Name: ______________________________

Analytic Geometry

Learning Goals:
We are learning to…

- use coordinates to determine and solve problems involving midpoints, slopes, and lengths of line segments
- determine the equation of a circle with centre (0,0)
- use properties of line segments to identify geometric figures and verify their properties

Analytic Geometry: Terms and Formulas

"Analytic Geometry" is using algebra to analyze geometric properties of shapes. The connection between the algebra and the geometry is through formulas which use the coordinates of points.

Some Terms

Line Segment – A part of a line between two points. For example

shows line segment $\overline{AB}$

Midpoint – The point in the middle of a line segment

$M_{\overline{AB}} = D(x, y)$

Median – A line segment in a triangle from one vertex to the midpoint of the opposite side

$\overline{AD}$ is a median of triangle *ABC*. *D* is the midpoint of $\overline{BC}$

Midsegment – A midsegment is a line segment inside a triangle which joins the midpoints of two sides of the triangle.

If *P* is the midpoint of $\overline{LM}$, and
Q is the midpoint of $\overline{MN}$, then
$\overline{PQ}$ is a midsegment of triangle *LMN*

Note: The slope of $\overline{PQ}$ is equal to the slope of $\overline{LN}$

Perpendicular Bisector – A line which cuts a line segment in half, and which is also perpendicular to that line segment.

Note that point *P* is the midpoint of $\overline{MN}$, and that the slope of line *l* is the negative reciprocal of the slope of $\overline{MN}$

Altitude – A line segment inside a triangle from one vertex, and perpendicular to the opposite side

$\overline{AD}$ is an altitude of triangle *ABC*

The slope of $\overline{AD}$ is the negative reciprocal of the slope of $\overline{BC}$

Formulas

Slope of a line (or line segment) – Given two points on a line $A(x_1, y_1)$ and $B(x_2, y_2)$, then

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation of a line –
The equation is:

$y = mx + b$
(slope-intercept form), or

$y - y_1 = m(x - x_1)$
(slope-point form)

Midpoint – Given a line segment $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$, then

$$M_{\overline{AB}} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Length of a line segment (or distance between two points) - Given a line segment $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$, then the length of $\overline{AB}$ is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Equation of a Circle – A circle centered at (h, k), and with radius r has the equation

$$x^2 + y^2 = r^2$$
(with centre (0,0))

Lesson 1- (2.0) Writing Equation of a Line

y-intercept

$y = mx + b$

slope

$m = \frac{Rise}{Run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Need 3 things (always):

1. Slope: m (given or calculated)
2. Point: (x, y) or (0,b) ***NOTE:* *(0,b) is y-intercept***
3. Formula to find equation of line: y = mx + b

Type 1 Problem:

$m = 4/5 \quad b = -7$

When you know the y-intercept, use the **Slope-Intercept Form, *y* = m*x* + b**

1. Identify the y-intercept (**b**) and slope (**m**)
2. Write the equation replacing the b and m

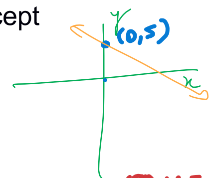
$\therefore$ EQUATION:

$y = \frac{4}{5}x - 7$

Type 2 Problem:

When you have 2 points, one of which is obviously y-intercept

1. Use the 2 points to calculate the slope (**m**)
2. Recognize that **(0,b)** is the y-intercept
3. Write the equation replacing **b** and **m**



$y = mx + b$

$P_1 \overset{x_1\, y_1}{(0, 5)} \quad P_2 \overset{x_2\, y_2}{(3, -3)}$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 5}{3 - 0} = \frac{-8}{3}$

(0, #)

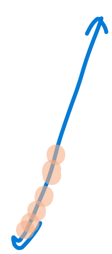
(0, b) $\quad \therefore b = 5$

EQUATION $\boxed{y = -\frac{8}{3}x + 5}$

Type 3 Problem: When you have 2 points, neither which are the y-intercept,

Slope-Intercept Form, *y* = m*x* + b

1. Use the 2 points to calculate the slope (**m**)
2. Sub in **m** and **a point for (x,y)**
3. Solve for **b**
4. Write the equation replacing the b and m



$\overset{x_1\, y_1}{(2, -5)} \quad \overset{x_2\, y_2}{(1, 3)}$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - -5}{1 - 2} = \frac{8}{-1} = -8$

$y = mx + b$

$3 = (-8)(1) + b$

$3 = -8 + b$

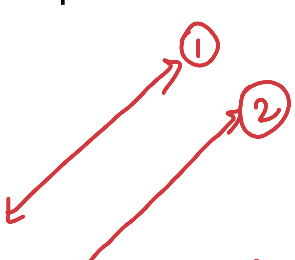
$3 + 8 = b \longrightarrow b = 11$

$\therefore$ EQUATION:

$y = -8x + 11$

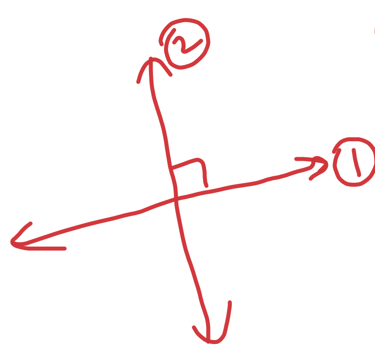
Also Note:

Slopes of Parallel Lines



$m_1 = m_2$

Slopes of Perpendicular Lines



NEGATIVE RECIPROCAL

1. FLIP fraction
2. SWITCH SIGNS

$m_{①} = \frac{a}{b}$

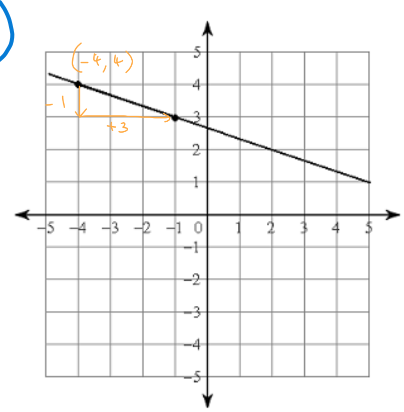
$m_{\perp} = m_{②} = \frac{-b}{a}$

$$y = mx + b$$

Let's Practice!

Find the equation of the following line

$m = \frac{-1}{3}$ $\qquad (x, y) = (-4, 4)$



$y = mx + b$

$4 = \frac{-1}{3}(-4) + b$

$4 = \frac{4}{3} + b$

$b = \frac{3 \times 4}{3 \times 1} - \frac{4}{3} = \frac{12 - 4}{3} = \frac{8}{3}$

∴ EQUATION of LINE

$$\boxed{y = -\frac{1}{3}x + \frac{8}{3}}$$

Determine the equation of the line that:

a) passes through $(\underset{x_1}{-1}, \underset{y_1}{7})$ and $(\underset{x_2}{2}, \underset{y_2}{14})$

$m = \frac{14 - 7}{2 - -1} = \frac{7}{3}$

$y = mx + b$

$7 = \frac{7}{3}(-1) + b$

$7 = -\frac{7}{3} + b$

$b = 7 + \frac{7}{3} = \frac{28}{3}$

∴ EQUATION: $\boxed{y = \frac{7}{3}x + \frac{28}{3}}$

b) is Perpendicular to $\boxed{y = -2x - 3}$ and passes through $(\underset{x,}{2}, \underset{y}{-5})$

$m = \frac{-2}{1}$

$\therefore m_{\perp} = \frac{1}{2}$

$y = m_{\perp}x + b$

$-5 = \frac{1}{2}(2) + b$

$-5 = 1 + b$

$-5 - 1 = b$

$b = -6$

∴ EQUATION: $y = \frac{1}{2}x - 6$

$$\boxed{y = 0.5x - 6}$$

More Practice:

c) passes through (0, 4) and (-2, -7)

$x_1\ y_1 \quad x_2\ y_2$

$b = 4$

$m = \frac{-7-4}{-2-0} = \frac{-11}{-2} = \frac{11}{2} = 5.5$

$\therefore$ EQUATION: $\boxed{y = 5.5x + 4}$

d) parallel to line having m = 3 and passes through (1, 3)

x, y

$m = 3$

$y = mx + b$

$3 = 3(1) + b$

$3 = 3 + b$

$b = 3 - 3 = 0$

$\therefore$ EQUATION:

$y = 3x + 0$

$\boxed{y = 3x}$

e) is perpendicular to $y = -2x - 3$ and passes (3, 4)

$m = -2 \quad x \quad y$

$m_\perp = \frac{1}{2} = 0.5$

$y = m_\perp x + b$

$4 = 0.5(3) + b$

$4 = 1.5 + b$

$4 - 1.5 = b$

$b = 2.5$

$\therefore$ EQUATION: $\boxed{y = 0.5x + 2.5}$

f) is parallel to $2x - 3y = 8$ and passes through (2, -5)

x, y

$2x - 3y = 8$

$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{8}{-3}$

$y = \frac{2}{3}x - \frac{8}{3}$

$m = \frac{2}{3}$

$y = mx + b$

$-5 = \frac{2}{3}(2) + b$

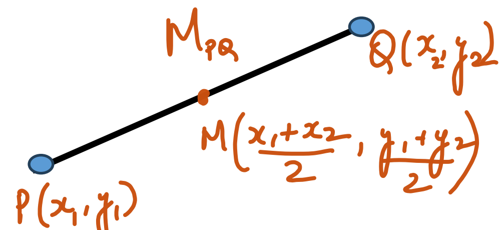
$-5 = \frac{4}{3} + b$

$\frac{3\times -5}{3\times 1} - \frac{4}{3} = b$

$b = \frac{-15-4}{3} = -\frac{19}{3}$

$\therefore$ EQUATION: $\boxed{y = \frac{2}{3}x - \frac{19}{3}}$

Lesson 2 (2.1) Midpoint



Find the Midpoint of a line – The point in the middle of a line segment

Question: If you scored a 70% on a test and then an 82% on the next test, how do you calculate the average of those tests?

$$\frac{70+82}{2} = 76\%$$

Similarly, the coordinates of the midpoint (M) of a line is the midpoint (average) of the x-values and the midpoint of the y-values $M_{\overline{AB}} = D(x, y)$

$$M_{\acute{A}B} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Examples

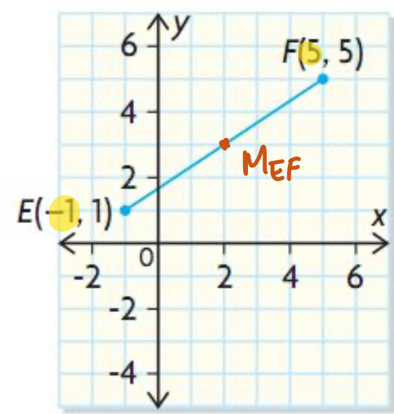
From your text: Pg. 78 #2a

Determine the coordinates of the midpoint of the line segment.

