Name: ______________________________

Similar Triangles and Trigonometry

Unit Outline:

a. Review of Angle Theorems
b. Similar Triangles
c. Right Angle Triangle Ratios
d. Solving Triangles using Primary Trigonometric Ratios SOH CAH TOA
e. Sine Law
f. Cosine Law

Formula Sheet

Pythagorean Theorem:

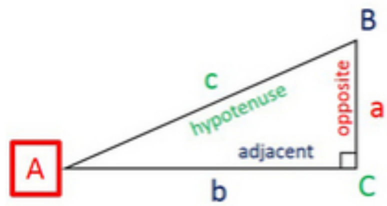
$$c^2 = a^2 + b^2$$

Right Angle Triangles SOHCAHTOA

$$\sin(A) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos(A) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan(A) = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$



Sine Law

For the Sine Law we need:
- An angle and its opposite side, and one other piece of information.
- OR SAS, ASA [and ASS (use with caution!)]

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The length of any side, divided by the Sine of its opposite angle is the same for all three pairs
If we are trying to **find an angle**, use the first form of the Sine Law (**angles on top**)
If we are trying to **find the length** of a side, use the second for of the law (**with sides on top**)

Cosine Law

For the Cosine Law we need:
- 2 sides and the included angle.
- 3 sides

To find a side (have SAS):

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

To find an angle (have SSS):

$$\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$$

$$\cos B = \frac{b^2 - a^2 - c^2}{-2ac}$$

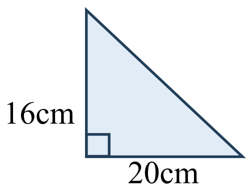
$$\cos C = \frac{c^2 - a^2 - b^2}{-2ab}$$

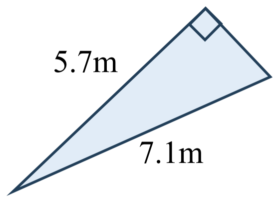
Lesson 3.0 - Trigonometry Prerequisite Skills

A Solving for *x*, to 2 decimal places if necessary.

$x^2 = 100$ $\qquad$ $x^2 = 3^2 + 4^2$ $\qquad$ $8^2 + x^2 = 12^2$

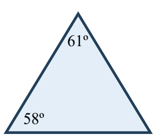
B The Pythagorean Theorem: Find the unknown side.

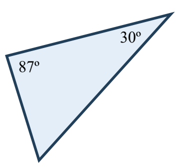




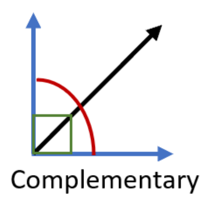
C Angle Theorems

Sum of a Triangle:
Angles in a triangle add up to ___________

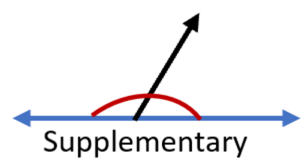


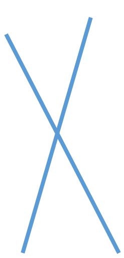


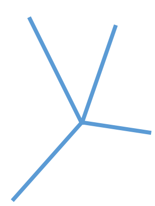
Comp $\angle s$ add to _____



Supp $\angle s$ add to _____

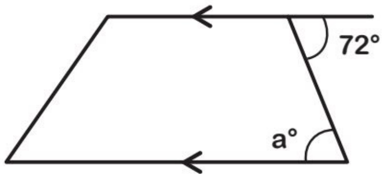




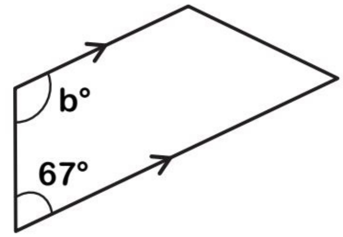


Let's now check if we remember any angle patterns from our elementary school geometry lessons ;)

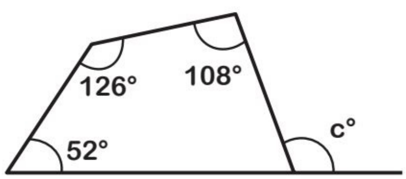
Calculate the missing angle and give a reason for your answer.



