Name: ______________________________

FACTORING

Unit Outline:

a. Polynomial Expansion
b. $ax^2 + bx + c$
c. Special Cases: a=1, Differences of Squares, and Perfect Squares

By the end of this unit:

- o I can expand a binomial multiplied by a binomial;
- o I can create simplified expressions for perimeters, areas, and volumes;
- o I can common factor a polynomial;
- o I can factor both simple and complex trinomials;
- o I can factor perfect square trinomials and differences of squares.
- o I can use factoring to determine expressions for dimensions; and
- o I can convert specific quadratic relations from standard form to factored form to graph.

Binomial Products

A binomial is a polynomial with two terms. The terms of a polynomial are separated by + sign or – sign. For example: $3x + y$ $5xy - 3$

When finding the product of two binomials, we can find the product by either of the following methods:

A. Repeated Distributive Property

B. FOIL Method– <u>First–Outside–Inside–Last</u>

Both the methods above are essentially the same. FOIL is just the acronym explaining the rule when multiplying two binomials.

Examples: FOIL

$(2x - 3)(4x + 1)$

$= 8x^2 + \boxed{2x - 12x} - 3$

$= 8x^2 - 10x - 3$

$(5a + 4)(7a + 2)$

$= 35a^2 + \boxed{10a + 28a} + 8$

$= 35a^2 + 38a + 8$

$(3x - 2y)(3x + 8y)$

$= 9x^2 + \boxed{24xy - 6xy} - 16y^2$

$= 9x^2 + 18xy - 16y^2$

$(2x - 3)^2$

$= (2x - 3)(2x - 3)$

$= 4x^2 - 6x - 6x + 9$

$= 4x^2 - 12x + 9$

$(2x - 3)(3x^2 - 4x + 1)$

$= 6x^3 - 8x^2 + 2x$

$\quad - 9x^2 + 12x - 3$

$6x^3 - 17x^2 + 14x - 3$

$(4x^2 - 3x + 3)(3x^2 - 7x + 3)$

$12x^4 - 28x^3 + 12x^2$

$\quad - 9x^3 + 21x^2 - 9x$

$\quad + 9x^2 - 21x + 9$

$12x^4 - 37x^3 + 42x^2 - 30x + 9$

Common Factoring

Factoring is the OPPOSITE of expanding. When expanding, we aim to remove brackets. However, when factoring, the attempt is to bring these brackets back.

Common Factoring is a very important factoring method and is the first rule of factoring whenever possible. When common factoring, you must look for the GREATEST COMMON FACTOR (GCF). Note that an algebraic expression has been fully factored once 1 or −1 is the only common factor left.

$\underbrace{-30x^2y^2}_{-10x} + \underbrace{20x}_{-10x}$ GCF = $-10x$

$= -10x(3xy^2 - 2)$

$\boxed{5x(x-3)} + \boxed{8(x-3)}$ $(x-3)$ $(x-3)$

$= (x-3)(5x+8)$

$\underbrace{5xy}_{x} + \underbrace{4x^3}_{x} + \underbrace{32x^2y}_{x}$

$= x(5y + 4x^2 + 32xy)$

$\underbrace{8x^4y^2}_{2x^2y} - \underbrace{18x^3y}_{2x^2y} + \underbrace{18x^2y}_{2x^2y}$

$= 2x^2y(4x^2y - 9x + 9)$

$\underbrace{-14n^6}_{-2n^3} - \underbrace{20n^5}_{-2n^3} + \underbrace{12n^3}_{-2n^3}$

$= -2n^3(7n^3 + 10n^2 - 6)$

$\underbrace{-8uv^5}_{-uv} - \underbrace{3u^2v}_{-uv} - \underbrace{2uv}_{-uv}$

$= -uv(8v^4 + 3u + 2)$

Factoring Trinomial Quadratics: $ax^2 + bx + c$

Let's begin the lesson with a Number Game. Your comfort in this game of finding the correct numbers to satisfy the given conditions, will decide how comfortable you will be with FACTORING TRINOMIALS

<u>**AIM of the GAME:**</u>

Find the SINGLE factor pair that satisfies the given conditions. (Note: M = multiply to, A = adds to)

M	A	S	B
+	+	+	+
+	−	−	−
−	−	+	−
−	+	−	+

a) M: 45
A: 18

45
1×45
$\boxed{3 \times 15}$

3, 15

b) M: −132
A: +1

132
-1×132
-2×66
-3×44
-4×33
-6×22
$\boxed{-11 \times 12}$

−11, 12

c) M: 60
A: -23

60
-1×-60
-2×-30
$\boxed{-3 \times -20}$

−3, −20

d) M: -216
A: -19

−216
1×-216
2×-108
3×-72
4×-54
6×-36
$\boxed{8 \times -27}$

8, −27

Why are we making these factor pairs?