Note: *Since you have been given a graph, midpoint can be found using two methods: graphically or algebraically*

$$M_{EF} = \left(\frac{-1+5}{2}, \frac{1+5}{2}\right)$$

$$M_{EF} = (2, 3)$$



Let's practice some more:

a) $C(9,8)$ and $D(3,22)$

$x_1\ y_1 \quad x_2\ y_2$

$$M_{CD} = \left(\frac{9+3}{2}, \frac{8+22}{2}\right)$$

$$M_{CD} = (6, 15)$$

b) $E(5.6,-3.3)$ and $F(-12.2,-3.3)$

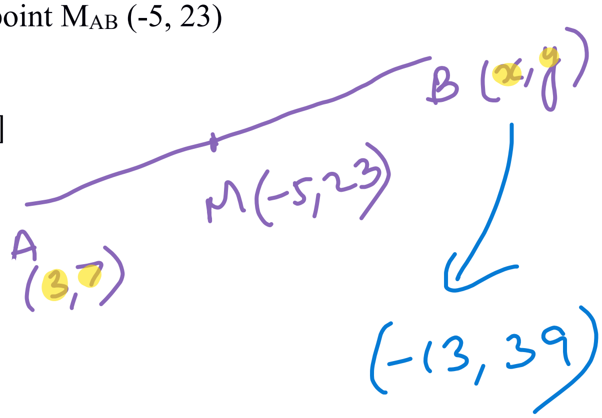
$x_1\ y_1 \quad x_2\ y_2$

$$M_{EF} = \left(\frac{5.6+-12.2}{2}, \frac{-3.3+-3.3}{2}\right)$$

$$M_{EF} = (-3.3, -3.3)$$

c) Line AB segment has the endpoint A (3, 7) and the Midpoint M_{AB} (-5, 23)
What are the coordinates of end point B ?

[*Hint: Draw the situation to understand the problem better*]



$$M = \left(\frac{x_1+x}{2}, \frac{y_1+y_2}{2}\right)$$

$$(-5, 23) = \left(\frac{3+x}{2}, \frac{7+y}{2}\right)$$

$-5 = \frac{3+x}{2}$

$-10 = 3+x$

$x = -10-3 = -13$

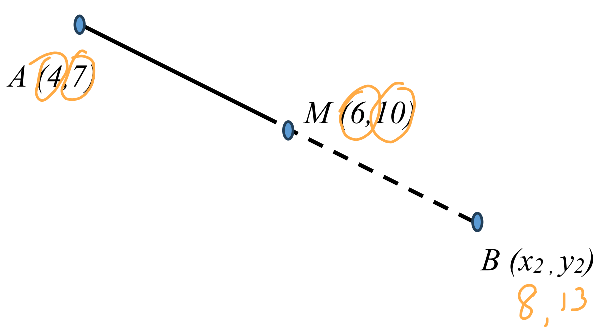
$23 = \frac{7+y}{2}$

$46 = 7+y$

$46-7 = y$

$39 = y$

Now, here's a quick method:
Just remember to "DOUBLE THE MIDPOINT AND SUBTRACT AN ENDPOINT."



$x_2 = 6(2)-4$

$y_2 = 10(2)-7$

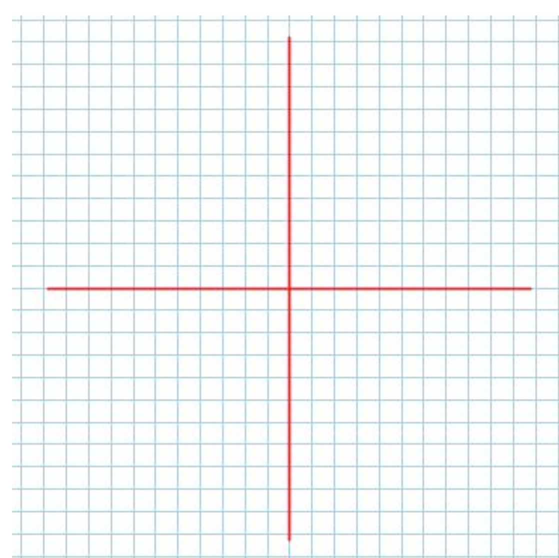
Practice:

1. X (2,5) M_{XY} (7, 11) Y (12 , 17)

2. A (-3, 4) M_{AB} (2, -6) B (7 , -16)

3. D (5,-6) M_{DE} (-4, 12) E (-13 , 30)

~~*Graph #3 to check.*~~



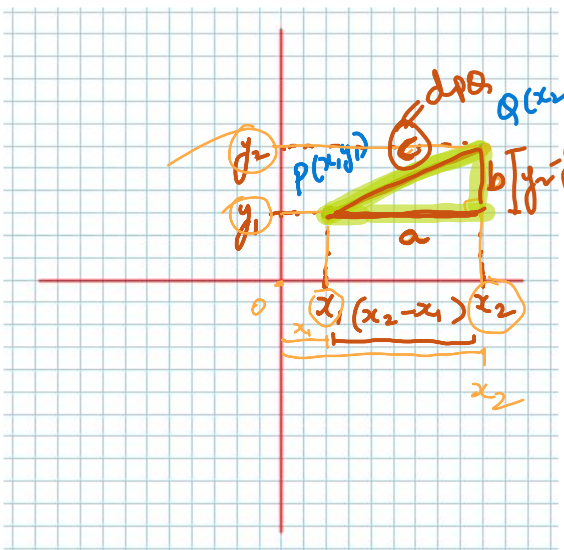
(2.2) Length of a line segment (distance between two points)

Given a line segment $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$, then the length of $\overline{AB}$ can be found by using formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Find the length of a line segment with endpoints A (1, 6) and B (-5, -6).

(*Let's also check our formula and our answer on the graph.*
Think of the Pythagorean Theorem $a^2 + b^2 = c^2$)



$c^2 = a^2 + b^2$

$c = \sqrt{a^2 + b^2}$

$d_{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

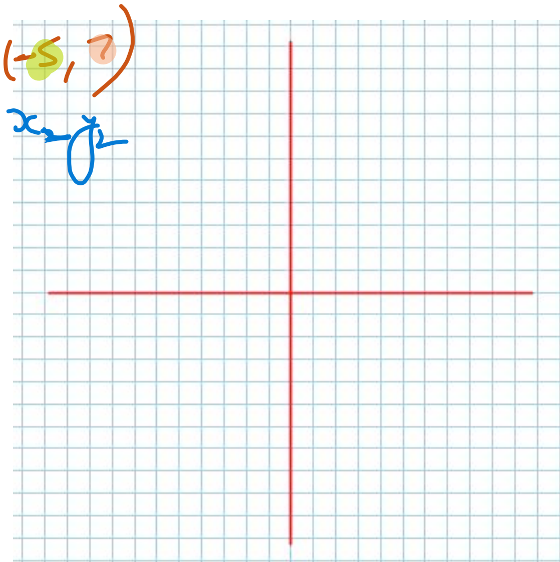
Example: Find the length of the line from (1, 2) to (-5 , 7) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

A B

$d_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

A (1, 2)
x_1 y_1

B(-5, 7)
x_2 y_2



$d_{AB} = \sqrt{(-5 - 1)^2 + (7 - 2)^2}$

$= \sqrt{36 + 25}$

$d_{AB} \approx 7.8$ units

Let’s practice some more:

