Name: ______________________________

Analytic Geometry

Learning Goals:
We are learning to…

- use coordinates to determine and solve problems involving midpoints, slopes, and lengths of line segments
- determine the equation of a circle with centre (0,0)
- use properties of line segments to identify geometric figures and verify their properties

Analytic Geometry: Terms and Formulas

"Analytic Geometry" is using algebra to analyze geometric properties of shapes. The connection between the algebra and the geometry is through formulas which use the coordinates of points.

Some Terms

Line Segment – A part of a line between two points. For example

shows line segment $\overline{AB}$

Midpoint – The point in the middle of a line segment

$$M_{\overline{AB}} = D(x, y)$$

Median – A line segment in a triangle from one vertex to the midpoint of the opposite side

$\overline{AD}$ is a median of triangle *ABC*. *D* is the midpoint of $\overline{BC}$

Midsegment – A midsegment is a line segment inside a triangle which joins the midpoints of two sides of the triangle.

If *P* is the midpoint of $\overline{LM}$, and
Q is the midpoint of $\overline{MN}$, then
$\overline{PQ}$ is a midsegment of triangle *LMN*

Note: The slope of $\overline{PQ}$ is equal to the slope of $\overline{LN}$

Perpendicular Bisector – A line which cuts a line segment in half, and which is also perpendicular to that line segment.

Note that point *P* is the midpoint of $\overline{MN}$, and that the slope of line *l* is the negative reciprocal of the slope of $\overline{MN}$

Altitude – A line segment inside a triangle from one vertex, and perpendicular to the opposite side

$\overline{AD}$is an altitude of triangle *ABC*

The slope of $\overline{AD}$ is the negative reciprocal of the slope of $\overline{BC}$

Formulas

Slope of a line (or line segment) – Given two points on a line $A(x_1, y_1)$and $B(x_2, y_2)$, then

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation of a line –
The equation is:

$y = mx + b$
(slope-intercept form), or

$y - y_1 = m(x - x_1)$
(slope-point form)

Midpoint – Given a line segment $\overline{AB}$ with endpoints $A(x_1, y_1)$and $B(x_2, y_2)$, then

$$M_{\overline{AB}} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Length of a line segment (or distance between two points) - Given a line segment $\overline{AB}$ with endpoints $A(x_1, y_1)$and $B(x_2, y_2)$, then the length of $\overline{AB}$ is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Equation of a Circle – A circle centered at (h, k), and with radius r has the equation

$$x^2 + y^2 = r^2$$
(with centre (0,0))

Lesson 1- (2.0) Writing Equation of a Line

SLOPE y-intercept.

$y = mx + b$ p(x,y)

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{Rise}{Run}$

Need 3 things (always):

1. Slope: m (given or calculated)
2. Point: (x, y) or (0,b) ***NOTE:* *(0,b) is y-intercept***
3. Formula to find equation of line: y = mx + b

Type 1 Problem:

m = 4/5 b = -7

When you know the y-intercept, use the **Slope-Intercept Form, *y* = mx + b**

1. Identify the y-intercept (**b**) and slope (**m**)
2. Write the equation replacing the b and m

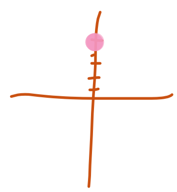
$y = mx + b$

$\boxed{y = \frac{4}{5}x - 7}$

Type 2 Problem:

When you have 2 points, one of which is obviously y-intercept

1. Use the 2 points to calculate the slope (**m**)
2. Recognize that **(0,b)** is the y-intercept
3. Write the equation replacing **b** and **m**



P_1 P_2 (0, #) (0, b)

x_1, y_1 x_2, y_2

(0, 5) (3, -3)

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-5}{3-0} = \frac{-8}{3}$

$b = 5$

$\boxed{y = \frac{-8}{3}x + 5}$

Type 3 Problem: When you have 2 points, neither which are the y-intercept,

Slope-Intercept Form, *y* = mx + b

1. Use the 2 points to calculate the slope (**m**)
2. Sub in **m** and **a point for (x,y)**
3. Solve for **b**
4. Write the equation replacing the b and m

P_1 P_2

x_1, y_1 x_2, y_2

(2, -5) (1, 3)

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - -5}{1-2} = \frac{8}{-1} = -8$

$y = mx + b$

$-5 = (-8)(2) + b$

$-5 = -16 + b$

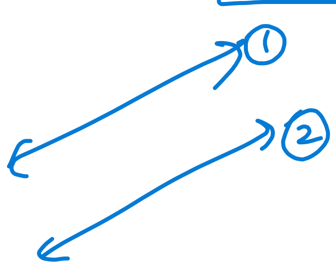
$-5 + 16 = b$

$\boxed{11 = b}$

$\boxed{y = -8x + 11}$

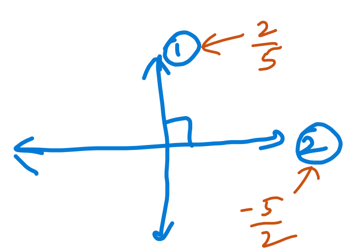
Also Note:

Slopes of Parallel Lines



$m_1 = m_2$

Slopes of Perpendicular Lines



NEGATIVE RECIPROCAL

1. Flip the fraction
2. Switch the sign

$m_1 = m = \frac{a}{b}$

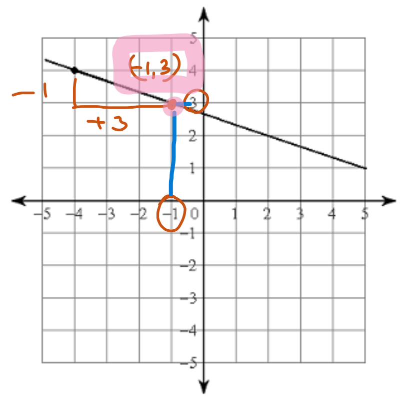
$m_2 = m_\perp = -\frac{b}{a}$

Let's Practice!

Find the equation of the following line

$y = mx + b$

$P(x, y) = (-1, 3)$



$m = \frac{Rise}{Run} = \frac{-1}{3}$

$3 = \frac{-1}{3}(-1) + b$

$3 = \frac{1}{3} + b$

$\frac{3 \times 3}{3} - \frac{1}{3} = b$

$\boxed{\frac{8}{3} = b}$

EQUATION:

$$\boxed{y = -\frac{1}{3}x + \frac{8}{3}}$$

Determine the equation of the line that:

a) passes through $(-1, 7)$ and $(2, 14)$

$x_1 \; y_1 \quad x_2 \; y_2$

$\boxed{y = mx + b}$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - 7}{2 - -1} = \frac{7}{3}$

$7 = \left(\frac{7}{3}\right)(-1) + b$

$7 = -\frac{7}{3} + b$

$\frac{3 \times 7}{3 \times 1} + \frac{7}{3} = b$

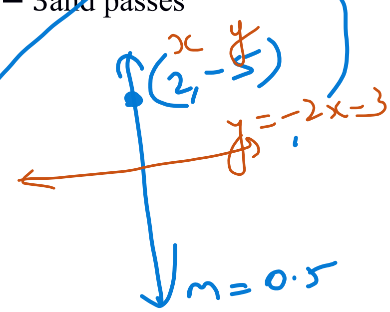
$\frac{21}{3} + \frac{7}{3} = b$

$\boxed{\frac{28}{3} = b}$

∴ EQUATION:

$$\boxed{y = \frac{7}{3}x + \frac{28}{3}}$$

b) is Perpendicular to $y = -2x - 3$ and passes through $(2, -5)$



$y = mx + b$

$m = -2$

$m_\perp = \frac{1}{2} = 0.5 = m$

$y = mx + b$

$-5 = (0.5)(2) + b$

$-5 = 1 + b$

$-5 - 1 = b$

$\boxed{-6 = b}$

∴ EQUATION:

$$\boxed{y = 0.5x - 6}$$

More Practice:

c) passes through (0, 4) and (-2, -7)

$x_1\ y_1 \quad x_2\ y_2$

$b = 4$

$m = \frac{-7-4}{-2-0} = \frac{-11}{-2} = \frac{11}{2} = 5.5$

$\therefore$ EQUATION: $\boxed{y = 5.5x + 4}$

d) parallel to line having m = 3 and passes through (1, 3)

