Name: ______________________________

Solving Quadratic Equations

Unit Outline:

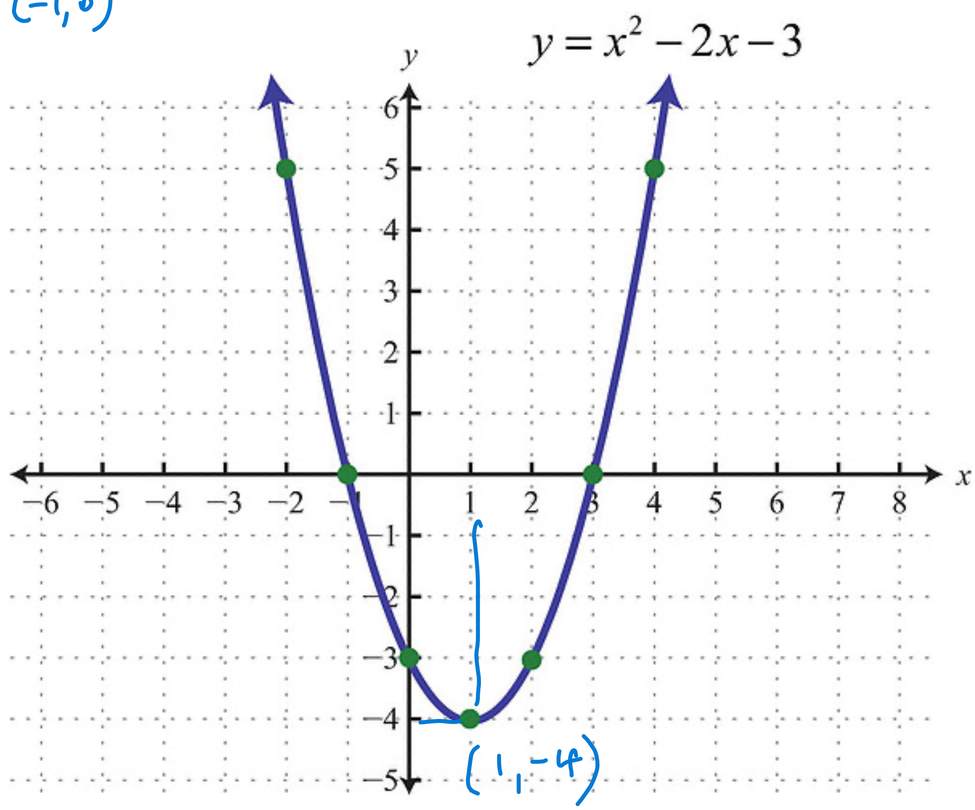
a. Solving by Factoring
b. Solving from Vertex Form
c. Solving using the Quadratic Formula
d. Real-Life Applications
e. Review

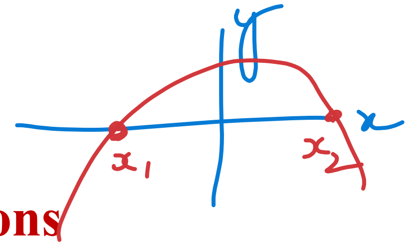
Standard Form	Zeros Form	Vertex Form
$y = ax^2 + bx + c$	$y = a(x-r)(x-s)$	$y = a(x-h)^2 + k$

$y = x^2 - 2x - 3$ $\qquad$ $y = (x-3)(x+1)$ $\qquad$ $y = (x-1)^2 - 4$

$-3 \quad 1$

Vertex	Zeros	y-int	Vertical Stretch	Horizontal Shift	Vertical Shift	Axis of Symmetry AoS	Max/Min Value
(h,k)	$(r,0)$ $(s,0)$	$(0,c)$	a	h	k	$x = h = \frac{r+s}{2}$	k
$(1,-4)$	$(3,0)$ $(-1,0)$	$(0,-3)$	1	1	-4	$x = 1$	-4





$x^2 - 4x = 5$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x - 5 = 0$

$\boxed{x = 5}$

$x + 1 = 0$

$\boxed{x = -1}$

Solving Quadratic Equations

A quadratic relation, $y = ax^2 + bx + c$, describes a relation between x and y.