a) $G(-4,10)$ and $H(8,12)$

$$d_{GH} = ?$$

$$= \sqrt{(8--4)^2 + (12-10)^2}$$

$$= \sqrt{144 + 4}$$

$$= \sqrt{148}$$

$$\approx 12.17 \approx 12.2 \text{ units}$$

b) $I(12,1)$ and $J(3,-6)$

$$d_{IJ} = ?$$

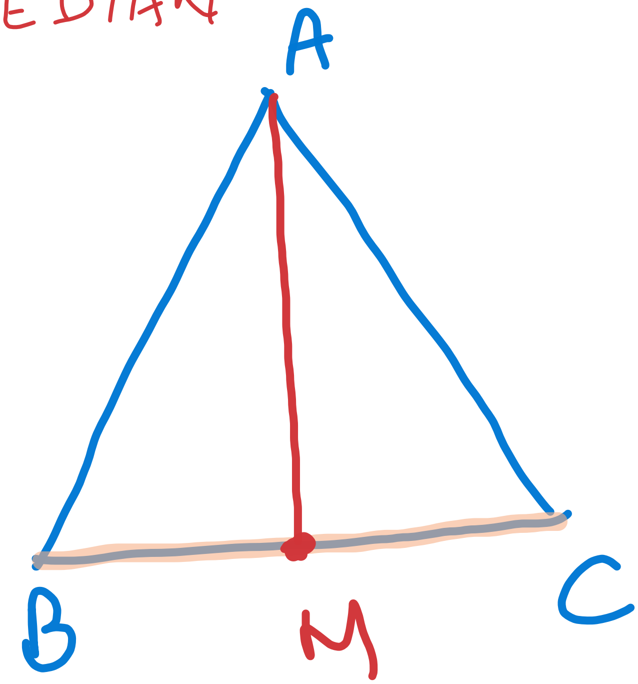
$$= \sqrt{(3-12)^2 + (-6-1)^2}$$

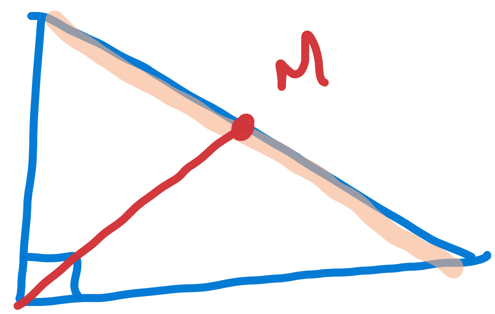
$$= \sqrt{81 + 49}$$

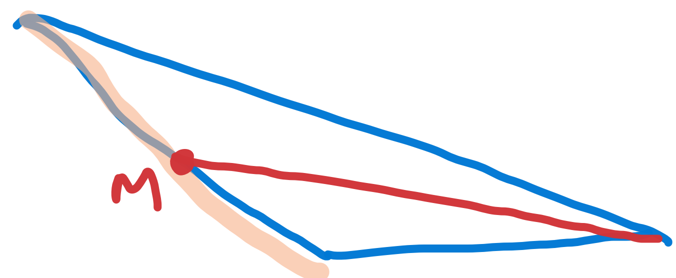
$$= \sqrt{130}$$

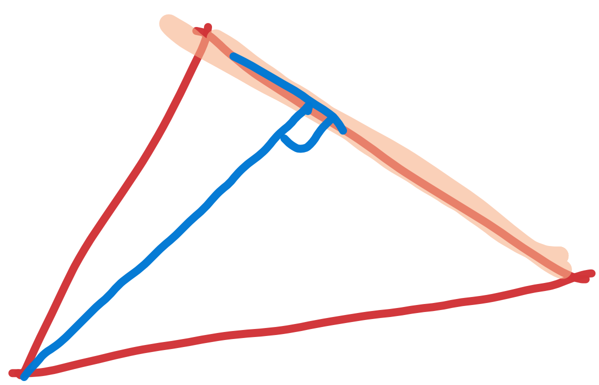
$$\approx 11.4 \text{ units}$$

MEDIAN

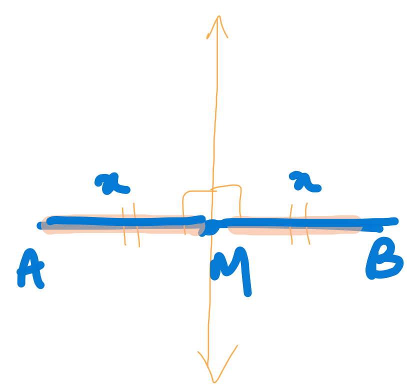




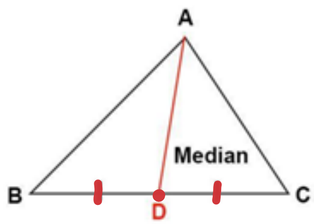
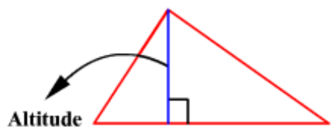
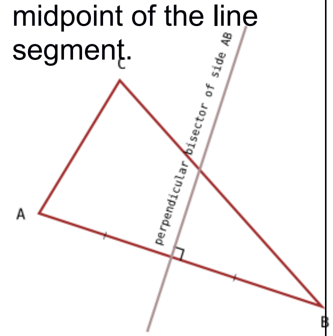
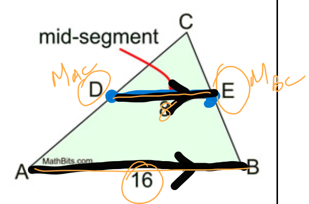




PERPENDICULAR BISECTOR



Lesson 3 (2.2) Equations of Lines found in Triangles

Median of a triangle: Line segment that joins a vertex of a triangle to the midpoint of the other side.  	Altitude of a triangle: Line segment from a vertex which meets the opposite side at a 90° angle.  	Perpendicular bisector of a line segment: line that is perpendicular to the line segment and passes through the midpoint of the line segment.  	Midsegment of a triangle: Line segment that connects two midpoints 

Find the equation of a MEDIAN

A Median is a line segment in a triangle from one vertex to the midpoint of the opposite side.

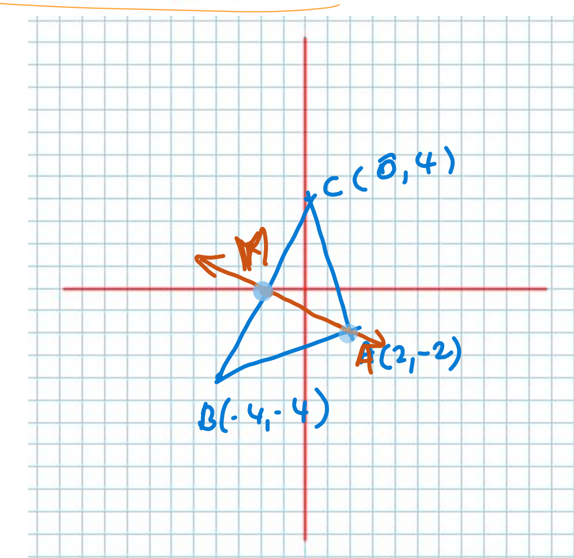
7. A triangle has vertices at $A(2, -2)$, $B(-4, -4)$, and $C(0, 4)$.

K a) Draw the triangle, and determine the coordinates of the midpoints of its sides.

b) Draw the median from vertex A, and determine its equation.

From your text: Pg. 79 #7

Let's begin by doing the drawings on the graph And thinking about the algebraic process to determine the midpoints and the equation of median.



Mid-points: $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Median: $\boxed{y = mx + b}$

Step 1: M_{BC}: (,)

Step 2: $m_{AM} = \frac{y_2-y_1}{x_2-x_1}$

Step 3:

$b = ?$ m_{AM} ; A

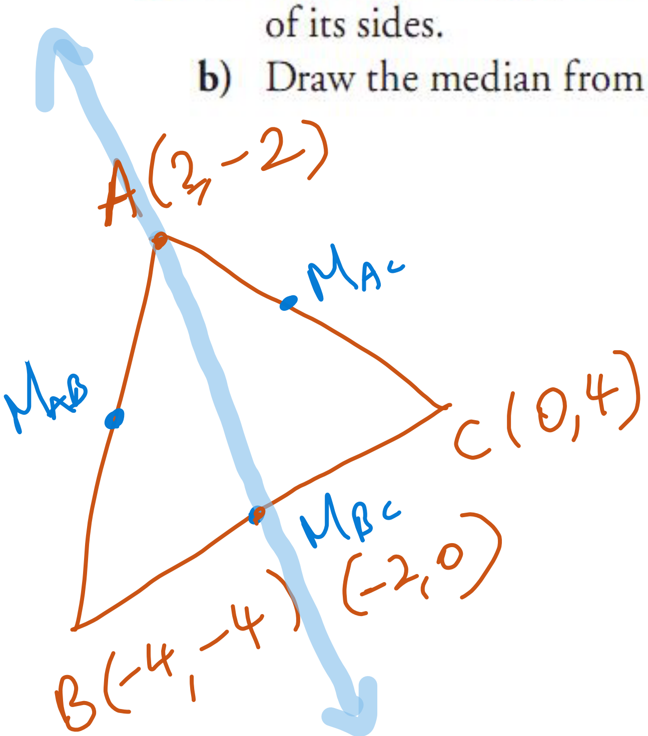
and now finally time to flex our algebraic muscles to determine

the midpoint coordinates and the median equation

7. A triangle has vertices at $A(2, -2)$, $B(-4, -4)$, and $C(0, 4)$.

K **a)** Draw the triangle, and determine the coordinates of the midpoints of its sides.

b) Draw the median from vertex A, and determine its equation.



$$M_{AB} = \left(\frac{2-4}{2}, \frac{-2-4}{2}\right) = (-1,-3)$$

$$M_{BC} = \left(\frac{-4+0}{2}, \frac{-4+4}{2}\right) = (-2,0)$$

$$M_{AC} = \left(\frac{2+0}{2}, \frac{-2+4}{2}\right) = (1,1)$$

$y = mx + b$

$$m = \frac{0 - -2}{-2-2} = \frac{2}{-4} = \frac{-1}{2} = -0.5$$

$A(2,-2)$

$\boxed{m = -0.5}$

$\overset{-2}{y} = \overset{-0.5}{m}\overset{2}{x} + b$

$-2 = (-0.5)(2) + b$

$-2 = -1 + b$

$-2 + 1 = b$

$\boxed{-1 = b}$

$\therefore$ EQUATION of MEDIAN:

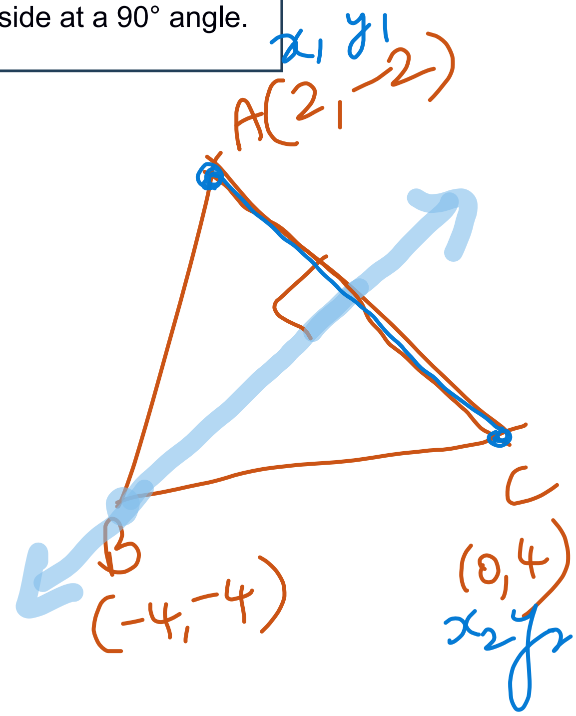
$y = -0.5x - 1$

Find the equation of an ALTITUDE

Let's use the same triangle ABC we used in the above question to determine the equation of an altitude from vertex B.

(Hint: Always start by drawing the diagram. This helps you visualize and to understand the problem better!!!)

> An Altitude is a line segment from a vertex which meets the opposite side at a 90° angle.