$m = 3$ $\quad x, y$

$y = mx + b$

$3 = 3(1) + b$

$3 = 3 + b$

$b = 3 - 3 = 0$

$\therefore$ EQUATION:

$y = 3x + 0$

$\boxed{y = 3x}$

e) is perpendicular to y= − 2x − 3and passes (3, 4)

$m = -2$ $\quad x\ y$

$m_\perp = \frac{1}{2} = 0.5$

$y = m_\perp x + b$

$4 = 0.5(3) + b$

$4 = 1.5 + b$

$4 - 1.5 = b$

$b = 2.5$

$\therefore$ EQUATION: $\boxed{y = 0.5x + 2.5}$

f) is parallel to 2x − 3y = 8 and passes through (2, -5)

x, y

$2x - 3y = 8$

$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{8}{-3}$

$y = \frac{2}{3}x - \frac{8}{3}$

$m = \frac{2}{3}$

$y = mx + b$

$-5 = \frac{2}{3}(2) + b$

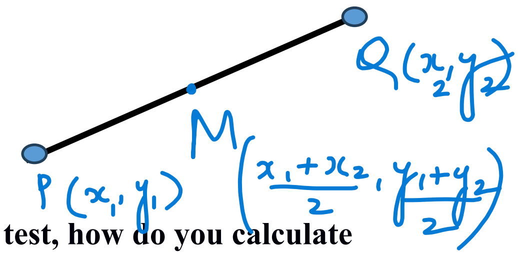
$-5 = \frac{4}{3} + b$

$\frac{-5 \times 3}{1 \times 3} - \frac{4}{3} = b$

$b = \frac{-15-4}{3} = -\frac{19}{3}$

$\therefore$ EQUATION: $\boxed{y = \frac{2}{3}x - \frac{19}{3}}$

Lesson 2 (2.1) Midpoint



Find the Midpoint of a line – The point in the middle of a line segment

Question: If you scored a 70% on a test and then an 82% on the next test, how do you calculate the average of those tests?

$\frac{70+82}{2} = 76\%$

Similarly, the coordinates of the midpoint (M) of a line is the midpoint (average) of the x-values and the midpoint of the y-values $M_{\overline{AB}} = D(x, y)$

$$\boxed{M_{\overline{AB}} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$$

Examples

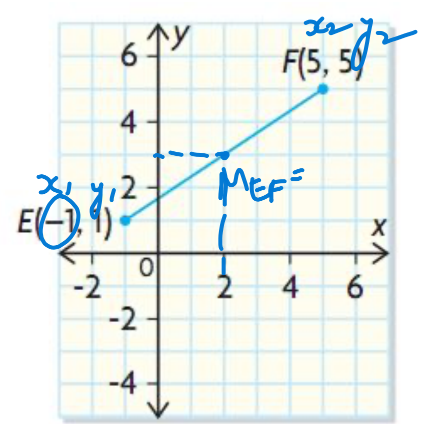
From your text: Pg. 78 #2a

Determine the coordinates of the midpoint of the line segment.

> Note: *Since you have been given a graph, midpoint can be found using two methods: graphically or algebraically*

$M_{EF} = \left(\frac{-1+5}{2}, \frac{1+5}{2}\right)$

$= (2, 3)$



Let's practice some more:

a) $C(9,8)$ and $D(3,22)$

$x_1\ y_1 \quad x_2\ y_2$

$M_{CD} = \left(\frac{9+3}{2}, \frac{8+22}{2}\right)$

$= (6, 15)$

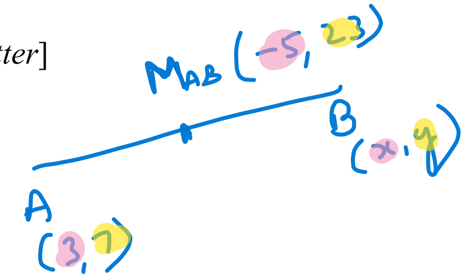
b) $E(5.6,-3.3)$ and $F(-12.2,-3.3)$

$M_{EF} = \left(\frac{5.6+-12.2}{2}, \frac{-3.3+-3.3}{2}\right)$

$= (-3.3, -3.3)$

c) Line AB segment has the endpoint A (3, 7) and the Midpoint M_{AB} (-5, 23)
What are the coordinates of end point B ?

[*Hint: Draw the situation to understand the problem better*]



$$M_{AB} = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$$

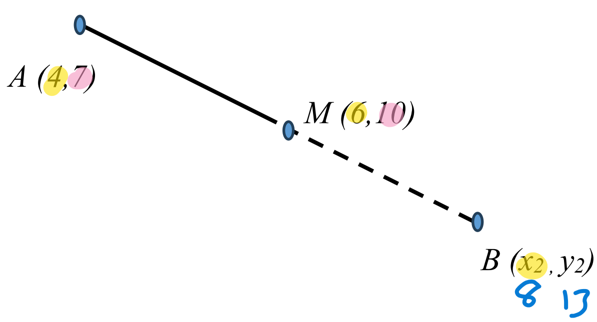
$$\frac{3+x}{2} = -5 \qquad\qquad \frac{7+y}{2} = 23$$

$$3+x = -10 \qquad\qquad 7+y = 46$$

$$x = -10-3 \qquad\qquad y = 46-7$$

$$\boxed{x = -13} \qquad\qquad \boxed{y = 39}$$

Now, here's a quick method:
Just remember to "DOUBLE THE MIDPOINT AND SUBTRACT AN ENDPOINT."



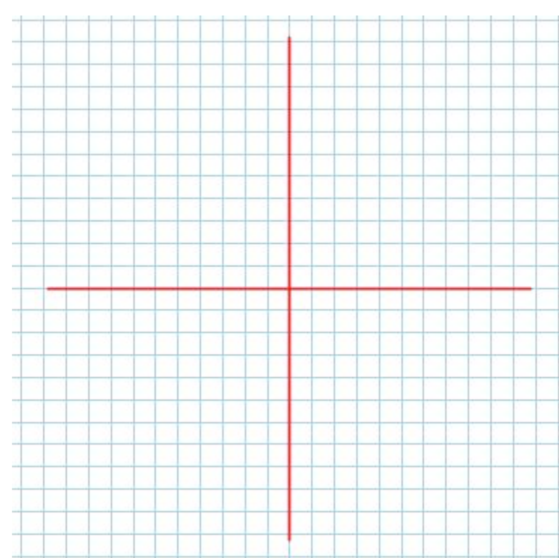
Practice:

1. *X (2,5) M_{XY} (7, 11) Y (* 12 *,* 7 *)*

2. *A (-3, 4) M_{AB} (2, -6) B (* 7 *,* -16 *)*

3. *D (5,-6) M_{DE} (-4, 12) E (* -13 *,* 30 *)*

~~*Graph #3 to check:*~~



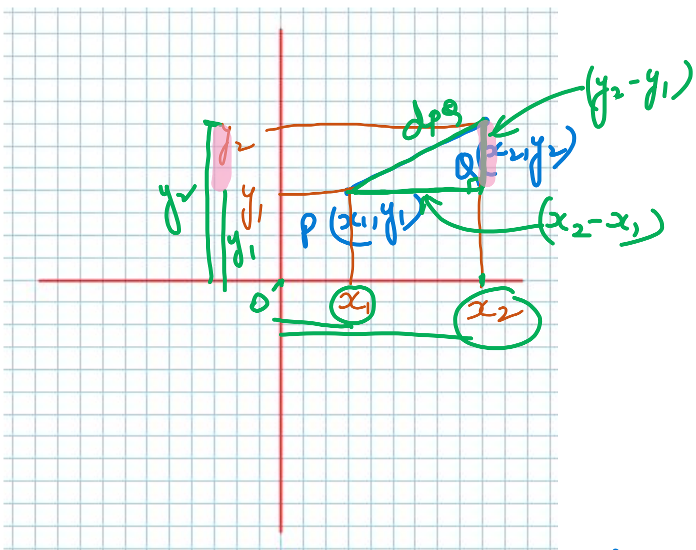
(2.2) Length of a line segment (distance between two points)

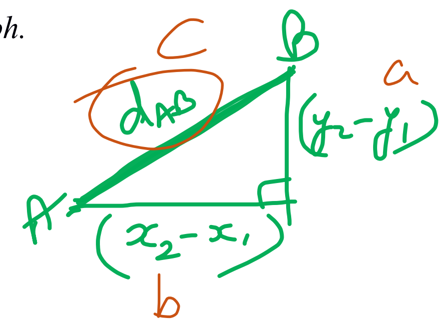
Given a line segment $\overline{AB}$ with endpoints $A(x_1, y_1)$and $B(x_2, y_2)$, then the length of $\overline{AB}$ can be found by using formula:

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Find the length of a line segment with endpoints A (1, 6) and B (-5, -6).