A quadratic equation, $ax^2 + bx + c = 0$, needs to be solved for the value of x.

When we solve an equation like 4x+2 =6, we are finding the value of *x*.

$4x + 2 = 6$

$4x = 6 - 2$

$\frac{4x}{4} = \frac{4}{4}$

$\boxed{x = 1}$

Try solving the equation: $x^2 - 4x = 5$

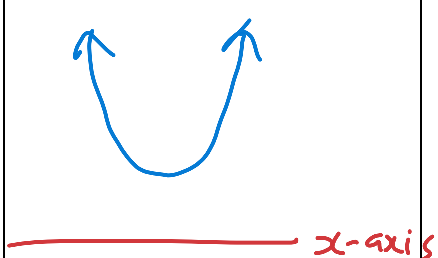
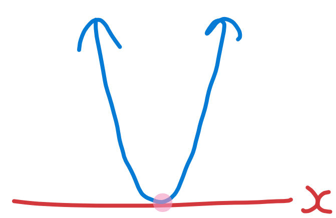
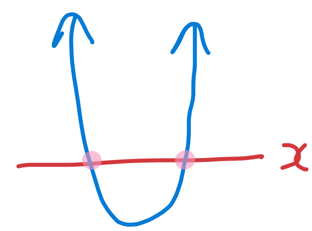
$\boxed{x = 5}$

Left $25 - 20 = 5 =$ Right

$\boxed{x = -1}$

Left $= (-1)^2 - 4(-1) = 1 + 4 = 5 =$ Right

"Solving" a quadratic equation means finding the point at which the parabola intersects with the y-axis.

None	One	Two
		

In this unit, we will learn 3 ways to Solve a Quadratic Equation:

1. Solving by **Factoring**
2. Solving by **Vertex Form**
3. Solving by **Quadractic Formula**

Solving a Quadratic Equation can be compared to finding the **Zeros** in a quadratic relation. Why?

This is because you are solving for x-values when $y = 0$ i.e. defn. of zeros or x-intercepts.

Solving by Factoring

from section 6.1

A quadratic expression can be solved by FACTORING. Another word for finding the "Solution" is finding the ZEROS.

Steps:

1. Put the equation into a form = 0 i.e. $ax^2+bx+c= 0$
2. Factor the Quadratic Expression i.e. $a(x-r)(x-s)= 0$
3. Set both parts of the product equal to 0 and then **solve!!!**

Examples:

$0 = x^2 + x - 12$

$4 \quad -3$

$0 = (x+4)(x-3)$

$x+4=0$

$\boxed{x_1=-4}$

$x-3=0$

$\boxed{x_2=3}$

$3x^2+9=-6x^2+10$

$3x^2+9+6x^2-10=0$

$9x^2-1=0$

$(3x)^2-(1)^2$

$(3x+1)(3x-1)=0$

$3x+1=0$

$3x=-1$

$\boxed{x_1=-\frac{1}{3}}$

$3x-1=0$

$3x=1$

$\boxed{x_2=\frac{1}{3}}$

$0= 2x^2+5x-12$

$M=-24$

$A=5$

$8 \quad -3$

$0=\underbrace{\boxed{2x^2+8x}}_{2x}\underbrace{\boxed{-3x-12}}_{-3}$

$0=(2x-3)(x+4)$

$2x-3=0$

$2x=3$

$\boxed{x_1=\frac{3}{2}}$

$x+4=0$

$\boxed{x_2=-4}$

Solving from Vertex Form

Let's solve from Vertex Form with some examples:

Remember SOLVE – SAMDEB

Calculate – BEDMAS

$37 = 2(x-3)^2 + 5$

$37-5 = 2(x-3)^2$

$\frac{\cancel{32}^{16}}{\cancel{2}} = \frac{\cancel{2}(x-3)^2}{\cancel{2}}$

$16 = (x-3)^2$

$\sqrt{16} = x-3$

$\pm 4 = x-3$

$4 = x-3$

$4+3 = x$

$\boxed{7 = x_1}$

$-4 = x-3$

$-4+3 = x$

$\boxed{-1 = x_2}$

$0 = \frac{1}{2}(x+5)^2 - 2$

$2 = \frac{1}{2}(x+5)^2$

$4 = (x+5)^2$

$\sqrt{4} = x+5$

$\pm 2 = x+5$

$\boxed{x_1 = -3}$ $\boxed{x_2 = -7}$

More complicated examples:

$0 = 3(x-1)^2 - 5$

$\frac{5}{3} = \frac{3(x-1)^2}{3}$

$\frac{5}{3} = (x-1)^2$

$\pm\sqrt{\frac{5}{3}} = (x-1)$

$\pm 1.29 = x-1$

$1.29 = x-1$

$1.29+1 = x_1$

$\boxed{x_1 = 2.29}$

$-1.29 = x-1$

$-1.29+1 = x_2$

$\boxed{x_2 = -0.29}$

$0 = -2(x+4)^2 + 5$

$\frac{-5}{-2} = \frac{-2(x+4)^2}{-2}$

$2.5 = (x+4)^2$

$\pm\sqrt{2.5} = x+4$

$\pm 1.58 = x+4$

$1.58 = x+4$

$1.58-4 = x_1$

$\boxed{-2.42 = x_1}$

$-1.58 = x+4$

$-1.58-4 = x_2$

$\boxed{-5.58 = x_2}$

Other methods of solving for *x*:

$2x^2 = 50$

$\frac{2x^2}{2} = \frac{50}{2}$

$x^2 = 25$

$x = \sqrt{25}$

$x = \pm 5$

$\boxed{x_1 = 5}$ $\boxed{x_2 = -5}$

$x(x-2) = 36 - 2x$

$x^2 - 2x = 36 - 2x$

$x^2 \cancel{-2x} - 36 \cancel{+2x} = 0$

$x^2 - 36 = 0$

$x^2 = 36$

$x = \sqrt{36}$

$x = \pm 6$

$\boxed{x_1 = 6}$ $\boxed{x_2 = -6}$

$(x+1)^2 - 16 = 0$

$(x+1)^2 = 16$

$x+1 = \sqrt{16}$

$x+1 = \pm 4$

$x_1 + 1 = 4$

$x_1 = 4 - 1$

$\boxed{x_1 = 3}$

$x_2 + 1 = -4$

$\boxed{x_2 = -5}$

$0.25(x-4)^2 - 4 = 0$

$\frac{0.25(x-4)^2}{0.25} = \frac{4}{0.25}$

$(x-4)^2 = 16$

$x-4 = \sqrt{16}$

$x-4 = \pm 4$

$x-4 = 4$

$\boxed{x_1 = 8}$

$x-4 = -4$

$\boxed{x_2 = 0}$

Solving using the Quadratic Formula

from section 6.4

Today we will use the Quadratic Formula to solve *ANY* Quadratic Equation.

The Quadratic Formula gives the solution to any quadratic equation given in standard form. The Quadratic Formula is:

$0 = ax^2 + bx + c$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}} \to \text{DISCRIMINANT}$$

Solve the following:

$3x^2 - 4x = 2$

$3x^2 - 4x - 2 = 0$

$a = 3,\ b = -4,\ c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{--4 \pm \sqrt{(-4)^2 - 4(3)(-2)}}{2(3)}$$

$$x = \frac{4 \pm \sqrt{16 + 24}}{6}$$

$$x = \frac{4 \pm \sqrt{40}}{6}$$

$$x = \frac{4 \pm 6.32}{6}$$

$$x_1 = \frac{4 + 6.32}{6}$$

$$\boxed{x_1 = 1.72}$$

$$x_2 = \frac{4 - 6.32}{6}$$

$$\boxed{x_2 = -0.39}$$

$2x^2 = 3x^2 - 4(x+3)$

$2x^2 = 3x^2 - 4x - 12$

$0 = 3x^2 - 4x - 12 - 2x^2$

$0 = x^2 - 4x - 12$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 + 48}}{2}$$

$$x = \frac{4 \pm \sqrt{64}}{2}$$

$$x = \frac{4 \pm 8}{2}$$

$$x_1 = \frac{4 + 8}{2} \qquad x_2 = \frac{4 - 8}{2}$$

$$\boxed{x_1 = 6} \qquad \boxed{x_2 = -2}$$

Note: $b^2 - 4ac$ is referred to as the Discriminant. It can determine how many *real solutions* there are to the Quadratic Equation (two, one or no real solutions)