Step 1: $m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$

Step 2: $m_\perp$ = NEGATIVE RECIPROCAL

Step 3: $b = ?$ $\quad m_\perp$, B(-4, -4)

$y = m_\perp x + b$

1.) $m_{AC} = \frac{4 - -2}{0 - 2} = \frac{6}{-2} = -3$

2.) $m_\perp = \frac{1}{3}$; B (-4, -4)

$\overset{-4}{\cancel{y}} = \underset{\frac{1}{3}}{\cancel{m}}\ \overset{-4}{\cancel{x}} + b$

$-4 = \frac{1}{3}(-4) + b$

$-4 = \frac{-4}{3} + b$

$\frac{3 \times}{3 \times 1} \quad -4 + \frac{4}{3} = b$

$\frac{-12 + 4}{3} = b$

$\boxed{\frac{-8}{3} = b}$

∴ EQUATION:

$\boxed{y = \frac{1}{3}x - \frac{8}{3}}$

Find the equation of a PERPENDICULAR BISECTOR to a line segment

Perpendicular Bisector is a line that is perpendicular to the line segment and passes through the midpoint of the line segment.

Note that the Perpendicular Bisector cuts a line segment in half, and which is also perpendicular to that line segment. Perpendiculars therefore have slopes which are the negative reciprocal of the slope of given line segment ie. $m_1 = \frac{3}{2}$, $m_2 = -\frac{2}{3}$

From your text: Pg. 80 #13a

13. Determine an equation for the perpendicular bisector of a line segment with each pair of endpoints.

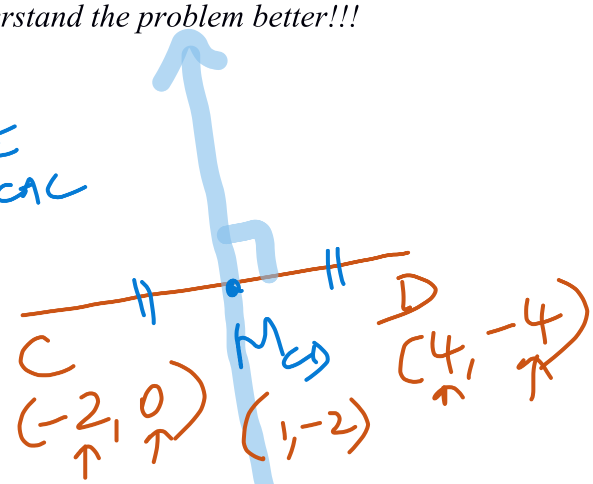
a) $C(-2, 0)$ and $D(4, -4)$

Always start by drawing the diagram. This helps you visualize and to understand the problem better!!!

Step 1: $M_{CD} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Step 2: $m_{CD} = \frac{y_2-y_1}{x_2-x_1}$; $m_\perp$ = NEGATIVE RECIPROCAL

Step 3: $b = ?$ $\quad m_\perp, M_{CD}$



$\boxed{y = m_\perp x + b}$

1.) $M_{CD} = \left(\frac{-2+4}{2}, \frac{0-4}{2}\right) = (1, -2)$

2.) $m_{CD} = \frac{-4-0}{4--2} = \frac{-4}{6} = -\frac{2}{3} \Rightarrow \boxed{m_\perp = \frac{3}{2}} = 1.5$

3.) $\overset{-2}{y} = \overset{\frac{3}{2}}{m}\overset{1}{x} + b$

$-2 = (1.5)(1) + b$

$-2 = 1.5 + b$

$-2 - 1.5 = b$

$\boxed{-3.5 = b}$

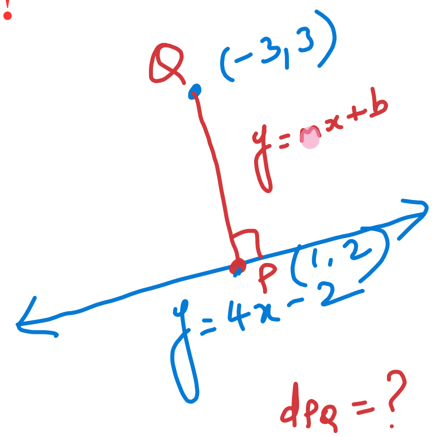
$\therefore$ EQUATION

$\boxed{y = 1.5x - 3.5}$

Now time for the BIGGEST QUESTION!!

Example (From your Text: Pg. 87 #12a)

Calculate the distance between the line y = 4x – 2 and the point (-3, 3)



***The shortest distance is a line perpendicular to y=4x - 2**

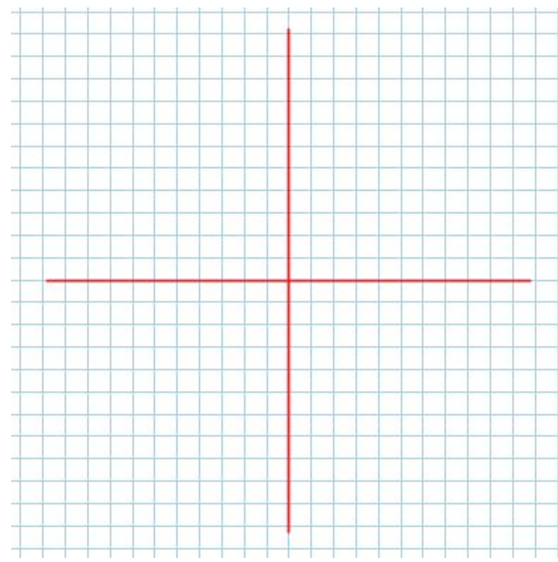
Before trying to work on the solution, CONQUER the PROBLEM!!!!

Step 1 find m_1 and m_2

Step 2 find the equation of the perpendicular line

$y = mx + b$ (slope-intercept form), or

~~$y - y_1 = m(x - x_1)$ (slope-point form)~~



Step 3 Find the POI of the two lines. Solve the system by substitution or elimination

Step 4 Find the length of the line from the POI to (-3, 3)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

1.) $m = 4$

$\therefore m_\perp = -\frac{1}{4}$

2.) $\cancel{y}^{3} = \cancel{m}^{-\frac{1}{4} = -0.25}\cancel{x}^{-3} + b$

$3 = (-0.25)(-3) + b$

$3 = 0.75 + b$

$3 - 0.75 = b$

$\boxed{2.25 = b}$

$\therefore$ EQUATION: $y = -0.25x + 2.25$

3.) $y = 4x - 2$

$y = -0.25x + 2.25$

$0 = 4.25x - 4.25$

$\frac{4.25}{4.25} = \frac{4.25x}{4.25}$

$\boxed{1 = x}$

$\therefore y = 4(1) - 2 = 4 - 2 = 2$

$POI = (1, 2)$

4.) d_{PQ} ; $P(\overset{x_1}{1}, \overset{y_1}{2})$
$Q(\underset{x_2}{-3}, \underset{y_2}{3})$

$d = \sqrt{(-3-1)^2 + (3-2)^2}$

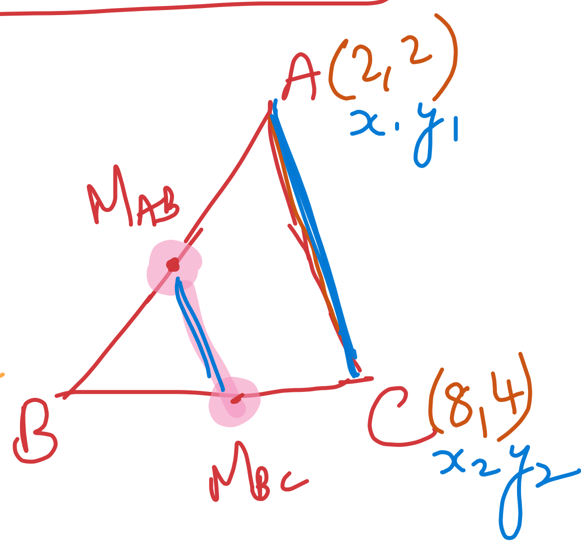
$d = \sqrt{16+1} = \sqrt{17}$

$\boxed{d \approx 4.12 \text{ units}}$

Midsegments

A midsegment is line segment formed by two midpoints.

Plot the triangle $A(2,2)$, $B(4,8)$, $C(8,4)$. Draw the midsegment from line AB to line BC. Calculate its length.



Step 1: $M_{AB} = (\overset{x_1}{3}, \overset{y_1}{5})$

$M_{BC} = (\overset{x_2}{6}, \overset{y_2}{6})$

$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Step 2: $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$d = \sqrt{(6-3)^2 + (6-5)^2} = \sqrt{9+1} = \sqrt{10} \approx 3.16 \text{ units}$
≈ 3

Now compare with the length of AC (for "fun").

$d_{AC} = \sqrt{(8-2)^2 + (4-2)^2}$

$= \sqrt{36+4} = \sqrt{40} \approx 6.3 \text{ units}$
≈ 6

$\therefore$ length of side AC = Twice length of mid segment.

Lesson 4 (2.3) The Equation of a Circle centered at (0, 0)

Analytic Definition of a Circle (i.e. the equation)

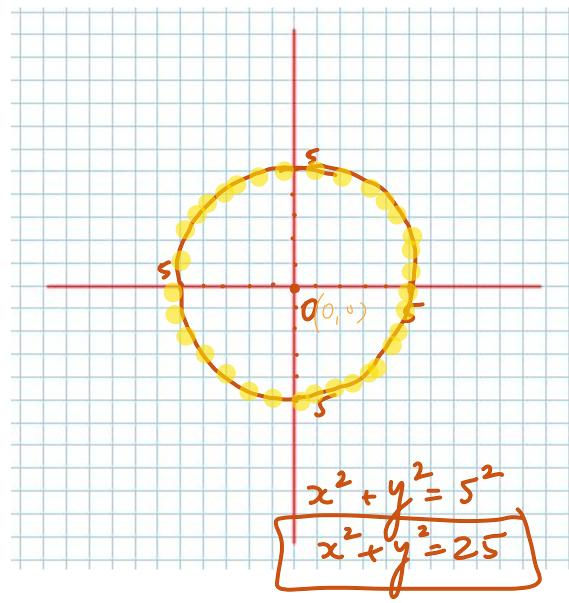
radius · origin

A **Circle** is a set of points which are all the same **distance** from a fixed central point.