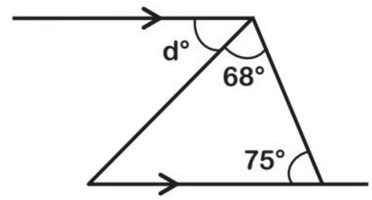
Angle a:	Reason:



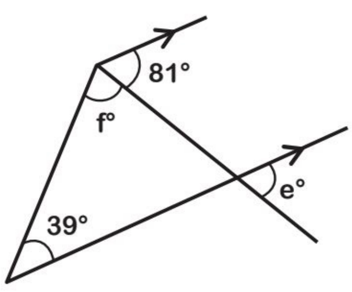
Angle b:	Reason:



Angle c:	Reason:

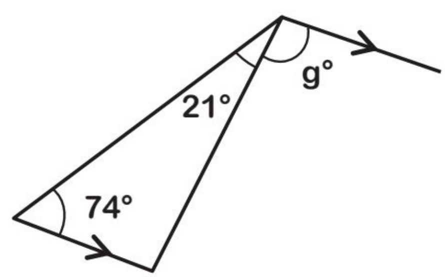


Angle d:	Reason:



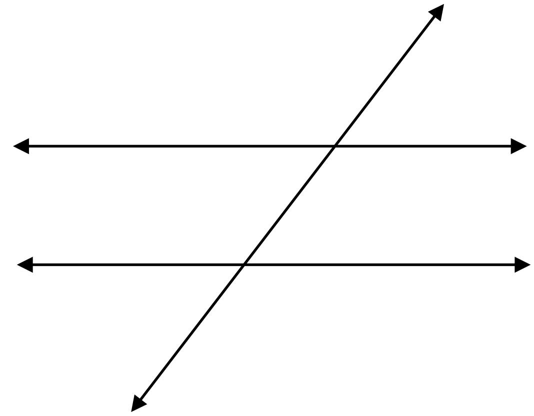
Angle e:	Reason:

Angle f:	Reason:



Angle g:	Reason:

Parallel Lines intersected by a TRANSVERSAL



Alternate Angles (Z) Co-Interior Angles (C) Corresponding Angles (F)

D Solving Proportions

$$\frac{3}{a} = \frac{5}{9}$$

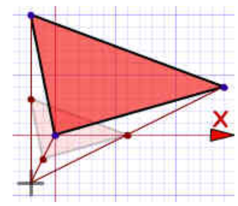
$$\frac{x}{19} = \frac{2.3}{3}$$

$$\frac{2}{x+7} = \frac{4}{11}$$

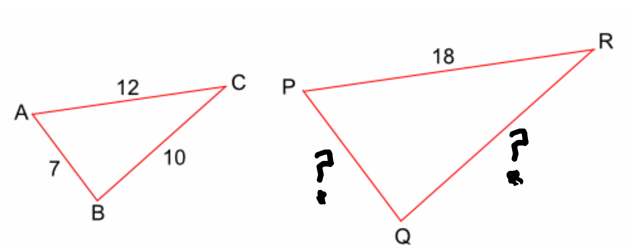
Lesson 3.1 Congruence and Similarity in Triangles

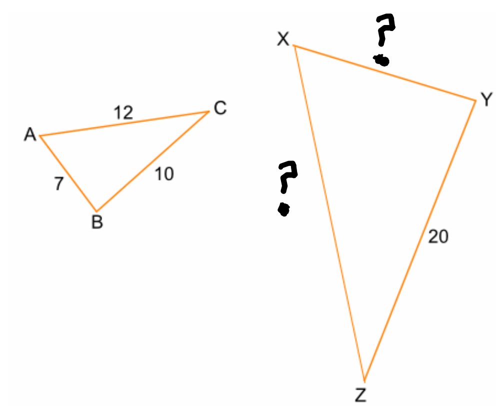
Congruent Triangles are triangles which

Similar triangles are triangles which have the exact same angle measures but the side lengths have a different scale i.e. side lengths may be resized. The scale factor which is a ratio of corresponding side lengths states how much bigger (or smaller) the second triangle is compared to the first triangle through proportions. You can use this proportion to solve for the unknown side lengths.



The following are similar triangles. Solve for unknown sides.





Mathematical Notation:

The ______ symbol is used to indicate similarity. The _______ symbol is used to indicate congruence.