(*Let's also check our formula and our answer on the graph. Think of the Pythagorean Theorem $a^2 + b^2 = c^2$*)





$c^2 = a^2 + b^2$

$d_{AB} = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$

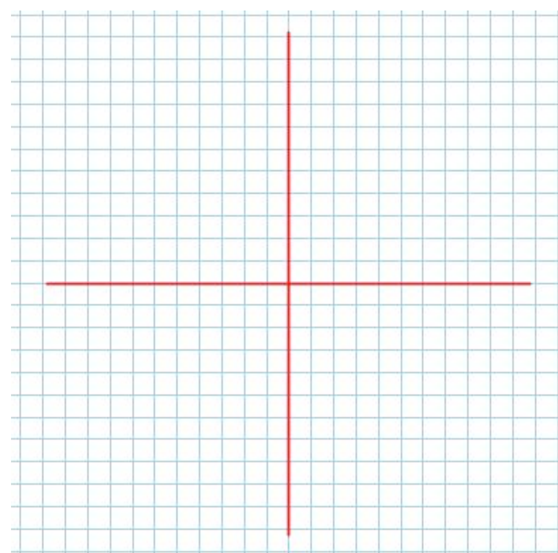
Example: Find the length of the line from $\underset{x_1,\,y_1}{\overset{A}{(1, 2)}}$ to $\underset{x_2,\,y_2}{\overset{B}{(-5, 7)}}$ $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d_{AB} = \sqrt{(-5-1)^2 + (7-2)^2}$

$= \sqrt{36+25}$

$= \sqrt{61}$

≈ 7.8 units



Let's practice some more:

a) $G(-4,10)$ and $H(8,12)$

$$d_{GH} = \sqrt{(8--4)^2 + (12-10)^2}$$

$$= \sqrt{144+4}$$

$$= \sqrt{148}$$

$$\approx 12.2 \text{ units}$$

b) $I(12,1)$ and $J(3,-6)$

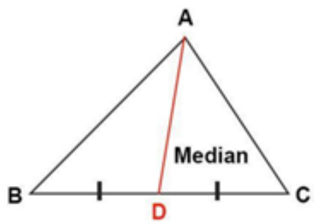
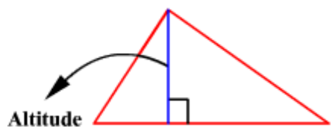
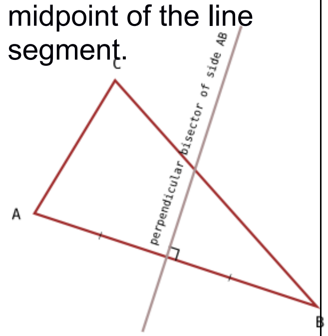
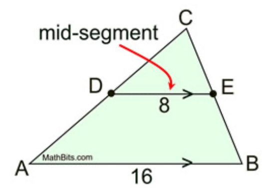
$$d_{IJ} = \sqrt{(3-12)^2 + (-6-1)^2}$$

$$= \sqrt{81+49}$$

$$= \sqrt{130}$$

$$\approx 11.4 \text{ units}$$

Lesson 3 (2.2) Equations of Lines found in Triangles

Median of a triangle: Line segment that joins a vertex of a triangle to the midpoint of the other side.	Altitude of a triangle: Line segment from a vertex which meets the opposite side at a 90° angle.	Perpendicular bisector of a line segment: line that is perpendicular to the line segment and passes through the midpoint of the line segment.	Midsegment of a triangle: Line segment that connects two midpoints
			

Find the equation of a MEDIAN

A Median is a line segment in a triangle from one vertex to the midpoint of the opposite side.

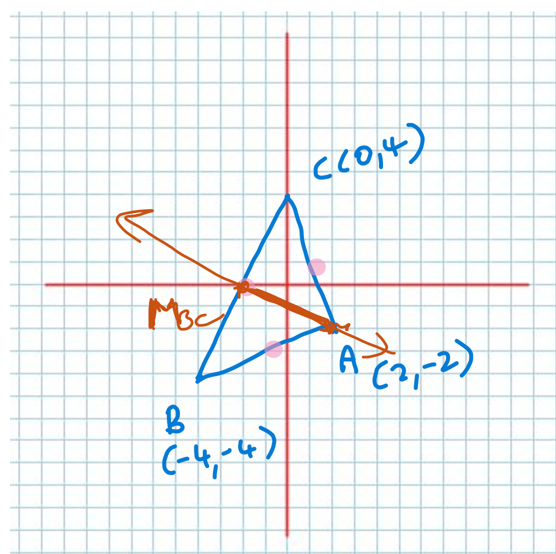
7. A triangle has vertices at $A(2, -2)$, $B(-4, -4)$, and $C(0, 4)$.

a) Draw the triangle, and determine the coordinates of the midpoints of its sides.

b) Draw the median from vertex A, and determine its equation.

From your text: Pg. 79 #7

Let's begin by doing the drawings on the graph And thinking about the algebraic process to determine the midpoints and the equation of median.



Mid-points: $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Median: EQUATION: $y = mx+b$

Step 1: $M_{BC} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Step 2: $m_{AM} = \frac{y_2-y_1}{x_2-x_1}$

Step 3:

$y = mx+b$

$b = ?$

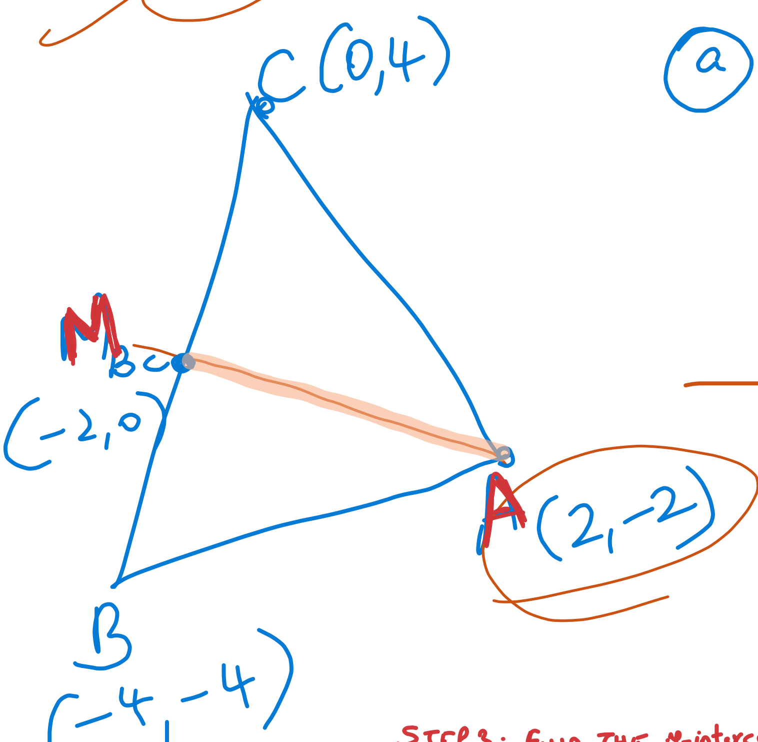
and now finally time to flex our algebraic muscles to determine

the midpoint coordinates and the median equation

7. A triangle has vertices at $A(2, -2)$, $B(-4, -4)$, and $C(0, 4)$.

K **a)** Draw the triangle, and determine the coordinates of the midpoints of its sides.

b) Draw the median from vertex A, and determine its equation.