$$\boxed{x^2+y^2=r^2}$$

$(x, y) \rightarrow$ points on the circle

$r \rightarrow$ radius



1. Determine the radius of the circle. $x^2 + y^2 = 25$. $= r^2$

$r = \sqrt{25} = 5$ units

2. Consider the sketch of a circle. Determine:

a) *x* intercepts

(2, 0) and (−2, 0)

b) *y* intercepts

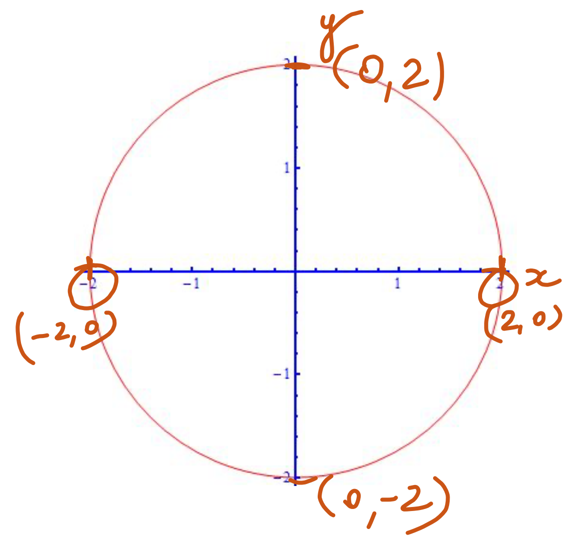
(0, 2) and (0, −2)

c) the radius of the circle

2 units

d) the equation of the circle

$x^2+y^2=4$



3. Determine the equation of a circle with radius $r = 5\frac{2}{3}$. $= \frac{17}{3}$

$x^2+y^2=\left(\frac{17}{3}\right)^2=\frac{17^2}{3^2}=\frac{289}{9}$

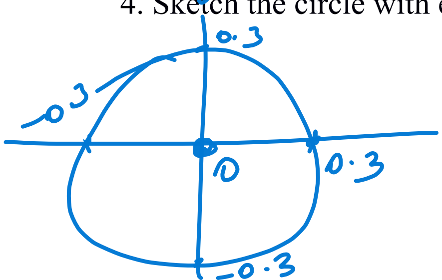
$\therefore$ EQUATION $\Rightarrow \boxed{x^2+y^2=\frac{289}{9}}$ $\rightarrow 9(x^2+y^2)=289$

$\Rightarrow 9x^2+9y^2=289$

Rough

4. Sketch the circle with equation $x^2 + y^2 = 0.09$

$r = \sqrt{0.09} = 0.3$

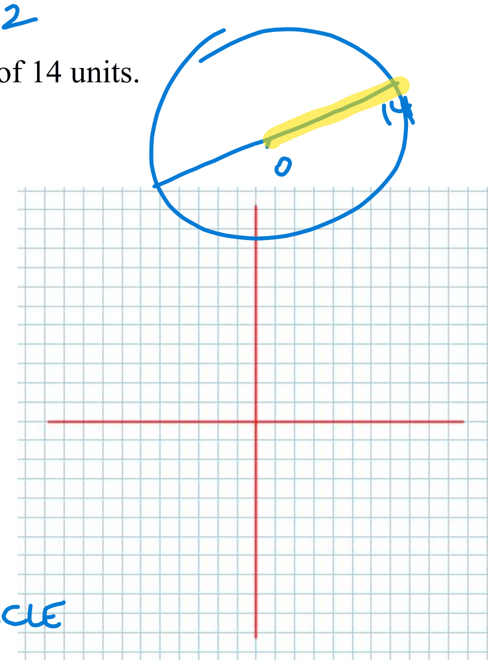


$r = \frac{14}{2} = 7$

5. Determine the equation of a circle with center at (0, 0) and a **diameter** of 14 units.

$x^2 + y^2 = 7^2$

$\boxed{x^2 + y^2 = 49}$



6. Determine whether the point (4.3, -2.6) is inside, outside or on the circle with equation $x^2 + y^2 = 25$

$x^2 + y^2 = (4.3)^2 + (-2.6)^2$

$x^2 + y^2 = 18.5 + 6.76$

$x^2 + y^2 = 25.26$

$\boxed{x^2 + y^2 = 25}$

∴ POINT IS OUTSIDE THE CIRCLE

On - if the answer is ___=___ to r^2 then the point is on the circle

Inside – if the answer is less than r^2, then the point is inside the circle

Outside - if the answer is more than r^2 than the point is ~~inside~~ outside the circle

What about the point (3, 4)? Is it on, in or outside the circle?

$x^2 + y^2 = 25$

$x^2 + y^2 = 3^2 + 4^2 = 9 + 16 = 25$

∴ (3,4) IS ON THE CIRCLE

7. Determine the equation of a circle with center (0, 0) which passes through the point (7, -3).

$\boxed{x^2 + y^2 = r^2}$

$7^2 + (-3)^2 = r^2$

$49 + 9 = r^2$

$58 = r^2$

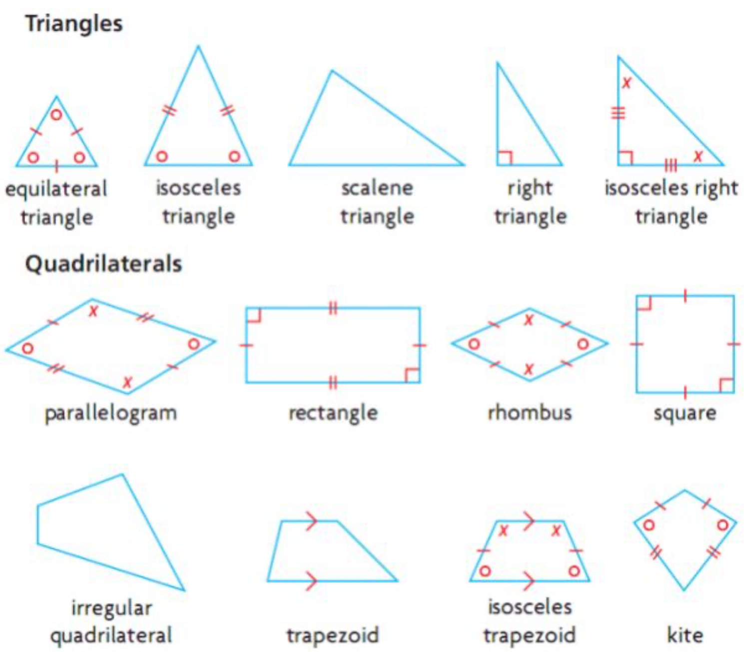
∴ EQUATION:

$x^2 + y^2 = 58$

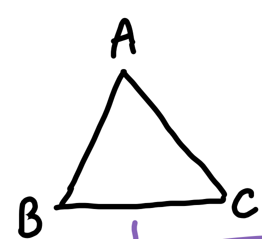
Lesson 5 (2.4) Classifying Geometric Figures

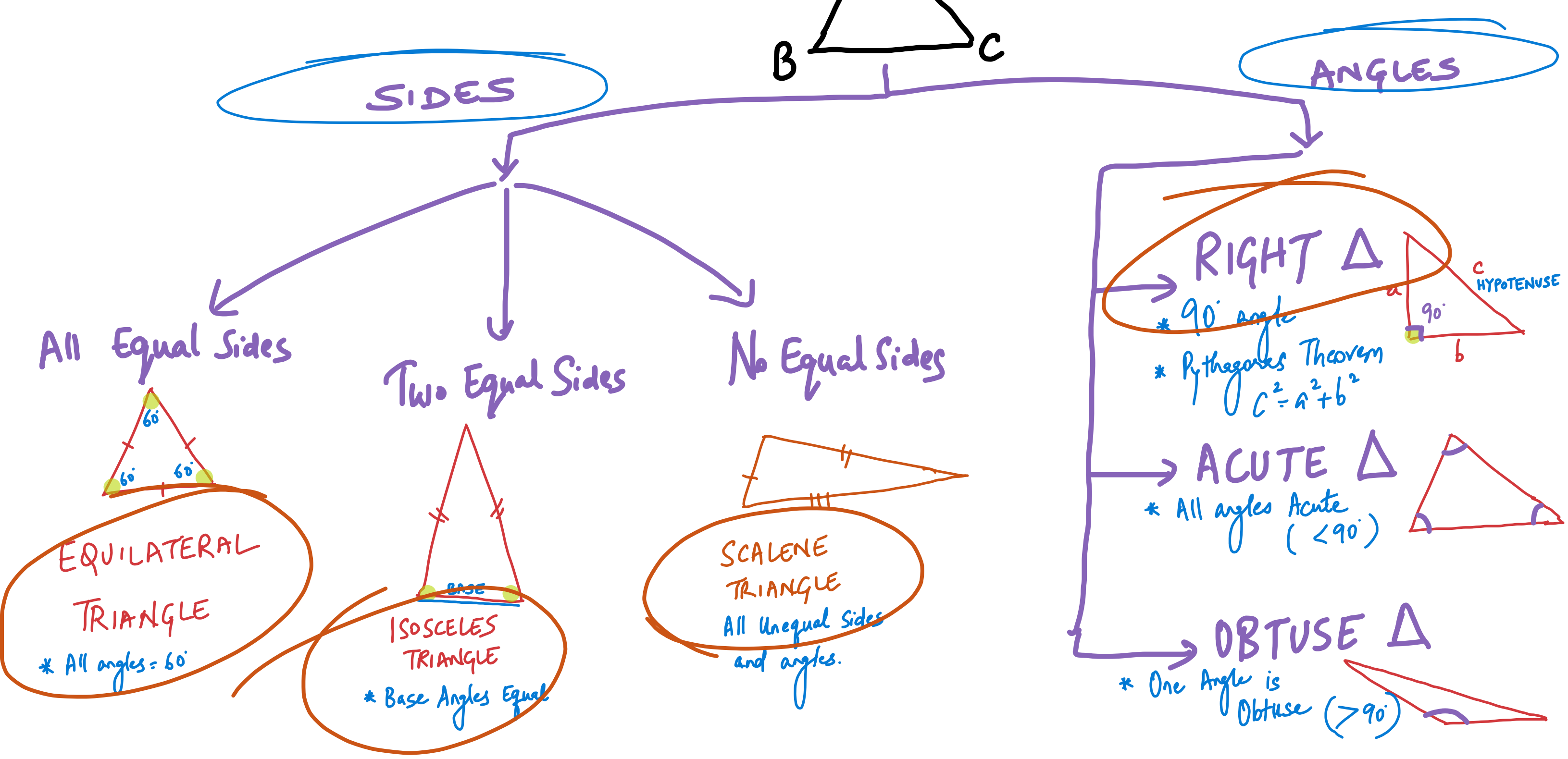
There are so many geometric figures that it's ridiculous. But we now know enough Analytic Geometry that we can easily do the "classification". We are really only going to worry about two "classes": Triangle and Quadrilaterals