> ***When naming triangles that are congruent or similar, the corresponding vertices must be listed in the same order.***

For example, if $\angle A = \angle D, \angle B = \angle E \; and \; \angle C = \angle F$, then $\Delta ABC \sim \Delta DEF$. (ABC ≠ EDF)

$\triangle ABC \sim \triangle RST$. Complete each statement.

a) $\angle ABC =$ ____

b) $\angle BCA =$ ____

c) $\dfrac{AB}{RS} =$ ____

d) $\triangle STR \sim$ ____

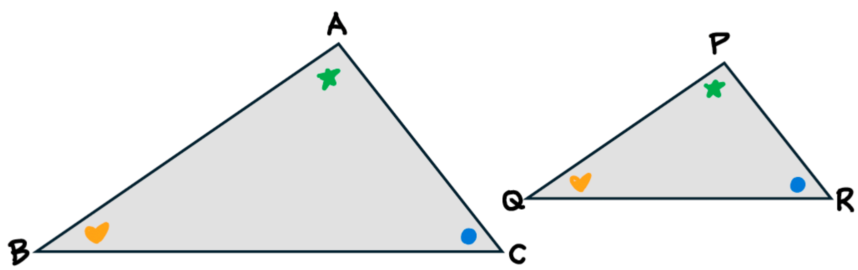
e) $\dfrac{ST}{BC} =$ ____

f) $\angle SRT =$ ____

But How Do You Know the given Triangles are SIMILAR?

Properties of Similar Triangles – Symbol: ________

- Similar triangles have the exact same shape BUT have a different scale



PROOF #1: Show that all three angles are equal (**AAA**)

$\angle ABC =$

$\angle BAC =$

$\angle ACB =$

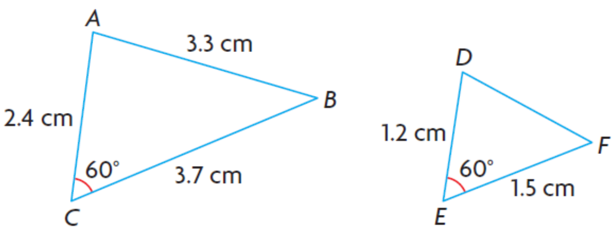
PROOF #2: Show that the ratios of the corresponding side lengths are equal (**SSS**)

$\dfrac{AB}{\quad} = \dfrac{BC}{\quad} = \dfrac{AC}{\quad}$

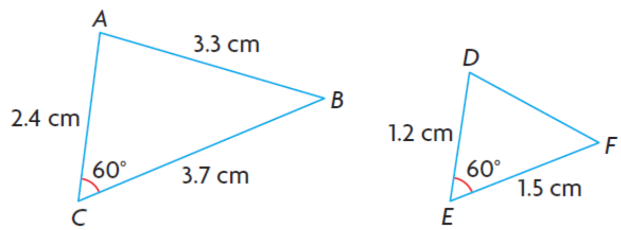
PROOF #3: (**SAS**) Show that 2 ratios of corresponding sides are equal and 1 angle is equal.

Similarity Proofs	
Angle-Angle (AA)	If two angles of one triangle are congruent to two angles of another triangle, the triangles are similar.
SSS proportional	If the three sets of corresponding sides of two triangles are in proportion, the triangles are similar.
SAS proportional	If an angle of one triangle is congruent to the corresponding angle of another triangle and the lengths of the sides including these angles are in proportion, the triangles are similar.

1. Is ΔDEF similar to ΔABC?

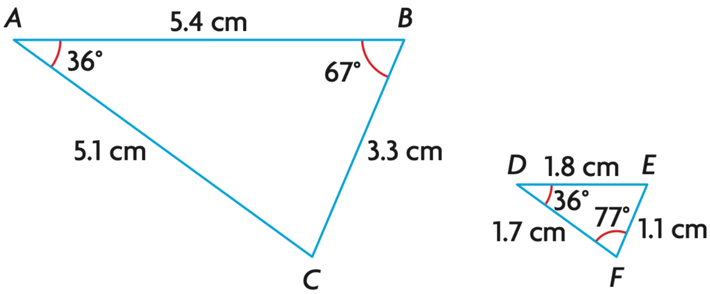


2. Is ΔDEF similar to ΔACB?
What is the length of side DF?
Provide Statement-Reason Proof.



Practice Example

1. Is $\triangle ABC \sim \triangle DEF$? Justify your answer.



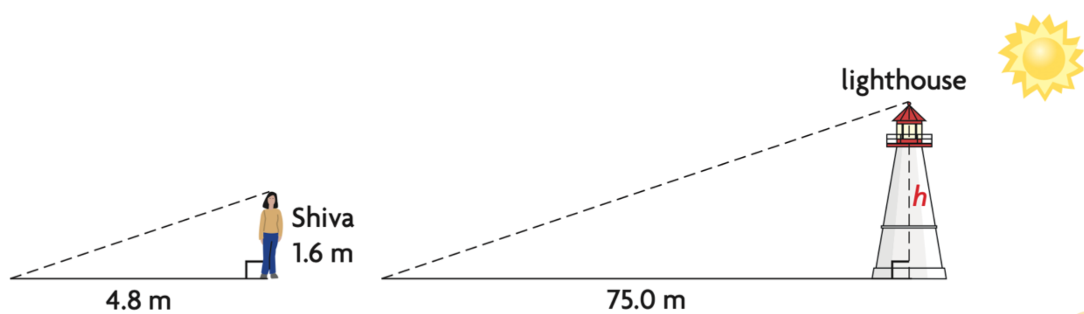
Example 2.