(a)

$$M_{BC} = \left(\frac{-4+0}{2}, \frac{-4+4}{2}\right) = (-2, 0)$$

$$M_{AC} = \left(\frac{2+0}{2}, \frac{-2+4}{2}\right) = (1, 1)$$

$$M_{AB} = \left(\frac{-4+2}{2}, \frac{-4+-2}{2}\right) = (-1, -3)$$

(b)

STEP 1: FIND THE MID POINT OF $\overline{BC}$.

$$M_{BC} = (-2, 0)$$

STEP 2: FIND THE SLOPE OF MEDIAN $\overline{AM}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 0}{2 - -2}$$

$$m = \frac{-2}{4} = -\frac{1}{2} = -0.5$$

STEP 3: FIND THE y-intercept (b) of the MEDIAN $\overline{AM}$

$$y = mx + b$$

$$-2 = -0.5(2) + b$$

$$-2 = -1 + b$$

$$-2 + 1 = b$$

$$\boxed{b = -1}$$

$\therefore$ EQUATION of the median through A

$$\Rightarrow \boxed{y = -0.5x - 1}$$

Find the equation of an ALTITUDE

Let's use the same triangle ABC we used in the above question to determine the equation of an altitude from vertex B.

(Hint: Always start by drawing the diagram. This helps you visualize and to understand the problem better!!!)

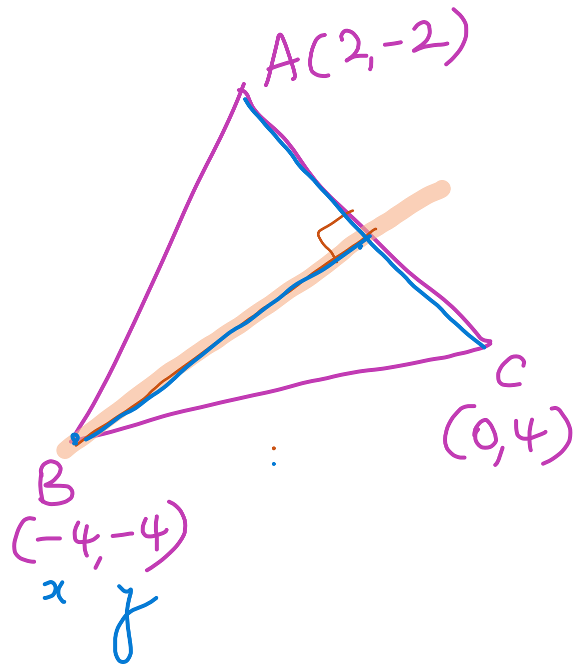
An Altitude is a line segment from a vertex which meets the opposite side at a 90° angle.

EQUATION: $y = mx + b$

Step 1: $m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$

Step 2: $m_{\perp}$ = NEGATIVE RECIPROCAL

Step 3: $b = ?$ use $m_{\perp}$ and $B(-4,-4)$



STEP 1: FIND THE SLOPE of AC

$m_{AC} = \frac{4 - -2}{0 - 2} = -3$

STEP 2: FIND THE SLOPE OF THE ALTITUDE

$\boxed{m_{\perp} = +\frac{1}{3}}$

STEP 3: FIND THE y-intercept of the ALTITUDE

$y = mx + b$

$-4 = \left(\frac{1}{3}\right)(-4) + b$

$-4 = -\frac{4}{3} + b$

$-4 + \frac{4}{3} = b$

$-\frac{12+4}{3} = b$

$\boxed{-\frac{8}{3} = b}$

∴ EQUATION:

$\boxed{y = \frac{1}{3}x - \frac{8}{3}}$

Find the equation of a PERPENDICULAR BISECTOR to a line segment

Perpendicular Bisector is a line that is perpendicular to the line segment and passes through the midpoint of the line segment.

Note that the Perpendicular Bisector cuts a line segment in half,
and which is also perpendicular to that line segment.
Perpendiculars therefore have slopes which are the negative reciprocal of the slope of given line segment ie. $m_1 = \frac{3}{2}$, $m_2 = -\frac{2}{3}$

From your text: Pg. 80 #13a

13. Determine an equation for the perpendicular bisector of a line segment with each pair of endpoints.

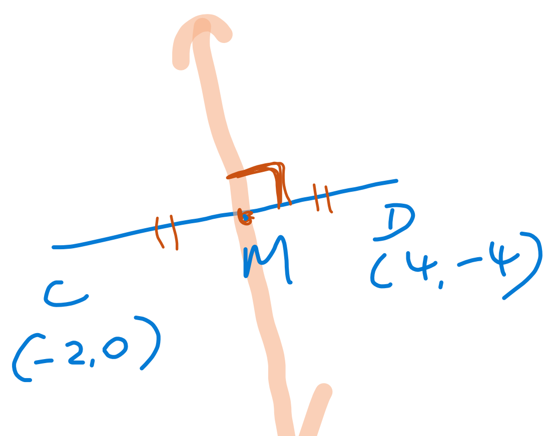
a) $C(-2, 0)$ and $D(4, -4)$

Always start by drawing the diagram. This helps you visualize and to understand the problem better!!!

Step 1: $M_{CD} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Step 2: $m_{CD} = \frac{y_2-y_1}{x_2-x_1}$; $m_\perp = ?$

Step 3: $b = ?$ $\overline{m_\perp}, \overline{M_{CD}}$



FIND MIDPOINT OF LINE SEGMENT $\overline{CD}$

STEP1: $M_{CD} = \left(\frac{-2+4}{2}, \frac{0+-4}{2}\right)$

$= (1, -2)$

FIND THE SLOPE OF $\overline{CD}$

STEP2: $m_{CD} = \frac{-4-0}{4--2} = \frac{-4}{6} = \frac{-2}{3}$

AND ITS PERPENDICULAR SLOPE ← (SLOPE of PERPENDICULAR BISECTOR)

$\therefore m_\perp = \frac{3}{2} = 1.5$

STEP3: FIND THE y-intercept

$y = mx + b$

$-2 = (1.5)(1) + b$

$-2 - 1.5 = b$

$\boxed{b = -3.5}$

EQUATION:

$y = 1.5x - 3.5$

Now time for the BIGGEST QUESTION!!

Example (From your Text: Pg. 87 #12a)

Calculate the distance between the line y = 4x – 2 and the point (-3, 3)

***The shortest distance is a line perpendicular to y=4x - 2**

Before trying to work on the solution, CONQUER the PROBLEM!!!!

Step 1 find m_1 and m_2 $= m_\perp$

Step 2 find the equation of the perpendicular line

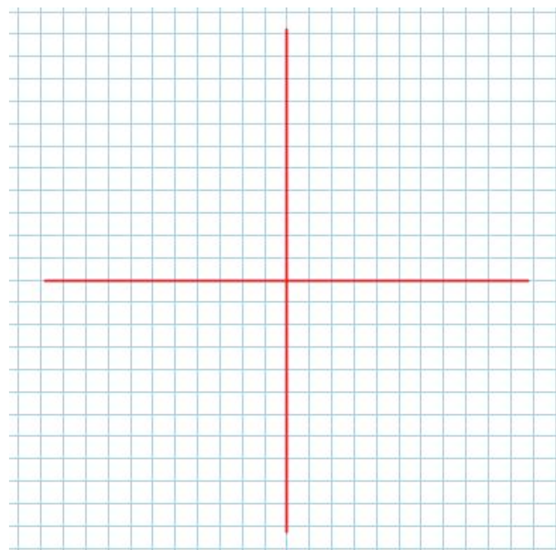
$$y = mx + b \text{ (slope-intercept form), or}$$

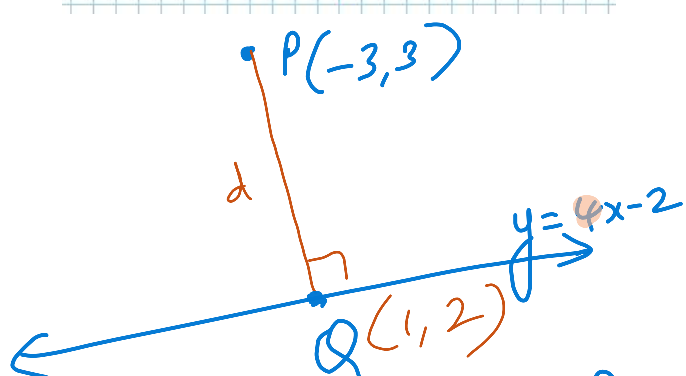
~~$y - y_1 = m(x - x_1)$ (slope-point form)~~