You need to know the following types of Triangles and Quadrilaterals:



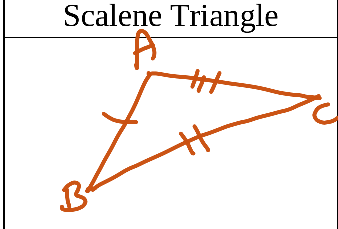
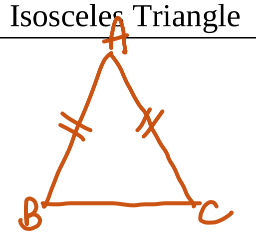
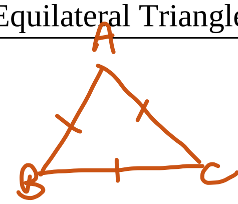
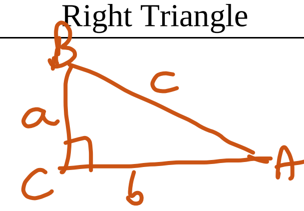
Triangles



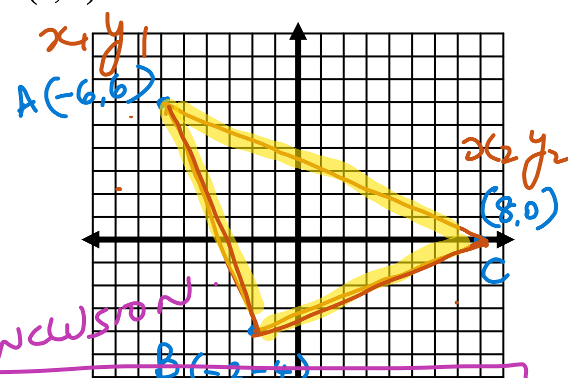


$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Properties of Triangles

Scalene Triangle	Isosceles Triangle	Equilateral Triangle	Right Triangle
			
Properties: All unequal sides	Properties: Two sides equal	Properties: All sides equal	Properties: $c^2 = a^2 + b^2$
How To Identify: $d_{AB} \neq d_{BC} \neq d_{AC}$	How To Identify: $d_{AB} = d_{AC} \neq d_{BC}$	How To Identify: $d_{AB} = d_{BC} = d_{CA}$	How To Identify: $(d_{AB})^2 = (d_{BC})^2 + (d_{AC})^2$ $m_1 = m$ $m_2 = m_\perp$

What type of triangle is formed by the points A(-6, 6), B(-2, -4), and C(8, 0)



Conclusion

$\therefore$ It is a right isosceles Δ.

$$d_{AB} = \sqrt{(-2 - -6)^2 + (-4 - 6)^2}$$

$$= \sqrt{(-2 + 6)^2 + (-4 - 6)^2}$$

$$= \sqrt{16 + 100} = \sqrt{116} = b$$

$$d_{BC} = \sqrt{(8 - -2)^2 + (0 - -4)^2}$$

$$= \sqrt{(8+2)^2 + (4)^2} = \sqrt{100 + 16} = \sqrt{116} = a$$

$$d_{AC} = \sqrt{(8 - -6)^2 + (0 - 6)^2} = \sqrt{(8+6)^2 + (-6)^2} = \sqrt{196 + 36}$$

$$= \sqrt{232} = c$$

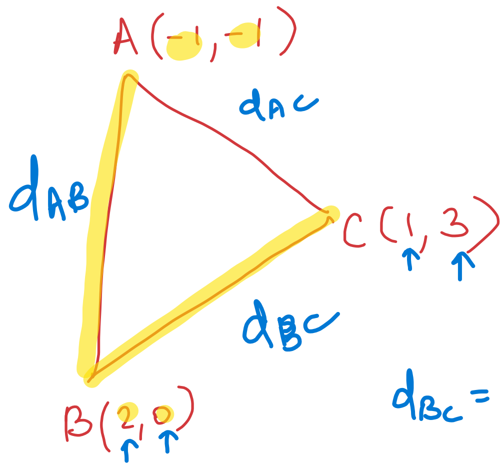
$$c^2 = (\sqrt{232})^2 = 232 \quad \Big| \quad a^2 + b^2 = (\sqrt{116})^2 + (\sqrt{116})^2$$

$$= 116 + 116 = 232$$

$$\therefore c^2 = a^2 + b^2$$

Practice:

A triangle has vertices at $A(-1,-1)$, $B(2,0)$, and $C(1,3)$. Using analytic geometry, determine what type of triangle it is.



$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d_{AB} = \sqrt{(2--1)^2+(0--1)^2} = \sqrt{3^2+1^2}$$

$$= \sqrt{9+1} = \sqrt{10} = a$$

$$d_{BC} = \sqrt{(1-2)^2+(3-0)^2} = \sqrt{(-1)^2+3^2} = \sqrt{1+9} = \sqrt{10} = b$$

$$d_{AC} = \sqrt{(1--1)^2+(3--1)^2} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20} = c$$

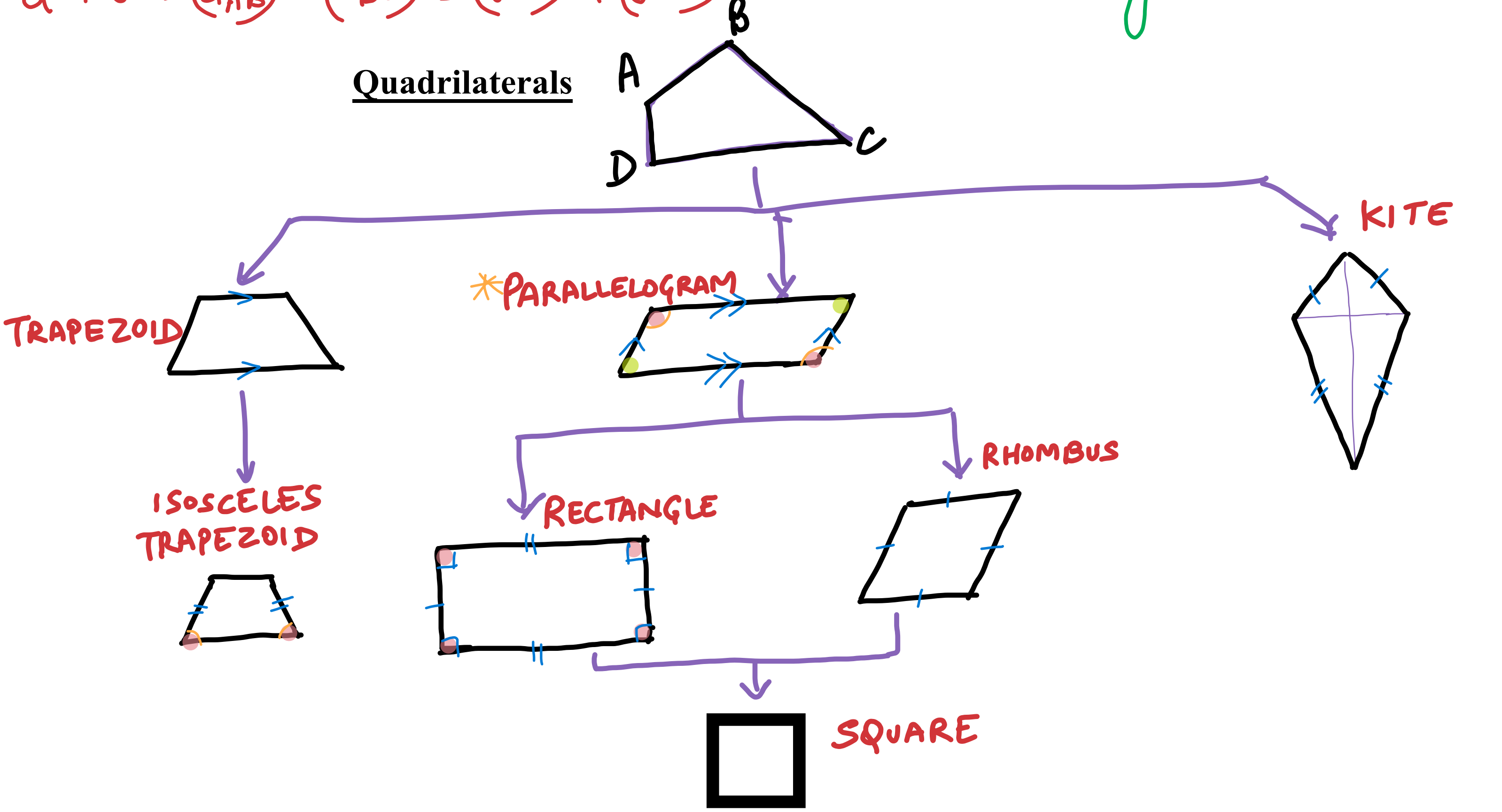
$$c^2 = (d_{AC})^2 = (\sqrt{20})^2 = 20$$

$$a^2+b^2 = (d_{AB})^2+(d_{BC})^2 = (\sqrt{10})^2+(\sqrt{10})^2 = 10+10 = 20$$

$c^2 = a^2+b^2$

∴ $\triangle ABC$ is a right isosceles $\triangle$.

Quadrilaterals

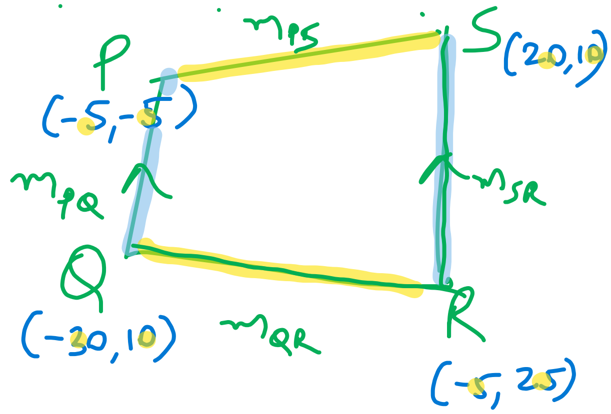


Note that all Geometric Shapes can be classified using the Side lengths and the Angles

Example 1:

Verify what type of quadrilateral is formed by the points P(-5,-5), Q(-30,10), R(-5,25), and S(20,10).