Suppose that $\triangle PQR \sim \triangle LMN$ and $\angle P = 90°$.

a) What angle in $\triangle LMN$ equals 90°? How do you know?

b) If $MN = 13$ cm, $LN = 12$ cm, $LM = 5$ cm, and $PQ = 15$ cm, what are the lengths of PR and QR?

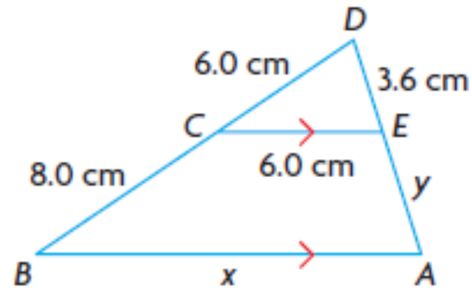
Example 3. Shiva is standing beside a lighthouse on a sunny day, as shown. She measures the length of her shadow and the length of the shadow cast by the lighthouse. Shiva is 1.6 m tall. How tall is the lighthouse?



Example 4

a) Show that the two triangles to the right are similar, with reasons.

b) Determine *x and y*



a)

Statement	*Reason*

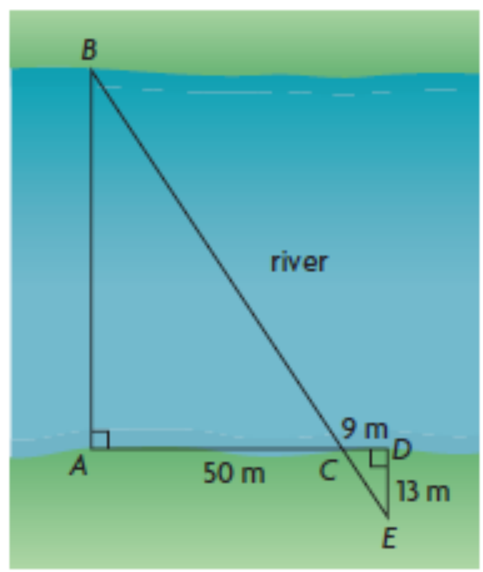
Example 5

A new bridge is going to be built across a river, but the width of the river cannot be measured directly. Surveyors set up posts at points A, B, C, D and *E*. Then they took measurements relative to the posts.

What is the width of the river?

a) Show that the two triangles in this diagram are similar.

b) Determine the width of the river



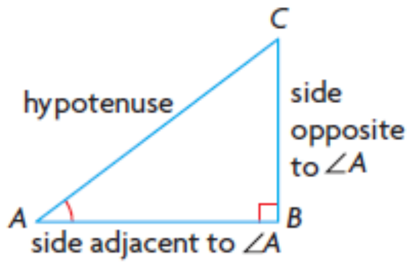
a)

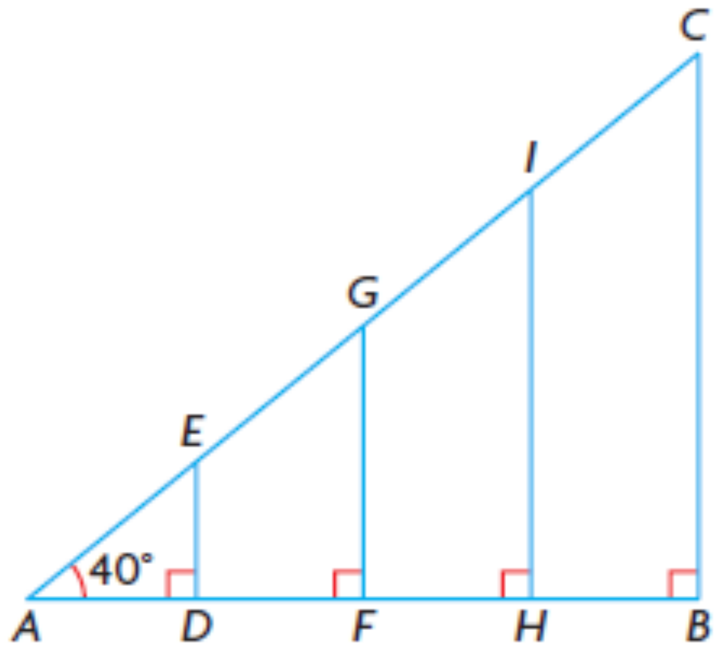
Statement	*Reason*

In order to solve "real world problems" you have to be SURE that the triangles you are working with are similar. All that is needed for proof of similarity is AA similarity.

