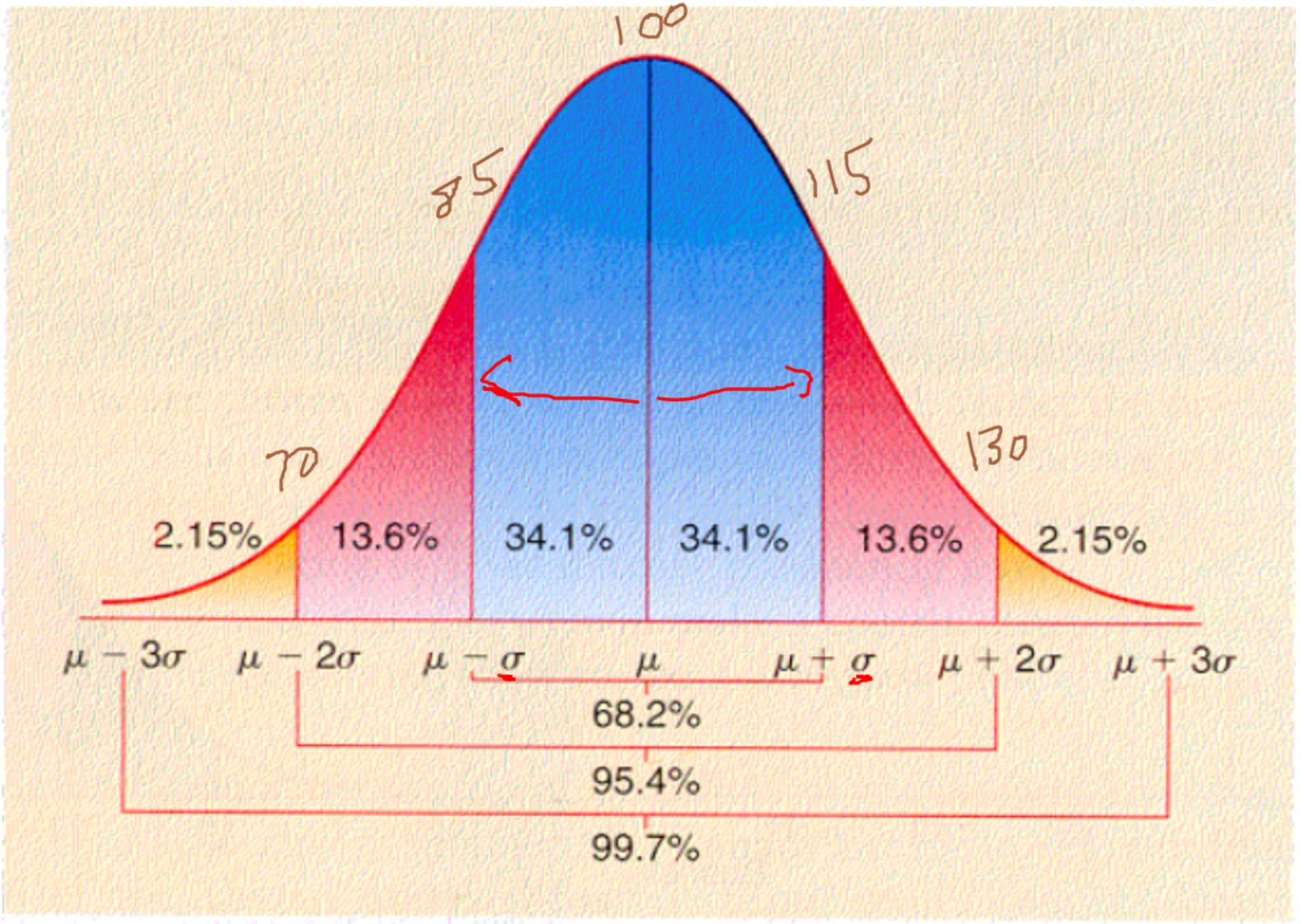
Z-Scores – Statistics and probability together at last!

Recall: standard deviation is the average distance the data points are with the mean.

Note: μ, "mu" = mean

σ, "sigma" = standard deviation

The normal curve

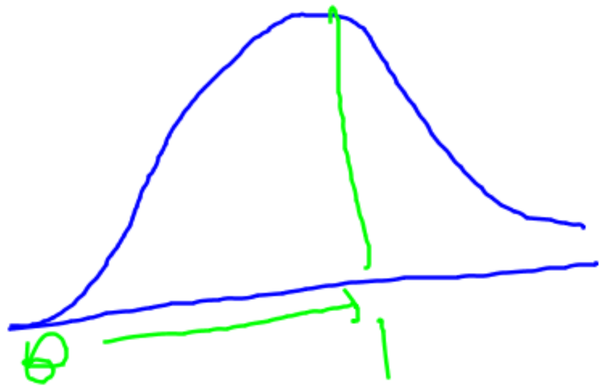


Z-score is the exact number of standard deviations a data point is from the mean.

$$Z = \frac{X - \mu}{\sigma}$$

Together with the normal curve, we can calculate certain probabilities. But first we need a chart:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177



A z-score of 1.08 give a probability of 0.8599

Example 1 Find the z-score corresponding to an IQ score of 132. IQ scores have a normal distribution with mean 100 and standard deviation 15.

$$z = \frac{132 - 100}{15} = 2.13$$

Example 2 Heights of young women are normally distributed with a mean (μ) 65.5 in and standard deviation (σ) 2.5 in.

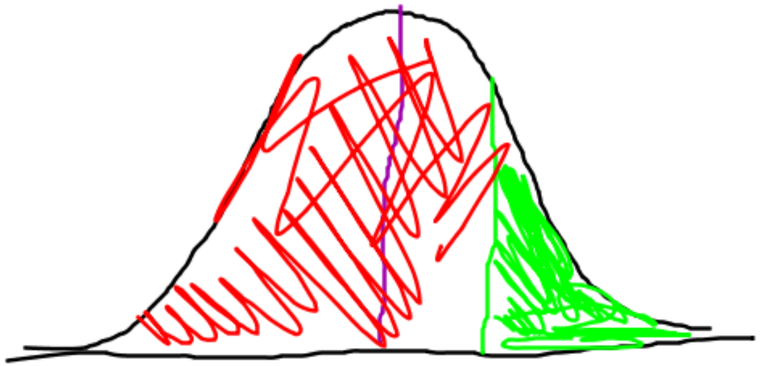
a) What is the probability that a women, chosen at random, is below 62 in?

$$z = \frac{62 - 65.5}{2.5} = -1.4$$

$P = 8.1\%$

b) What is the probability that a women, chosen at random, is above 68 in.?

$$Z = \frac{68 - 65.5}{2.5} = 1$$



$$P = 1 - 0.84 = 16\%$$

c)What is the probability that a woman, chosen at random is between 60 and 72 in.?

$$Z = \frac{72 - 65.5}{25} = 2.6 \qquad Z = \frac{60 - 65.5}{2.5} = -2.2$$

$$P = 0.9953 - 0.0139$$

$$= 98\%$$

d) How tall does a woman have to be to be taller than 90% of women? (*Hint: Use the chart to find the z-score associated with 90%*)

$$2 = \frac{x - 65.5}{2.5}$$

90% ⇒ 1.28

$$1.28 = \frac{x - 65.5}{2.5}$$

$+65.5$

$\times 2.5$

$$68.7 = x$$