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$



$m_{PQ} = \frac{10 - -5}{-30 - -5} = \frac{10+5}{-30+5} = \frac{15}{-25} = \frac{-3}{5}$

$m_{SR} = \frac{25-10}{-5-20} = \frac{15}{-25} = \frac{-3}{5}$

$\rightarrow PQ \parallel SR$

$\rightarrow$ PQRS is a PARALLELOGRAM.

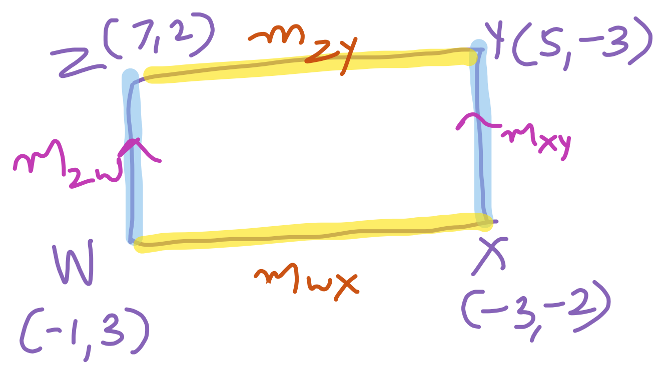
$m_{QR} = \frac{25-10}{-5 - -30} = \frac{15}{-5+30} = \frac{15}{25} = \frac{3}{5}$

$m_{PS} = \frac{10 - -5}{20 - -5} = \frac{10+5}{20+5} = \frac{15}{25} = \frac{3}{5}$

$\rightarrow QR \parallel PS$

Example 2:

Your friend claims that the quadrilateral with vertices at W(-1, 3), X(-3, -2), Y(5, -3), and Z(7, 2) form a rectangle.
Is your friend correct? Fully justify your answer.



$m_{ZW} = \frac{3-2}{-1-7} = \frac{1}{-8} = -\frac{1}{8}$

$m_{XY} = \frac{-2 - -3}{-3-5} = \frac{-2+3}{-3-5} = \frac{1}{-8} = -\frac{1}{8}$

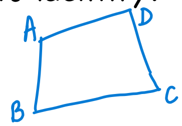
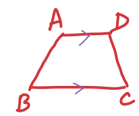
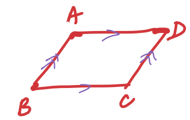
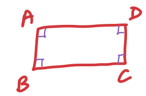
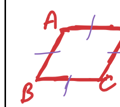
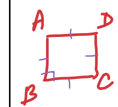
$m_{WX} = \frac{-2-3}{-3 - -1} = \frac{-2-3}{-3+1} = \frac{-5}{-2} = \frac{5}{2}$

$m_{ZY} = \frac{-3-2}{5-7} = \frac{-5}{-2} = \frac{5}{2}$

$ZW \parallel XY$ AND $WX \parallel ZY$

$\therefore$ WXYZ is a PARALLELOGRAM

But it is Not a RECTANGLE because there are no 90° angles.

Properties of Quadrilaterals		How to identify? 	 Trapezoid	 Parallelogram	 Rectangle	 Rhombus	 Square
Sides	All sides are equal in length	$d_{AB}=d_{BC}=d_{CD}=d_{AD}$					
	Opposite sides are equal in length	$d_{AD}=d_{BC}$; $d_{AB}=d_{CD}$					
	Opposite sides are parallel	$m_{AD}=m_{BC}$; $m_{AB}=m_{CD}$					
Angles	All angles are equal=90°	NEGATIVE RECIPROCAL slopes of adjacent sides					
	Opposite angles are equal						

A few tips to identify the quadrilateral when given all the four vertices:

Step 1: Find the slopes of all sides.

	Conclusion:
1. One pair of opposite sides with same slope	TRAPEZOID
2. Both pair of opposite sides with the same slope	PARALLELOGRAM (or RHOMBUS)
3. Both pair of opposite sides with the same slope and one of the slopes is negative reciprocal of the other	RECTANGLE (or SQUARE)

Step 2: Find the length of all sides.

		Conclusion
2. Both pair of opposite sides have the same slope	2. a.) All sides equal	RHOMBUS
	2. b.) Only ~~one~~ pair of opposite sides equal two	PARALLELOGRAM
3. Both pair of opposite sides have the same slope and one of the slopes is negative reciprocal of the other	3. a.) All sides equal	SQUARE
	3. b.) Only ~~one~~ pair of opposite sides equal two.	RECTANGLE

A few tips to identify the triangle when given all the three vertices:

Step 1: Find the lengths of all sides (*-How?*.... $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ ....)

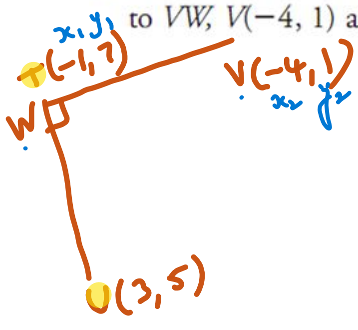
Step 2: Check if the sides satisfy Pythagoras theorem........ $c^2=a^2+b^2$ ....

Some Textbook Questions

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Example 2.4.1 From your text – Pg. 101 #2

2. Show that *TU*, *T*(−1, 7) and *U*(3, 5), is perpendicular to *VW*, *V*(−4, 1) and *W*(−1, 7).

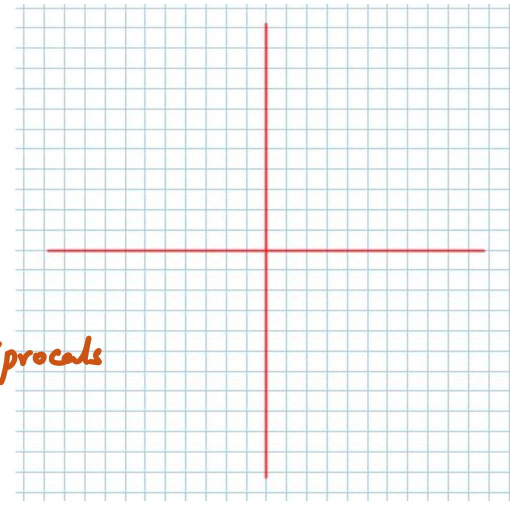


$m_{TU} = \dfrac{5-7}{3--1} = \dfrac{-2}{4} = -\dfrac{1}{2}$

$m_{WV} = \dfrac{1-7}{-4--1} = \dfrac{-6}{-3} = +\dfrac{2}{1}$

because the slopes are negative reciprocals

$\therefore TU \perp WV$.

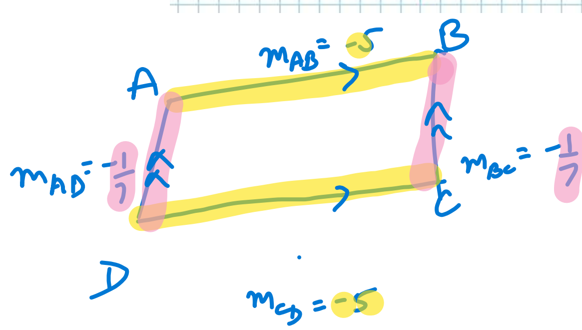


Example 2.4.2 From your text – Pg. 101 #3

3. The sides of quadrilateral *ABCD* have the following slopes.

Side	*AB*	*BC*	*CD*	*AD*
Slope	−5	$-\frac{1}{7}$	−5	$-\frac{1}{7}$

What types of quadrilateral could *ABCD* be? What other information is needed to determine the exact type of quadrilateral?

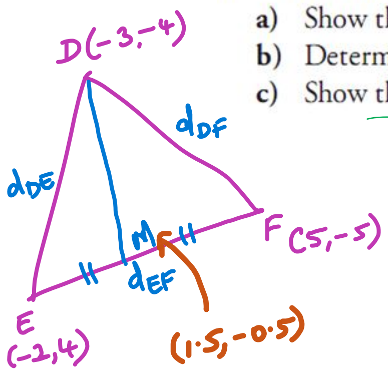


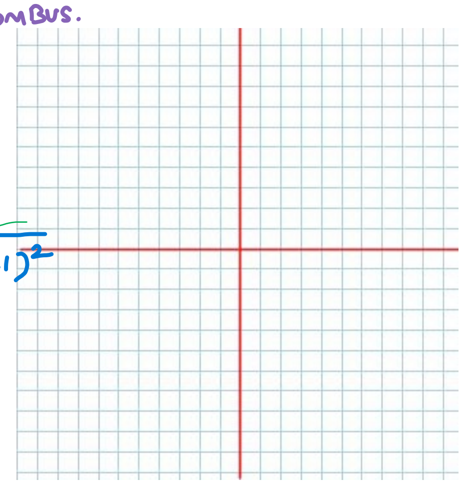
Since both pair of opposite sides are PARALLEL, the quadrilateral ABCD is a PARALLELOGRAM.

If the length (distance) of all sides are known, we can also check if it is a RHOMBUS.