Lesson 3.2a Exploring Similar Right Angle Triangles

ΔADE ~ ΔAFG therefore AD ~ AF
AE~ AG
DE ~ FG





Use a ruler and measure the side lengths then calculate the ratios.

				Trigonometric Ratios		
Triangle	Side OPPOSITE to ∠A	Side ADJACENT to ∠A	HYPOTENUSE	OPPOSITE / HYPOTENUSE	ADJACENT / HYPOTENUSE	OPPOSITE / ADJACENT
ΔABC						
ΔADE						
ΔAFG						
ΔAHI						

So, instead of saying $\frac{opp}{hyp}$, we call this ratio ________, and for $\frac{adj}{hyp}$ __________ and for $\frac{opp}{adj}$ _________

Mathematicians have calculated the side ratios for each possible angle and programmed the algorithms necessary into the scientific calculators. Even triangles with angles to the hundredth decimal place can be solved!

Having observed now that sides and angles of triangles are connected and related in so many different ways, we are ready to formally move into a brand-new branch of Mathematics called TRIGONOMETRY 😊

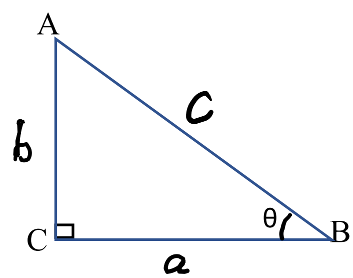
TRIGONOMETRY is the branch of Mathematics that deals with the properties of triangles and calculations based on these properties. As you will find out soon, Trigonometry has a lot of advantages in helping us solve real world problems.

Angle	Sine	Cosine	Tangent
1°	.0175	.9998	.0175
2°	.0349	.9994	.0349
3°	.0523	.9986	.0524
4°	.0698	.9976	.0699
5°	.0872	.9962	.0875
6°	.1045	.9945	.1051
7°	.1219	.9925	.1228
8°	.1392	.9903	.1405
9°	.1564	.9877	.1584
10°	.1736	.9848	.1763
11°	.1908	.9816	.1944
12°	.2079	.9781	.2126
13°	.2250	.9744	.2309
14°	.2419	.9703	.2493
15°	.2588	.9659	.2679
16°	.2756	.9613	.2867
17°	.2924	.9563	.3057
18°	.3090	.9511	.3249
19°	.3256	.9455	.3443
20°	.3420	.9397	.3640
21°	.3584	.9336	.3839
22°	.3746	.9272	.4040
23°	.3907	.9205	.4245
24°	.4067	.9135	.4452
25°	.4226	.9063	.4663
26°	.4384	.8988	.4877
27°	.4540	.8910	.5095
28°	.4695	.8829	.5317
29°	.4848	.8746	.5543
30°	.5000	.8660	.5774
31°	.5150	.8572	.6009
32°	.5299	.8480	.6249
33°	.5446	.8387	.6494
34°	.5592	.8290	.6745
35°	.5736	.8192	.7002
36°	.5878	.8090	.7265
37°	.6018	.7986	.7536
38°	.6157	.7880	.7813
39°	.6293	.7771	.8098
40°	.6428	.7660	.8391
41°	.6561	.7547	.8693
42°	.6691	.7431	.9004
43°	.6820	.7314	.9325
44°	.6947	.7193	.9657
45°	.7071	.7071	1.0000