1. $D = b^2 - 4ac > 0 \to$ 2 SOLUTIONS
2. $D = b^2 - 4ac = 0 \to$ 1 SOLUTION
3. $D = b^2 - 4ac < 0 \to$ 0 SOLUTION

Use the quadratic formula to solve for the roots of the following equations: zeros

1.) $y = 2x^2 + 8x - 5$

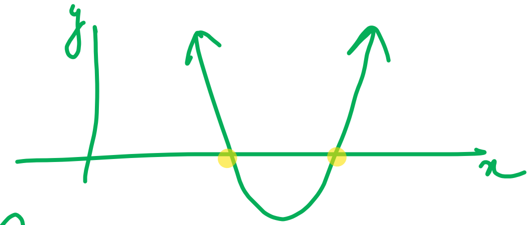
$0 = 2x^2 + 8x - 5$

$$x = \frac{-8 \pm \sqrt{64 - 4(2)(-5)}}{2(2)}$$

$$x = \frac{-8 \pm \sqrt{64+40}}{4}$$

$$x = \frac{-8 \pm \sqrt{104}}{4} = \frac{-8 \pm 10.2}{4}$$

$$\boxed{x_1 = \frac{-8+10.2}{4} = 0.55} \qquad \boxed{x_2 = \frac{-8-10.2}{4} = -4.55}$$



∴ 2 SOLUTIONS.

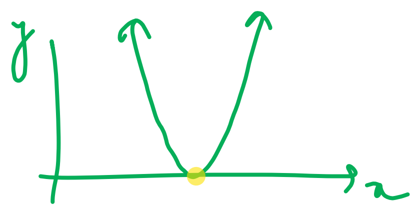
2.) $y = 43x^2 - 1204x + 8428$

$$\frac{0}{43} = \frac{43x^2}{43} - \frac{1204x}{43} + \frac{8428}{43}$$

$0 = x^2 - 28x + 196$

$$x = \frac{28 \pm \sqrt{784 - 784}}{2} \quad = 0$$

$$x = \frac{28}{2} = 14 \qquad \boxed{x = 14}$$



∴ 1 SOLUTION

3.) $y = 6x^2 + 4x + 9$

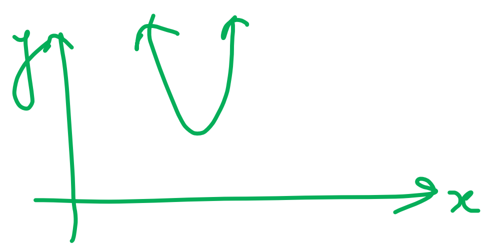
$0 = 6x^2 + 4x + 9$

$$x = \frac{-4 \pm \sqrt{16 - 4(6)(9)}}{2(6)}$$

$$x = \frac{-4 \pm \sqrt{16-216}}{12} = \frac{-4 \pm \sqrt{-200}}{12}$$ Not possible

No SOLUTIONS.

∴ 0 SOLUTIONS.



Real-Life Applications:

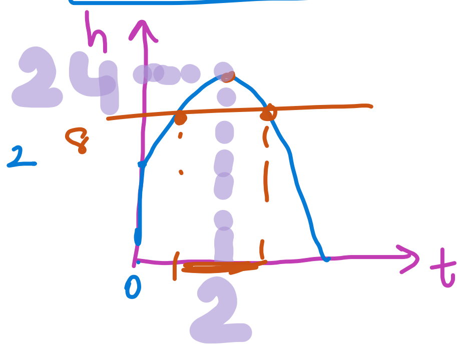
Let's start by applying our learning of solving quadratics with an example. Even if you are someone who doesn't enjoy Word Problems, I am sure you will find patterns that make these Word Problems super fun!!

A ball is thrown and follows the path of a parabola. It follows the path of $\boxed{h = -5t^2 + 20t + 4}$.

Where h is the height in metres and t is time in seconds.

a) Complete the Square to find the vertex.