Step 3 Find the POI of the two lines. Solve the system by substitution or elimination

Step 4 Find the length of the line from the POI to (-3, 3)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$





STEP1: $m = 4 \therefore \boxed{m_\perp = -\frac{1}{4}}$

STEP2: $b = ?$ $m_\perp = -\frac{1}{4}$, $P(-3, 3)$

$y = mx + b$

$3 = \left(-\frac{1}{4}\right)(-3) + b$

$3 = \frac{3}{4} + b$

$3 - \frac{3}{4} = b$

$\boxed{\frac{9}{4} = b}$

$\therefore$ EQUATION: $y = -\frac{1}{4}x + \frac{9}{4}$

$\Leftrightarrow y = -0.25x + 2.25$

STEP3: $POI = Q(x, y) = ?$

$y = 4x - 2$

$-\ y = -0.25x + 2.25$

$0 = 4.25x - 4.25$

$4.25 = 4.25x$

$\boxed{x = 1}$ $\therefore y = 4(1) - 2$

$\boxed{y = 2}$

POI $(1, 2)$

STEP 4: $d_{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $(x_1, y_1) = (-3, 3)$

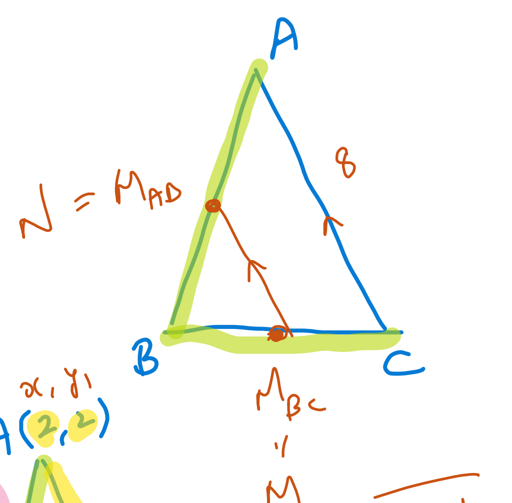
$(x_2, y_2) = (1, 2)$

$$= \sqrt{(1 - -3)^2 + (2-3)^2}$$

$$= \sqrt{16 + 1}$$

$$= \sqrt{17}$$

Midsegments



A midsegment is line segment formed by two midpoints.

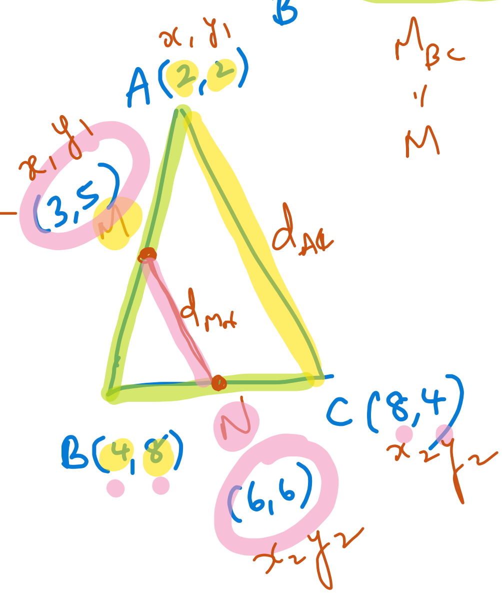
Draw a rough diagram for ~~Plot~~ the triangle $A(2,2)$, $B(4,8)$, $C(8,4)$. Draw the midsegment from line AB to line BC. Calculate its length.

Step 1: $M = ?$ $N = ?$ $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$M = \left(\frac{2+4}{2}, \frac{2+8}{2}\right) = (3, 5)$

$N = \left(\frac{4+8}{2}, \frac{8+4}{2}\right) = (6, 6)$

Step 2: $MN = ? = d_{MN} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



$\overline{M_{AB} M_{BC}}$

$\overline{MN}$

$d_{MN} = \sqrt{(6-3)^2 + (6-5)^2}$

$= \sqrt{9 + 1} = \sqrt{10} \approx 3.2$ units

≈ 3

Now compare with the length of AC (for "fun").

$d_{AC} = \sqrt{(8-2)^2 + (4-2)^2}$

$= \sqrt{36 + 4} = \sqrt{40} \approx 6.32$

≈ 6

$\overline{AC} = 2(\overline{MN})$

Lesson 4 (2.3) The Equation of a Circle centered at (0, 0)

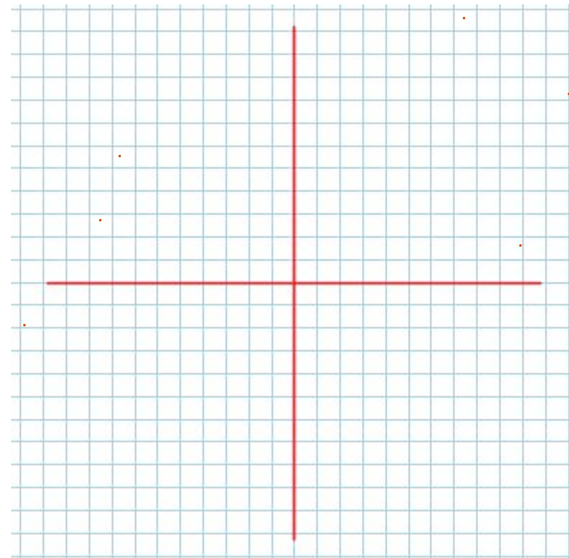
Analytic Definition of a Circle (i.e. the equation)

A **Circle** is a set of points which are all the same **distance** from a fixed central point.

$$\boxed{x^2 + y^2 = r^2}$$

P (x, y) → point on circle

r → radius.

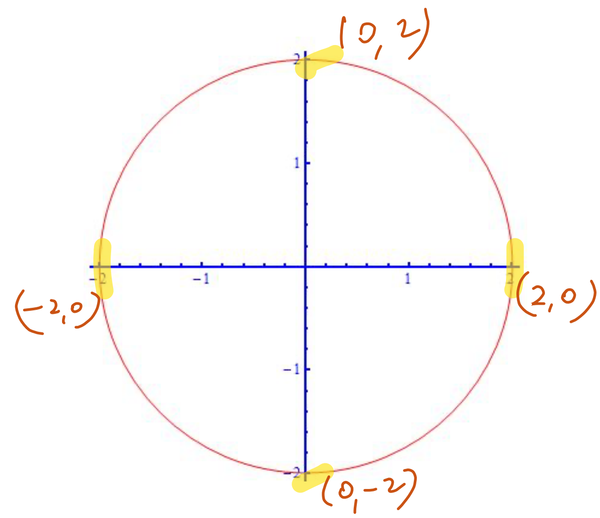


1. Determine the radius of the circle. $x^2 + y^2 = 25$.

$x^2 + y^2 = r^2$

$r = \sqrt{25} = 5$

2. Consider the sketch of a circle. Determine:



a) *x* intercepts

(2, 0) , (-2, 0)

b) *y* intercepts

(0, 2) , (0, -2)

c) the radius of the circle

$r = 2$

d) the equation of the circle

$x^2 + y^2 = 4$

3. Determine the equation of a circle with radius $r = 5\frac{2}{3}$ $= \frac{17}{3}$

$\boxed{x^2 + y^2 = r^2}$

$r^2 = \left(\frac{17}{3}\right)^2 = \frac{289}{9}$

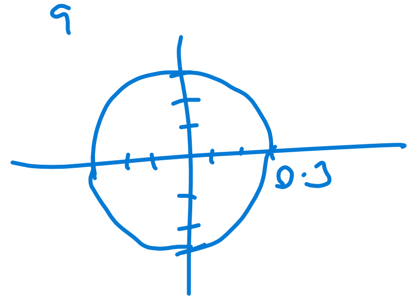
$x^2 + y^2 = \frac{289}{9}$

4. Sketch the circle with equation $x^2 + y^2 = 0.09$

Rough

r^2

$r = \sqrt{0.09} = 0.3$



5. Determine the equation of a circle with center at (0, 0) and a **diameter** of 14 units.

$x^2+y^2=49$

$d=14$
$r=7$
$r^2=49$

6. Determine whether the point (4.3, -2.6) is inside, outside or on the circle with equation $x^2 + y^2 = 25$

$x^2+y^2 = 4.3^2 + (-2.6)^2$
$= 18.49 + 6.76$
$= 25.25 > 25$

$\therefore$ OUTSIDE

On - if the answer is EQUAL **to r^2 then the point is on the circle**

Inside – if the answer is Less than **r^2 then the point is inside the circle**

Outside - if the answer is more than **r^2 then the point is outside the circle**

What about the point (3, 4)? Is it on, in or outside the circle?