Remember that factoring is the opposite of expanding. When we FOILed a few lessons ago, we ended up with trinomials. Therefore, factoring is the opposite of FOILing. Our answers will always be (*binomial*)(*binomial*).
This method we use is called "decomposition"

Let's say our trinomial looks like: $Ax^2 + Bx + C$

How to Set-Up Factoring by Decomposition

A. Multiply the first coefficient (A) and the constant term (C) together. This is the number that you are finding the factors of.
B. The specific factor pair that you are looking for, must add up to the middle coefficient (B) term.
C. Decompose the middle term
D. Common factor the first half and second half of the decomposed expression
E. Remember the leftovers and the brackets

-150

-1×150

-2×75

-3×50

1) $8x^2+6x-5$

$M=-40$

$A=6=10-4$

$= 8x^2+10x \;\; -4x-5$

$= 2x(4x+5)-1(4x+5)$

$= (4x+5)(2x-1)$

2) $5n^2+47n-30$

$M=-150$

$A=47=-3+50$

$= 5n^2-3n \;\; +50n-30$

$= n(5n-3)+10(5n-3)$

$= (5n-3)(n+10)$

3) $2n^2+n-36$

$M=-72$

$A=1$

$= \underbrace{2n^2+9n}_{n}\underbrace{-8n-36}_{-4}$

$=(n-4)(2n+9)$

4) $9a^2-47a+10$

$M=90$

$A=-47$

$= \underbrace{9a^2-45a}_{9a}\underbrace{-2a+10}_{-2}$

$=(9a-2)(a-5)$

5) n^2+5n+6

$M=6$

$A=5$

$= \underbrace{n^2+2n}_{n}+\underbrace{3n+6}_{3}$

$=(n+3)(n+2)$

6) $9n^2-34n-8$

$M=-72$

$A=-34$

$= \underbrace{9n^2-36n}_{9n}+\underbrace{2n-8}_{2}$

$=(9n-2)(n-4)$

7) $2r^2-5r-3$

$M=-6=(-6)(1)$

$A=-5=-6+1$

$= \underbrace{2r^2-6r}_{2r}+\underbrace{1r-3}_{1}$

$=(2r+1)(r-3)$

8) $x^2-12x+27$

$M=27$

$A=-12$

$\underbrace{x^2-9x}_{x}\underbrace{-3x+27}_{-3}$

$(x-3)(x-9)$

9) $3a^2-18a-48$ (each term divided by 3)

$=3(a^2-6a-16)$

$M=-16=(-8)(2)$

$A=-6$

$=3(\underbrace{a^2-8a}_{a}+\underbrace{2a-16}_{2})$

$=3(a+2)(a-8)$

10) $24k^2-100k+56$ (each term divided by 4)

$=4(6k^2-25k+14)$

$M=84=(-4)(-21)$

$A=-25=(-4)+(-21)$

$=4(\underbrace{6k^2-4k}_{2k}\underbrace{-21k+14}_{-7})$

$=4(2k-7)(3k-2)$

Special Cases

A. Factoring Trinomial Quadratics: $x^2 + bx + c$

A quadratic expression in the form $x^2 + bx + c$ can be factored into two binomials of the form $(x + s)$ and $(x + t)$, where $s+t = b$ **=A** and $s.t= c$ **=M**.

Note that this is **the simplest case of decomposition** since here, a=1 in the **quadratic** form ax^2+bx+c.