Example 2.4.3 From your text – Pg. 101 #4

4. △*DEF* has vertices at *D*(−3, −4), *E*(−2, 4), and *F*(5, −5).
   a) Show that △*DEF* is isosceles.
   b) Determine the length of the median from vertex *D*. → DM
   c) Show that this median is perpendicular to *EF*.





a) $d_{DF} = \sqrt{(5--3)^2+(-5--4)^2} = \sqrt{8^2+(-1)^2}$
$= \sqrt{64+1} = \sqrt{65}$

$d_{EF} = \sqrt{(5--2)^2+(-5-4)^2} = \sqrt{7^2+(-9)^2}$
$= \sqrt{49+81} = \sqrt{130}$

$d_{DE} = \sqrt{(-3--2)^2+(-4-4)^2} = \sqrt{(-1)^2+(-8)^2}$
$= \sqrt{1+64} = \sqrt{65}$

DF = DE $\therefore$ △DEF is isosceles.

b) $M = M_{EF} = \left(\dfrac{5+-2}{2}, \dfrac{-5+4}{2}\right) = \left(\dfrac{3}{2}, \dfrac{-1}{2}\right) = (1.5, -0.5)$

$\therefore d_{DM} = \sqrt{(1.5--3)^2+(-0.5--4)^2} = \sqrt{(4.5)^2+(3.5)^2} = \sqrt{20.25+12.25} = \sqrt{32.5} = 5.7$ (approx)

c) To Show: $DM \perp EF$ $\therefore$ Find slope of DM and EF

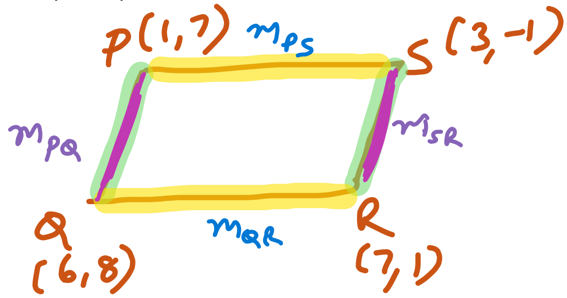
$m_{DM} = \dfrac{-4--0.5}{-3-1.5} = \dfrac{-3.5}{-4.5} = \dfrac{35}{45} = \dfrac{7}{9}$ and $m_{EF} = \dfrac{-5-4}{5--2} = \dfrac{-9}{7}$

m_{DM} and m_{EF} are NEGATIVE RECIPROCALS $\Rightarrow DM \perp EF$

Practice Problems

1. Quadrilateral PQRS has vertices at P(1, 7), Q(6, 8), R(7, 1), and S(3, -1).

Is PQRS a parallelogram? Explain how you know.



$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

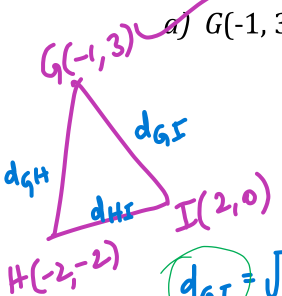
$m_{PQ} = \frac{8-7}{6-1} = \frac{1}{5}$; $m_{SR} = \frac{1--1}{7-3} = \frac{2}{4} = \frac{1}{2}$

PQ is not parallel to SR

∴ PQRS is not a parallelogram because both pair of opposite sides are Not parallel.

2. The following points are the vertices of triangles. Determine whether each triangle is scalene, isosceles, or equilateral. Calculate each side length to check your prediction.

a) $G(-1, 3)$, $H(-2, -2)$, $I(2, 0)$



All sides unequal ∴ SCALENE Δ.

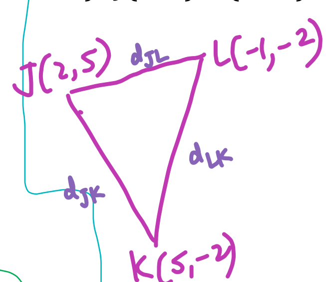
$d_{GI} = \sqrt{(2--1)^2 + (0-3)^2}$

$= \sqrt{3^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}$

$d_{HI} = \sqrt{(2--2)^2 + (0--2)^2} = \sqrt{4^2+2^2} = \sqrt{16+4} = \sqrt{20}$

$d_{GH} = \sqrt{(-2--1)^2 + (-2-3)^2} = \sqrt{(-1)^2 + (-5)^2} = \sqrt{1+25} = \sqrt{26}$

b) $J(2, 5)$, $K(5, -2)$, $L(-1, -2)$



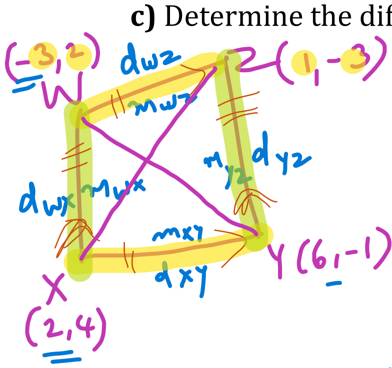
$d_{JL} = \sqrt{(2--1)^2 + (5--2)^2} = \sqrt{3^2 + 7^2} = \sqrt{58}$

$d_{LK} = \sqrt{(5--1)^2 + (-2--2)^2} = \sqrt{6^2 + 0^2} = \sqrt{36} = 6$

$d_{JK} = \sqrt{(5-2)^2 + (-2-5)^2} = \sqrt{3^2 + (-7)^2} = \sqrt{58}$

∴ JL = JK ⇒ ΔJKL is an ISOSCELES Δ.
(two sides equal)

3. A quadrilateral has vertices at $W(-3, 2)$, $X(2, 4)$, $Y(6, -1)$, and $Z(1, -3)$
a) Determine the length and slope of each side of the quadrilateral.
b) Based on your calculations for part a), what type of quadrilateral is *WXYZ*? Explain.
c) Determine the difference in the lengths of the two diagonals of *WXYZ*.



a) $d_{WZ} = \sqrt{(-3-1)^2 + (2--3)^2} = \sqrt{16+25} = \sqrt{41}$

$d_{YZ} = \sqrt{(6-1)^2 + (-1--3)^2} = \sqrt{25+4} = \sqrt{29}$

$d_{XY} = \sqrt{(2-6)^2 + (4--1)^2} = \sqrt{16+25} = \sqrt{41}$

$d_{WX} = \sqrt{(2--3)^2 + (4-2)^2} = \sqrt{25+4} = \sqrt{29}$

$m_{WZ} = \frac{-3-2}{1--3} = \frac{-5}{4}$

$m_{YZ} = \frac{-3-1}{1-6} = \frac{-2}{-5} = \frac{2}{5}$

$m_{XY} = \frac{-1-4}{6-2} = \frac{-5}{4}$

$m_{WX} = \frac{2-4}{-3-2} = \frac{-2}{-5} = \frac{2}{5}$

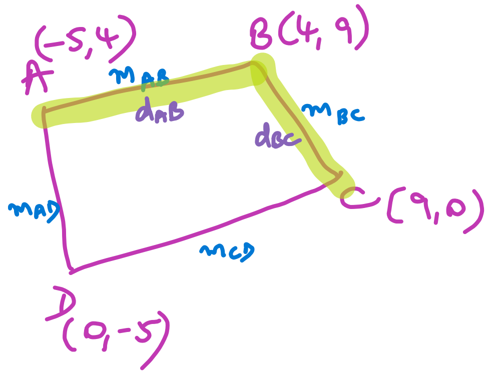
b) WXYZ is a parallelogram. bcz both pair of opposite sides are parallel.

c) $d_{XZ} = \sqrt{(2-1)^2 + (4--3)^2} = \sqrt{1+49} = \sqrt{50} \approx 7.07$

$d_{WY} = \sqrt{(-3-6)^2 + (2--1)^2} = \sqrt{81+9} = \sqrt{90} \approx 9.49$

∴ Approx difference = 9.49 − 7.07 = 2.42

4. A surveyor is marking the corners of a building lot. If the corners have coordinates $A(-5, 4)$, $B(4, 9)$, $C(9, 0)$, and $D(0, -5)$, what shape is the building lot? Include your calculations in your answer.



$m_{AB} = \frac{9-4}{4--5} = \frac{5}{9}$

$m_{BC} = \frac{0-9}{9-4} = \frac{-9}{5}$

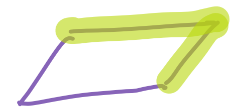
$m_{CD} = \frac{0--5}{9-0} = \frac{5}{9}$

$m_{AD} = \frac{-5-4}{0--5} = \frac{-9}{5}$

$\Rightarrow$ $AB \parallel CD$; $BC \parallel AD$

$AB \perp BC$

$\therefore$ ABCD is a RECTANGLE



$d_{AB} = \sqrt{(-5-4)^2+(4-9)^2}$
$= \sqrt{81+25} = \sqrt{106}$

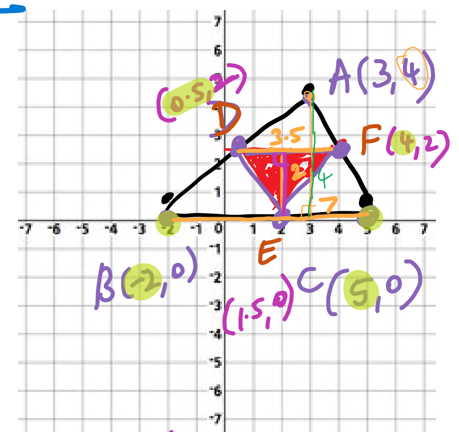
$d_{BC} = \sqrt{(9-4)^2+(0-9)^2}$
$= \sqrt{25+81} = \sqrt{106}$

$\therefore$ It is a SQUARE because it is a rectangle with all equal sides. ☺

5. ABC has vertices at $A(3, 4)$, $B(-2, 0)$, and $C(5, 0)$. Prove that the area of the triangle formed by joining the midpoints of is one-quarter the area of ABC

the sides

$ar(\Delta) = \frac{\text{base} \times \text{height}}{2}$



TO PROVE: $\text{area}(\Delta DEF) = \frac{1}{4}\ \text{area}(\Delta ABC)$

$D = M_{AB} = \left(\frac{3+-2}{2}, \frac{4+0}{2}\right) = (0.5, 2)$

$E = M_{BC} = \left(\frac{-2+5}{2}, \frac{0+0}{2}\right) = (1.5, 0)$

$F = M_{AC} = \left(\frac{3+5}{2}, \frac{4+0}{2}\right) = (4, 2)$

$ar(\Delta DEF) = \frac{(3.5)(\cancel{2})}{\cancel{2}} = 3.5$ sq. units.

$ar(\Delta ABC) = \frac{(7)(\cancel{4})^{2}}{\cancel{2}_{1}} = 14$ sq. units.

$\therefore \frac{1}{4} ar(\Delta ABC) = \frac{14}{4} = 3.5$ sq. units

$\therefore ar(\Delta DEF) = \frac{1}{4} ar(\Delta ABC)$

Classifying Geometric Figures

Shape	What are you looking for when trying to classify each geometric shape?	What formulas/calculations would you use to prove it?
Equilateral Triangles		
Isosceles Triangle		
Scalene Triangles		
Right angle Triangles		
Parallelogram		
Rectangle		
Rhombus		
Square		
Irregular quadrilateral		
Trapezoid		
Isosceles Trapezoid		
Kite		