$3^2+4^2 = 9+16 = 25 = 25$

$\therefore$ ON THE CIRCLE

7. Determine the equation of a circle with center (0, 0) which passes through the point (7, -3).

x y

$x^2+y^2=r^2$
$7^2+(-3)^2=r^2$
$49+9=r^2$
$58=r^2$

$\therefore$ $\boxed{\text{EQUATION: } x^2+y^2=58}$

2.3.2 General Form of the Equation of a Circle:

$$(\underline{\ x\ } - \underline{\ h\ })^2 + (\underline{\ y\ } - \underline{\ k\ })^2 = \underline{\ r\ }^2$$

Center: $(\underline{\ h\ }, \underline{\ k\ })$ and radius = $\underline{\ r\ }$

*A. Given the **center and radius**, write the equation.*

1. **C (5, 2) r = 7**

(__ - ___)^2 + (___ - ____)^2 = ___^{2} Equation: ____________________________

2. **C (-3, 4) r =** 25

(__ - ___)^2 + (___ - _____)^2 = ___^{2} Equation: ____________________________

*B. Given the **center and another point on the circle**, write the equation.*

To find r^2 either plug in the point or use the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

3. C (4, -7) and P (5, 3) Find r^2 by plugging in the point

(__ - ___)^2 + (___ - _____)^2 = ___^{2} Equation: ________________________

4. C (0,0) and P (-5, 2) Find r using the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

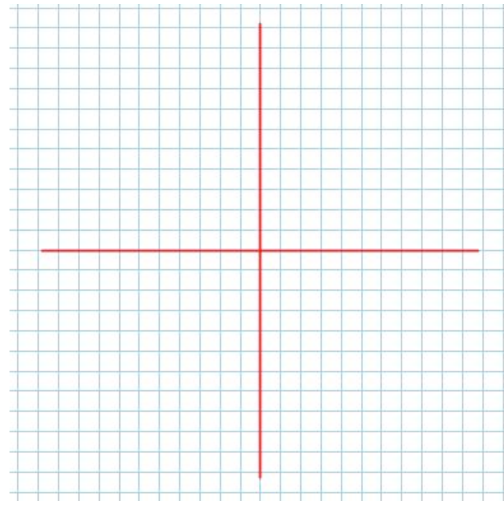
Equation: ________________________

Graphing Circles

1. $(x)^2 + (y)^2 = 36$

C = (____ , ____)

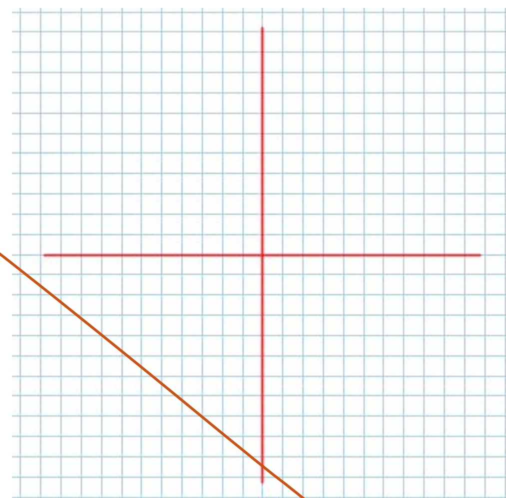
r = __________



2. $(x-3)^2 + (y-4)^2 = 25$

C = (____ , ____)

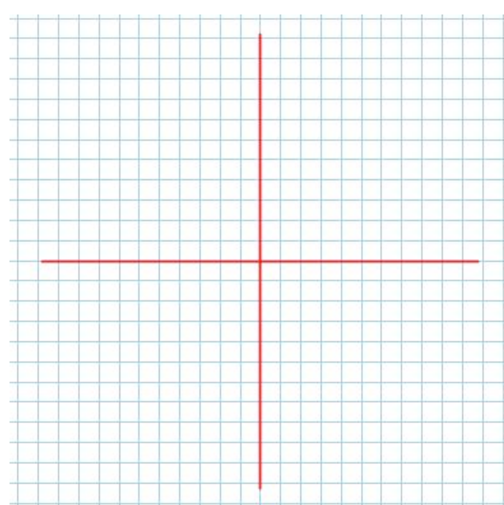
r = __________



3. $(x-5)^2 + (y+4)^2 = 41$

C = (____ , ____)