Angle	Sine	Cosine	Tangent
46°	.7193	.6947	1.0355
47°	.7314	.6820	1.0724
48°	.7431	.6691	1.1106
49°	.7547	.6561	1.1504
50°	.7660	.6428	1.1918
51°	.7771	.6293	1.2349
52°	.7880	.6157	1.2799
53°	.7986	.6018	1.3270
54°	.8090	.5878	1.3764
55°	.8192	.5736	1.4281
56°	.8290	.5592	1.4826
57°	.8387	.5446	1.5399
58°	.8480	.5299	1.6003
59°	.8572	.5150	1.6643
60°	.8660	.5000	1.7321
61°	.8746	.4848	1.8040
62°	.8829	.4695	1.8807
63°	.8910	.4540	1.9626
64°	.8988	.4384	2.0503
65°	.9063	.4226	2.1445
66°	.9135	.4067	2.2460
67°	.9205	.3907	2.3559
68°	.9272	.3746	2.4751
69°	.9336	.3584	2.6051
70°	.9397	.3420	2.7475
71°	.9455	.3256	2.9042
72°	.9511	.3090	3.0777
73°	.9563	.2924	3.2709
74°	.9613	.2756	3.4874
75°	.9659	.2588	3.7321
76°	.9703	.2419	4.0108
77°	.9744	.2250	4.3315
78°	.9781	.2079	4.7046
79°	.9816	.1908	5.1446
80°	.9848	.1736	5.6713
81°	.9877	.1564	6.3138
82°	.9903	.1392	7.1154
83°	.9925	.1219	8.1443
84°	.9945	.1045	9.5144
85°	.9962	.0872	11.4301
86°	.9976	.0698	14.3007
87°	.9986	.0523	19.0811
88°	.9994	.0349	28.6363
89°	.9998	.0175	57.2900

Lesson 3.2b The Primary Trigonometric Ratios

Given the Right ΔABC



We use θ (Theta) to indicate angle in geometry.
*For any one of the two **non-right angles** θ, we have an **adjacent side** and an **opposite side**.*

In the given triangle,

c -

a -

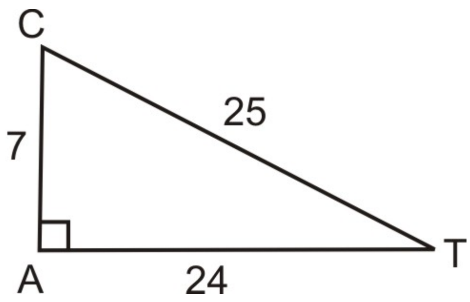
b -

The Trig Ratios

Sine: sin θ =

Cosine: cos θ =

Tangent: tan θ =



Examples ΔACT

Sine: sin T =

Cosine: cos T =

Tangent: tan T =

What is the Value of $\angle T$?

Sine *Cosine* *Tangent*

SOH CAH TOA

Let's Practice on our Scientific Calculators some more:

1. Find the value of each trigonometric ratio to the nearest ten-thousandth (_______ decimal places)

    a) $\sin 45=$ b) $cos38 =$ c) $tan80 =$

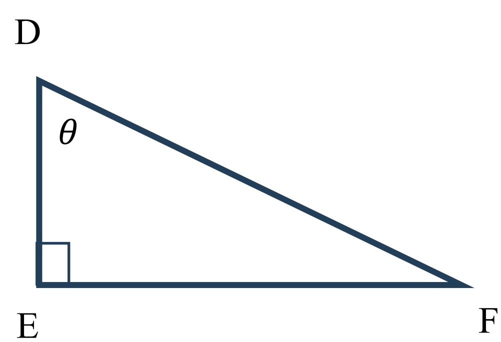
2. Find each angle measure to the nearest degree

    a) $sin\theta = 0.422618$ b) $cos\theta = 4.393267$ c) $tan\theta = 0.176327$

SOH CAH TOA

Given Right ΔDEF, label

- The Hypotenuse side
- The Adjacent side
- The Opposite side



The 3 TRIG RATIOS:

Sine: $\sin\theta =$	Cosine: $\cos\theta =$	Tangent: $\tan\theta =$

Lesson 3.3 Solving Right Angle Triangles

Solving for Sides using the Primary Trigonometric Ratios

Solve for the unknown in the following:

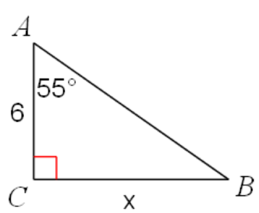
$\sin 35 = \frac{x}{8}$

$\tan 62 = \frac{3}{y}$

Notice: Pay close attention to when the unknown is in the numerator and when the unknown is in the denominator.