Why does this quick factoring work? Look at what happens when we `expand` the following expression:

$(x+4)$ $(x+5)$

$(x + A)(x + B)$

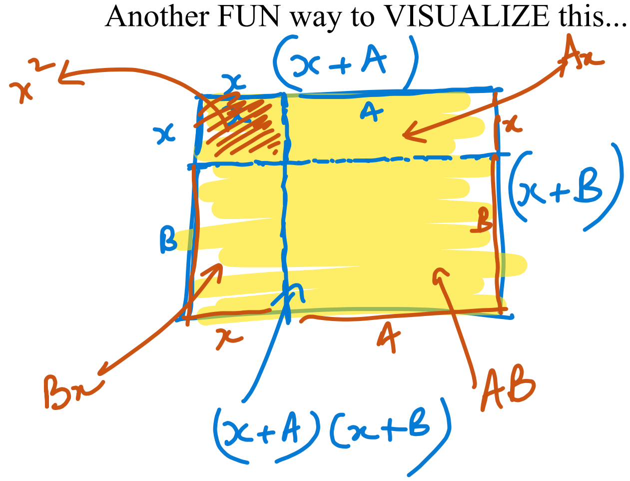
$x^2 + \boxed{Bx + Ax} + AB$

$x^2 + (A+B)x + AB$

$(4+5)$ $(4)(5)$

$x^2 + 9x + 20$

Another FUN way to VISUALIZE this....



Factor the following expressions:

$x^2 + \overset{A}{9}x + \overset{M}{20}$

4 5

1x20
2x10
$\boxed{4x5}$

$(x+4)(x+5)$

$a^2 - \overset{A}{10}a + \overset{M}{24}$

−6 −4

$(a-6)(a-4)$

$2n^2 - 22n - 24$

$\frac{2n^2}{2} - \frac{22n}{2} - \frac{24}{2}$

$= 2(n^2 - \overset{A}{11}n - \overset{M}{12})$

−12 1

$= 2(n-12)(n+1)$

B Differences of Squares

Let's look at expanding first:

$(5n - 3)(5n + 3)$

$25n^2 + \cancel{15n} - \cancel{15n} - 9$

$25n^2 - 9$

$(7x - 4)(7x + 4)$

$49x^2 \cancel{+ 28x} \cancel{- 28x} - 16$

$49x^2 - 16$

(A+B)(A−B)

$A^2 \cancel{- AB} \cancel{+ AB} - B^2$

$A^2 - B^2$

Since we clearly notice a pattern in the examples above, try figuring out how to work backwards from the last step to the first step. Remember factoring is the opposite of expanding!! :)

Difference of Squares– the polynomial must be a BINOMIAL

– the 1st and 2nd terms must be perfect squares

– there must be a — sign between the two terms

– The factor form of $A^2 - B^2$ is $(A+B)(A-B)$

$49n^2 - 4$

$(7n)^2 - (2)^2$

$(7n+2)(7n-2)$

$x^2 - 25$

$x^2 - 5^2$

$(x+5)(x-5)$

$75n^4 - 27$

$\frac{75n^4}{3} - \frac{27}{3}$

$= 3(25n^4 - 9)$

$(5n^2)^2 \quad 3^2$

$= 3(5n^2+3)(5n^2-3)$

C. Perfect Squares

Let's look at expanding First:

$(5n+3)^2$

$(5n+3)(5n+3)$

$25n^2+15n+15n+9$

$25n^2+30n+9$

$(5n)^2+2(5n)(3)+(3)^2$

$(7x-4)^2$

$(7x-4)(7x-4)$

$49x^2-28x-28x+16$

$49x^2-2(28x)+16$

$(7x)^2-2(7x)(4)+4^2$

$(A+B)^2$

$(A+B)(A+B)$

$A^2+AB+AB+B^2$

$A^2+2AB+B^2$

$(A-B)^2$

$(A-B)(A-B)$

$A^2-AB-AB+B^2$

$A^2-2AB+B^2$

In the examples above, **we again notice PATTERNS!!! :)**

Try figuring out how to work backwards from the last step to the first step.

– the polynomial must be a trinomial of the form $A^2+2AB+B^2$
or $A^2-2AB+B^2$

– Remember to check the middle term by expanding to see if it can take the form $+2AB$ or $-2AB$

$4k^2-12k+9$

$(2k)^2 \qquad 3^2$

$-2(2k)(3)$

$=(2k-3)^2$

$16m^2 + 8m + 1$

$(4m)^2$ 1^2

$2(4m)(1)$

$= (4m+1)^2$

$36\text{m}^2 + 60\text{m} + 25$

$(6m)^2$ 5^2

$2(6m)(5)$

$= (6m+5)^2$

$9a^2 + 42a + 49$

$(3a)^2$ $(7)^2$

$+2(3a)(7)$

$= (3a+7)^2$

$49a^2 - 140a + 100$

$7a$ 10

$-2(7a)(10)$

$= (7a-10)^2$