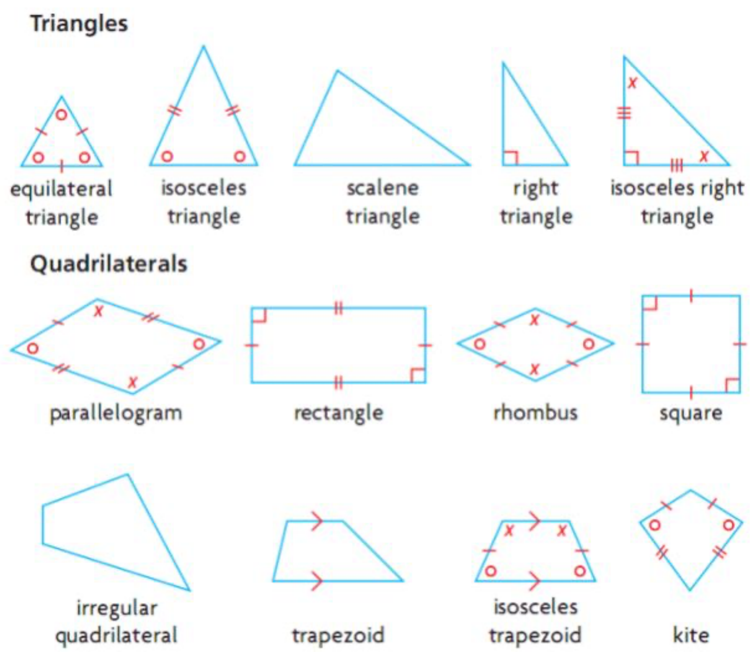
r = __________



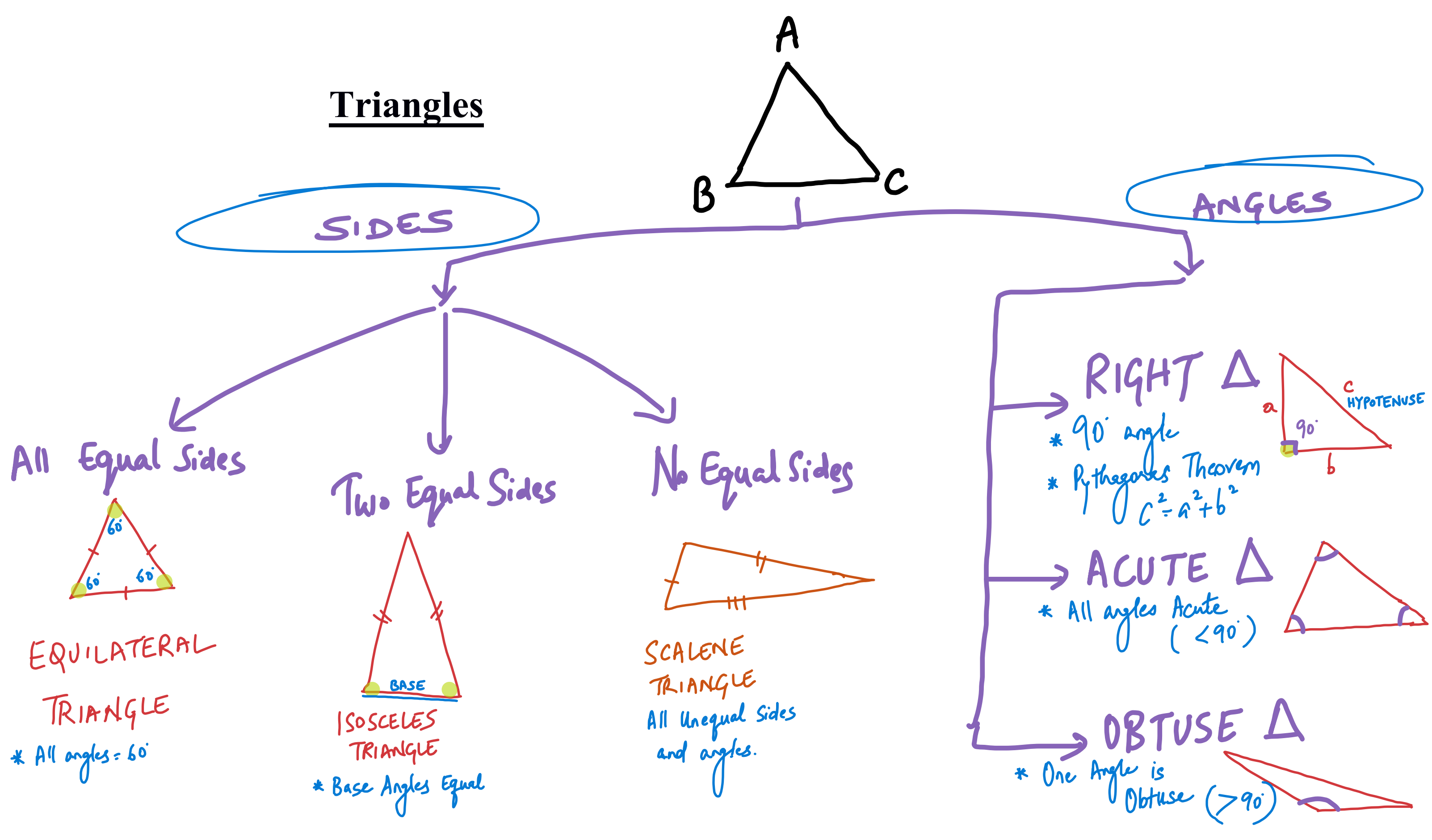
Lesson 5 (2.4) Classifying Geometric Figures

There are so many geometric figures that it's ridiculous. But we now know enough Analytic Geometry that we can easily do the "classification". We are really only going to worry about two "classes": Triangle and Quadrilaterals

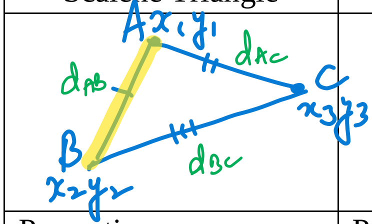
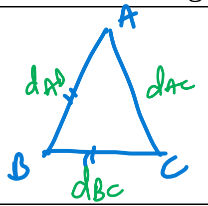
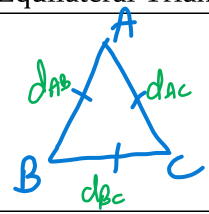
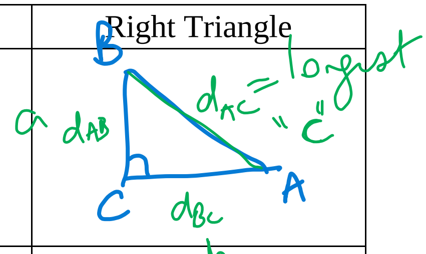
You need to know the following types of Triangles and Quadrilaterals:



Triangles

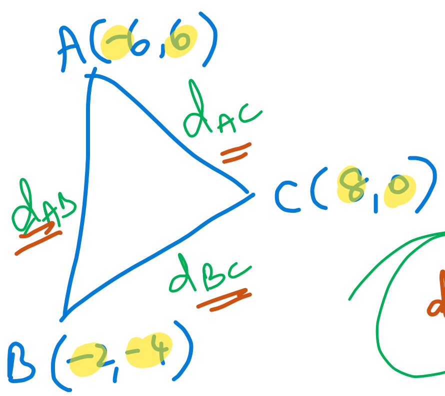


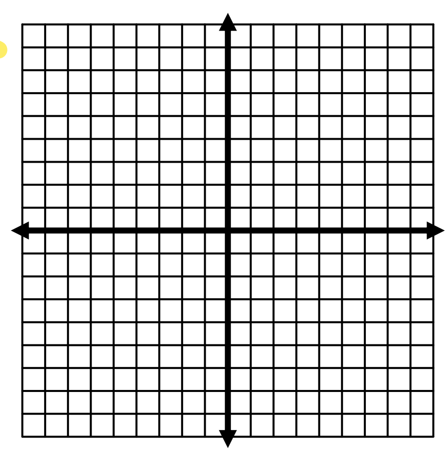
Properties of Triangles

Scalene Triangle	Isosceles Triangle	Equilateral Triangle	Right Triangle
			
Properties: All sides unequal	Properties: 2 sides EQUAL	Properties: All sides EQUAL	Properties: $c^2 = a^2 + b^2$
How To Identify: $d_{AB} \neq d_{BC} \neq d_{AC}$	How To Identify: $d_{AB} = d_{BC} \neq d_{AC}$	How To Identify: $d_{AB} = d_{BC} = d_{AC}$	How To Identify: $(d_{AC})^2 = (d_{AB})^2 + (d_{BC})^2$

In essence, side lengths are a powerful way to determine the type of triangle. It has an additional advantage to check for the presence of a right angle using Pythagoras Theorem.

What type of triangle is formed by the points A(-6, 6), B(-2, -4), and C(8, 0)





Graph not needed but provided in case you want to check your result graphically.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d_{AB} = \sqrt{(-2 - -6)^2 + (-4 - 6)^2}$$
$$= \sqrt{16 + 100} = \sqrt{116} = b \rightarrow \boxed{b^2 = 116} *$$

$$d_{BC} = \sqrt{(8 - -2)^2 + (0 - -4)^2}$$
$$= \sqrt{100 + 16} = \sqrt{116} = a \rightarrow \boxed{a^2 = 116} *$$

$$d_{AC} = \sqrt{(8 - -6)^2 + (0 - 6)^2}$$
$$= \sqrt{196 + 36}$$
$$= \sqrt{232} = c \rightarrow \boxed{c^2 = 232} *$$

*

CHECK Pythagoras Thm:

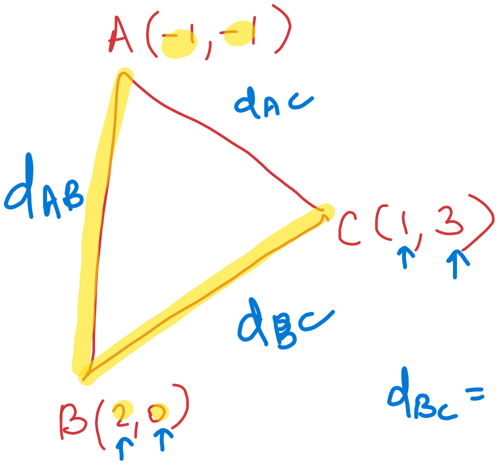
$c^2 = a^2 + b^2$

$232 = 116 + 116.$

∴ ΔABC is an isosceles Right Δ.

Practice:

A triangle has vertices at $A(-1,-1)$, $B(2,0)$, and $C(1,3)$. Using analytic geometry, determine what type of triangle it is.