Steps to Solve:
1. Identify the given angle you are solving.
2. Identify 1 known side and one unknown side.
3. Write the appropriate Trig Ratio using #1 and 2 and solve

Solve for the unknown side in the following examples



SOH CAH TOA

SOH CAH TOA

Solving for Angles using the Primary Trigonometric Ratios

To solve for the angle, you must use the **INVERSE** function, which is $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$

Solve for θ in the following examples

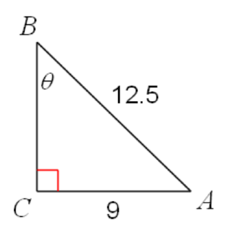
$\sin\theta=0.4782$ $\tan\theta=2.01$ $\cos\theta=\frac{3}{5}$

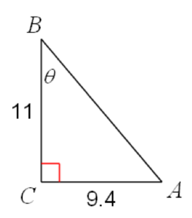
$\theta = \sin^{-1} 0.4782$

$\theta=$

Steps to Solve:

1. Identify the angle you are solving.
2. Identify 2 known sides.
3. Write the appropriate Trig Ratio using #2 and solve





Solve the Triangle (Find ALL missing measurements)

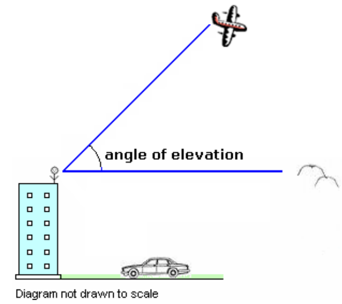
In ΔDEF, $\measuredangle E = 90^{\circ}$, $d = 7.2cm$, and $f = 5.8cm$. Solve the triangle.

(Remember to draw Rough Figures always!! Math is a visual subject and Drawings are your best Friends!)

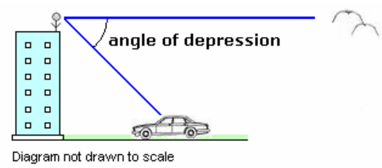
Lesson 3.4 Solving Right Triangle Real World Problems

Angle of Elevation vs Angle of Depression

The word "elevation" means "rise" or "move up". Angle of elevation is the angle between the horizontal and the line of sight to an object above the horizontal.



The word "depression" means "fall" or "drop". Angle of depression is the angle between the horizontal and the line of sight to an object beneath the horizontal.



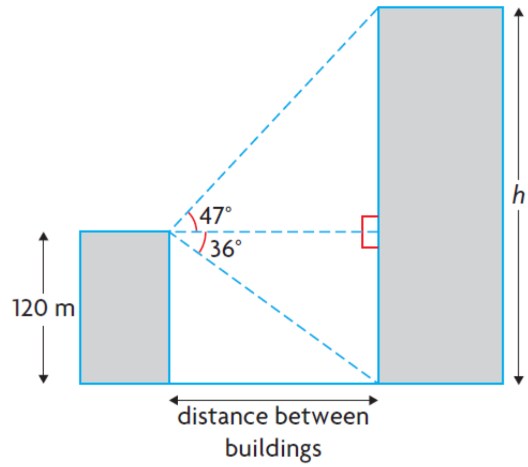
Word Problems

#4 A tree that is 9.5 m tall casts a shadow that is 3.8 m long. What is the angle of elevation of the Sun?

#6 A building code states that a set of stairs cannot rise more than 72 cm for each 100 cm of run. What is the maximum angle at which the stairs can rise?

#8 Firefighters dig a triangular trench around a forest fire to prevent the fire from spreading. Two of the trenches are 800 m long and 650 m long. The angle between them is 36°. Determine the area that is enclosed by these trenches.

#15 A video camera is mounted on top of a building that is 120 m tall. The angle of depression from the camera to the base of another building is 36°. The angle of elevation from the camera to the top of the same building is 47°a) How far apart are the two buildings?
b) How tall is the building viewed by the camera?



Understand that solving problems involves drawing a picture and then developing a plan to solve for the unknown.
This may take several steps, PATIENCE, and PRACTICE.

---*we've just concluded the first part of Trigonometry ☺*

Optional Extra Practice: Textbook Chapter 7.6 pg. 412, # 5, 7, 9-13, 16, 17

Lesson 3.5 Sine Law *(Super useful when it is NOT a Right Triangle!!)*

For the Sine Law we need: - An angle and its opposite side, and one other piece of information.

