff = 0"

$-2x \; -7 \; -2x \; -7$

$4x^2 - 10x + 3 = 0$

$a = 4 \quad b = -10 \quad c = 3$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(4)(3)}}{2(4)}$$

$$x = \frac{10 \pm \sqrt{100 - 48}}{8}$$

$$x = \frac{10 \pm \sqrt{52}}{8}$$

$$x_1 = \frac{10 + \sqrt{52}}{8} \qquad x_2 = \frac{10 - \sqrt{52}}{8}$$

$x_1 = 2.151 \qquad x_2 = 0.349$

Example 4:

The profit on a school drama production is modelled by the quadratic equation $\underbrace{\cancel{P(x)}}_{P}=-60x^2+790x-1000,$ where *P(x)* is the profit in dollars and *x* is the price of the ticket, also in dollars.

a) Use the quadratic formula to determine the break-even price for the tickets. (zeros)

b) At what price should the drama department set the tickets to maximize their profit? (vertex)

$a=-60 \quad b=790 \quad c=-1000$

$$x=\frac{-(790)\pm\sqrt{(790)^2-4(-60)(-1000)}}{2(-60)}$$

$$x=\frac{-790\pm\sqrt{384,100}}{-120}$$

$$x_1=\frac{-790+\sqrt{384,100}}{-120} \qquad x_2=\frac{-790-\sqrt{384,100}}{-120}$$

$$x_1=1.419 \qquad x_2=11.748$$

The break even price is \$1.40, or \$11.75

b) AoS: $\dfrac{1.419+11.748}{2}$

$x=6.5835$

\$6.58

The price should be: \$6.50.

Success Criteria

- I can use the quadratic formula to solve a quadratic equation in the form: $ax^2+bx+c=0$
- I can recognize that there may be 2, 1, or 0 solutions to my quadratic equation
- I can understand that the solutions to the quadratic formula represent the zeros, or x-intercepts, of the quadratic function