$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d_{AB} = \sqrt{(2--1)^2+(0--1)^2} = \sqrt{3^2+1^2}$$

$$= \sqrt{9+1} = \sqrt{10} = a$$

$$d_{BC} = \sqrt{(1-2)^2+(3-0)^2} = \sqrt{(-1)^2+3^2} = \sqrt{1+9} = \sqrt{10} = b$$

$$d_{AC} = \sqrt{(1--1)^2+(3--1)^2} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20} = c$$

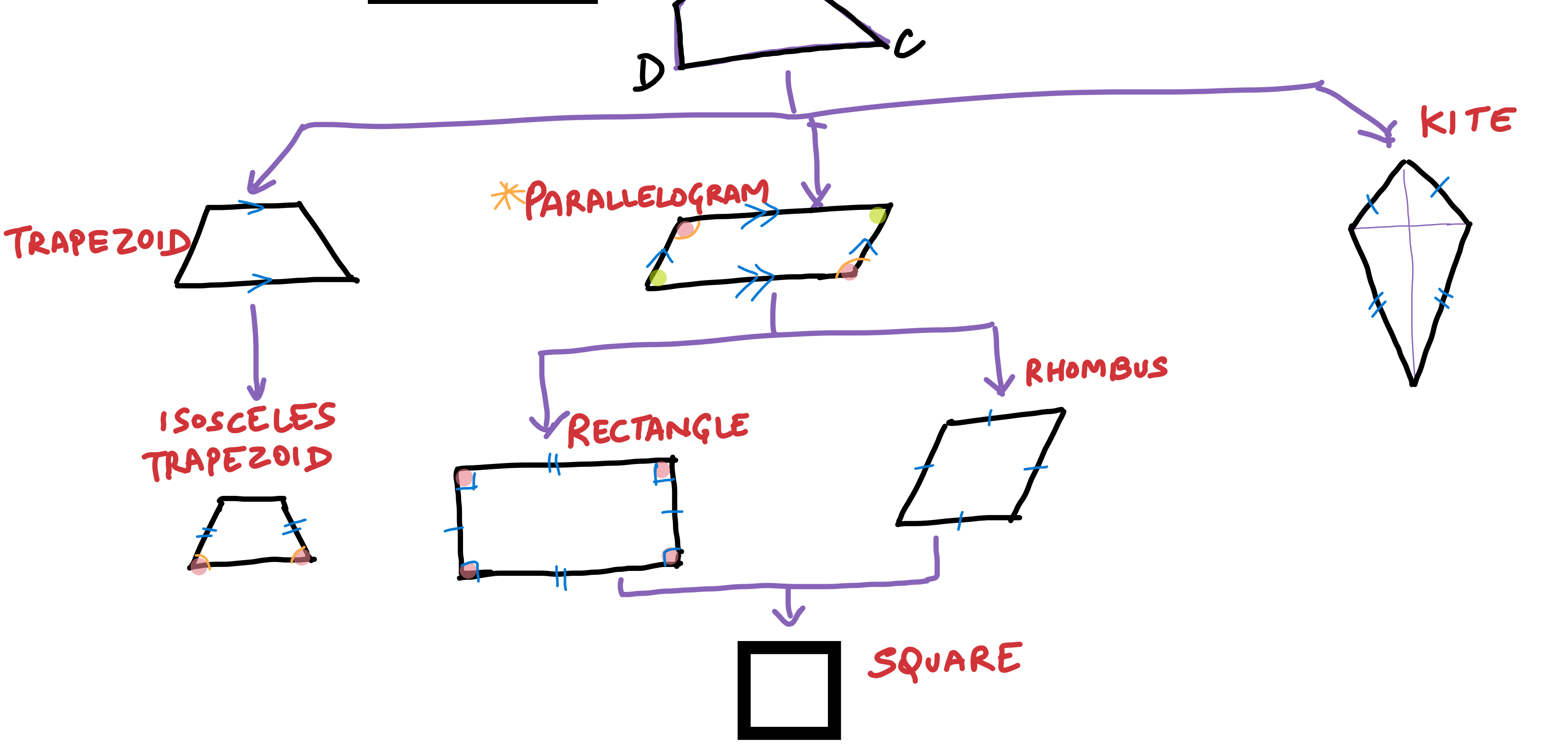
$$c^2 = (d_{AC})^2 = (\sqrt{20})^2 = 20$$

$$a^2+b^2 = (d_{AB})^2+(d_{BC})^2 = (\sqrt{10})^2+(\sqrt{10})^2 = 10+10 = 20$$

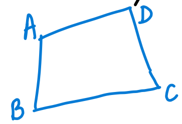
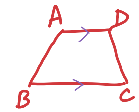
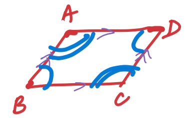
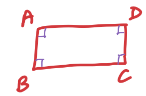
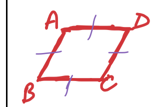
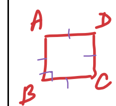
$\rightarrow c^2 = a^2+b^2$

$\therefore$ ΔABC is a right isosceles Δ.

Quadrilaterals



Note that all Geometric Shapes can be classified using the Side lengths and the Angles

Properties of Quadrilaterals		How to identify? 	 Trapezoid	 Parallelogram	 Rectangle	 Rhombus	 Square
Sides	All sides are equal in length	$d_{AB} = d_{BC} = d_{CD} = d_{AD}$	X	X	X	✓	✓
	Opposite sides are equal in length	$d_{AD} = d_{BC}$; $d_{AB} = d_{CD}$	X	✓	✓	✓	✓
	Opposite sides are parallel	$m_{AD} = m_{BC}$; $m_{AB} = m_{CD}$	X	✓	✓	✓	✓
Angles	All angles are equal=90°	NEGATIVE RECIPROCAL slopes of adjacent sides	X	X	✓	X	✓
	Opposite angles are equal		X	✓	✓	✓	✓

A few tips to identify the quadrilateral when given all the four vertices:

Step 1: Find the slopes of all sides.

	Conclusion:
1. ONLY One pair of opposite sides with same slope	TRAPEZOID
2. ONLY Both pair of opposite sides with the same slope	PARALLELOGRAM, RHOMBUS
3. Both pair of opposite sides with the same slope and one of the slopes is negative reciprocal of the other	RECTANGLE, SQUARE

Step 2: Find the length of all sides.

		Conclusion
2. ONLY Both pair of opposite sides have the same slope	2. a.) All sides equal	RHOMBUS
	2. b.) Only opposite sides equal	PARALLELOGRAM
3. Both pair of opposite sides have the same slope and one of the slopes is negative reciprocal of the other	3. a.) All sides equal	SQUARE
	3. b.) Only opposite sides equal	RECTANGLE.

In essence, slopes are a powerful tool for determining whether a quadrilateral belongs to the Parallelogram Family.
It has an additional advantage of identifying the presence of right angles which means Rectangle/Square.

Example 1:

Verify what type of quadrilateral is formed by the points P(-5,-5), Q(-30,10), R(-5,25), and S(20,10).

Example 2:

Your friend claims that the quadrilateral with vertices at W(-1, 3), X(-3, -2), Y(5, -3), and Z(7, 2) form a rectangle.
Is your friend correct? Fully justify your answer.

Example 3.
A surveyor is marking the corners of a building lot. If the corners have coordinates $A(-5, 4)$, $B(4, 9)$, $C(9, 0)$, and $D(0, -5)$, what shape is the building lot? Include your calculations in your answer.

Lesson 6 Centroid; Circumcentre; Orthocentre

In any given triangle,

i. All the Medians intersect at the same point called the **CENTROID**.

ii. All the perpendicular bisectors intersect at the same point called the **CIRCUMCENTRE.**

iii. All the altitudes intersect at the same point, called the **ORTHOCENTRE.**

Example 1: Triangle PQR has vertices at P(12, 6), Q(4, 0), and R(8, 6). Use analytic geometry to determine the coordinates of the centroid (the point where the medians intersect).

Example 2: Triangle JKL has vertices at J(2, 0), K(2, 8), and L(7, 3). Use analytic geometry to determine the coordinates of the circumcentre (the point where the perpendicular bisectors intersect).

Example 3: Triangle DEF has vertices at D(2, 8), E(6, 2), and F(3, 2). Use analytic geometry to determine the coordinates of the orthocentre (the point where the altitudes intersect).

Example 4: Triangle LMN has vertices at L(3, 4), M(4, 3), and N(4, 1). Use analytic geometry to determine the area of the triangle.