The Sine Law:

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC} \text{ or } \frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$$

where a is the side opposite $\angle A$, b is the side opposite $\angle B$, and c is the side opposite $\angle$C

If we are trying to **find an angle**, use the first form of the Sine Law (**angles on top**)

If we are trying to **find the length** of a side, use the second form of the law (**with sides on top**)

Ex 1. Calculate

$$a = (\frac{3}{sin60})sin72$$

Ex 2. Calculate:

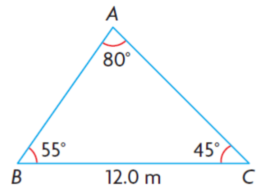
$$sinA = (\frac{sin72}{15})12$$

Ex. 3 – Solve for the given variable (correct to 1 decimal place) in each of the following:

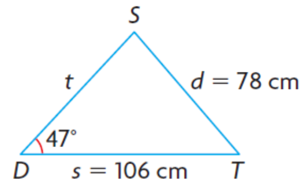
(a) $\frac{a}{\sin 55°} = \frac{12}{\sin 30°}$

(b) $\frac{35}{\sin 65°} = \frac{b}{\sin 38°}$

Ex 4 - Find the Length of Side b to the nearest tenth



Ex. 5 – Find ∠S to the nearest degree.



A telephone pole is supported by two wires on opposite sides. At the top of the pole, the wires form an angle of 60º. On the ground, the ends of the wires are 15.0 m apart. One wire makes a 45º angle with the ground. How long are the wires, and how tall is the pole?

Optional Extra Practice: Textbook Pg. 433 – 434 #2b, 3acdf, 4 – 8, 10, 11

Lesson 3.6 Cosine Law

Which Law do I Use?

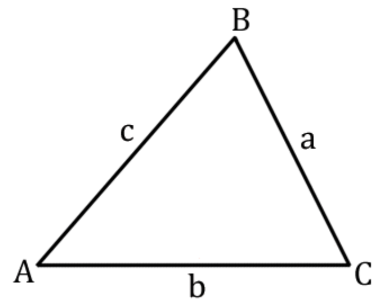
Do I have a right angle triangle? **YES - ** use SOH CAH TOA

↓ **NO**

Do I have a CORRESPONDING angle and side pair in the triangle? **YES** - use SINE LAW

↓ **NO**

Cannot use SOHCAHTOA or SINE Law Must use the **The COSINE LAW**



For the Cosine Law we need:
- 2 sides and the included angle.
- 3 sides

The cosine law is an extension of the Pythagorean theorem to triangles with no right angle.

To find a side (have SAS):

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

To find an angle (have SSS):

$$\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$$

$$\cos B = \frac{b^2 - a^2 - c^2}{-2ac}$$

$$\cos C = \frac{c^2 - a^2 - b^2}{-2ab}$$

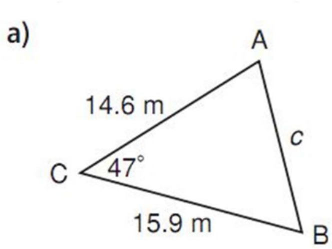
Ex 1. Calculate *a* if

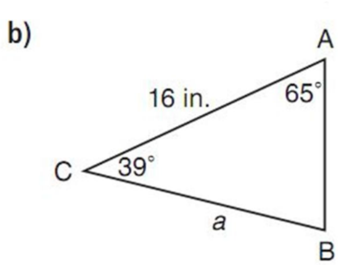
$$a^2 = 4^2 + 6^2 - 2(4)(6)\cos 56$$

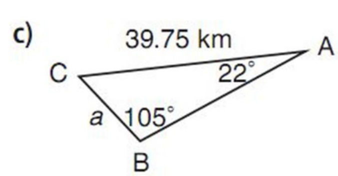
Ex 2. Calculate ∠X if:

$$\cos X = \frac{4.5^2 - 3.2^2 - 4.6^2}{-2(3.2)(4.6)}$$

Determine if you need to use the SINE law or the COSINE law for the following triangles:



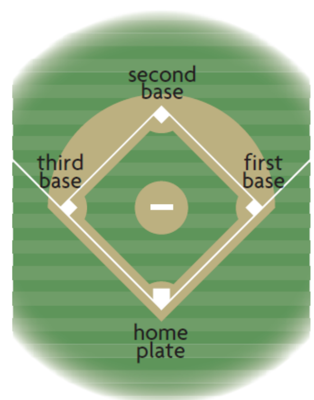




Draw the triangle and then find ∠A to the nearest tenth if a = 3m, b = 5m, and c = 4.7m

Draw the triangle then find side b, to the nearest tenth if ∠B = 68°, a = 17 cm, and c = 22 cm

Ex. 5 The bases in a baseball diamond are 90 ft apart. A player picks up a ground ball 11 ft from third base, along the line from second base to third base. Determine the angle that is formed between first base, the player's present position, and home plate.



Optional Extra Practice: Textbook page 443: 7, 9, 12 and page 449: 1 – 10, 15