$\frac{4}{2} = 2$



$h = -5t^2 + 20t + 4$

-5

$h = -5(t^2 - 4t + 2^2 - 2^2) + 4$

$h = -5((t-2)^2 - 4) + 4$

$h = -5(t-2)^2 + 20 + 4$

$\boxed{h = -5(t-2)^2 + 24}$

VERTEX $(2, 24)$
t h

b) When is the height of the ball above 8 m? Solve using "Solving from Vertex Form".

$t = ?$ $h = 8\text{m}$

$8 = -5(t-2)^2 + 24$

$8 - 24 = -5(t-2)^2$

$-16 = -5(t-2)^2$

$\frac{+16}{+5} = (t-2)^2$

$\sqrt{\frac{16}{5}} = t - 2$

$\pm 1.79 = t - 2$

$1.79 = t_1 - 2$

$t_1 = 1.79 + 2 = 3.79$ sec

$-1.79 = t_2 - 2$

$t_2 = 2 - 1.79$

$= 0.21$ sec.

$\therefore$ The ball is above 8m from 0.21 sec to 3.79 sec.

c) When does the ball hit the ground? Solve using the "Quadratic Formula.

$t = ?$ $h = 0$

$0 = -5t^2 + 20t + 4$

$a = -5$, $b = 20$, $c = 4$

$t = \frac{-20 \pm \sqrt{400 - 4(-5)(4)}}{2(-5)}$

$t = \frac{-20 \pm \sqrt{400 + 80}}{-10} = \frac{-20 \pm \sqrt{480}}{-10}$

$\approx \frac{-20 \pm 21.91}{-10}$

$t_1 = \frac{-20 + 21.91}{-10}$

$t_1 = -0.191$ Rejected

Time is not NEGATIVE

$t_2 = \frac{-20 - 21.91}{-10}$

$\boxed{t_2 = 4.191}$

$\therefore$ The ball hits the ground in 4.191 sec.

Students really dislike word problems, no matter what type they are.

Things to Remember: There are two types of word problems in Quadratics: equations given and equations not given. However, no matter the situation, once you have an equation, there are 4 things you can do:

1. You are given an x value. Plug it in and work it out.
2. You are asked, in some way, to find the zeros. Solve it by whichever method works.
3. You are given a y value. Plug it into the y, then bring it over to the other side, then solve it using whichever method works.
4. Find the maximum or minimum. Complete the Square, or if you have the zeros, find the axis of symmetry, then plug that into the original equation.

None of this is new! You just need to apply it the skills you have acquired over the past few weeks!
Let's do 4 examples. Two with equations given, 2 without.

1. An automated hose on a tower sprays water on a forest fire. The height of the water, h, in metres, can be modelled by the relation $\boxed{h = -2.25x^2 + 4.5x + 6.75}$, where x is the horizontal distance of the water from the hose, in decametres (1dam=10m).

a) What is the maximum height of the water?

$h = -2.25x^2 + 4.5x + 6.75$

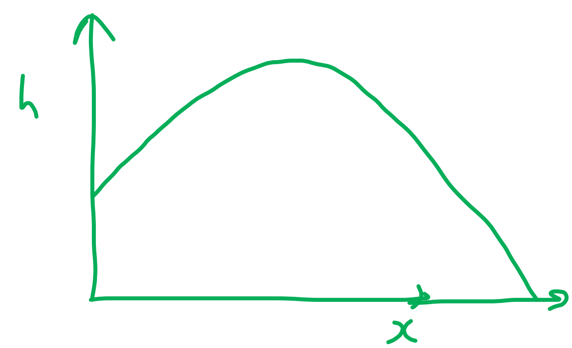
-2.25

$h = -2.25(x^2 - 2x + 1^2 - 1^2) + 6.75$

$h = -2.25((x-1)^2 - 1) + 6.75$

$h = -2.25(x-1)^2 + 2.25 + 6.75$

$h = -2.25(x-1)^2 + 9$



$\therefore$ Max height is 9m at 1dam

b) When will the water hit the ground?

$h = 0$

$0 = -2.25(x-1)^2 + 9$

$-9 = -2.25(x-1)^2$

$-2.25 \qquad -2.25$

$4 = (x-1)^2$

$\sqrt{4} = x - 1$

$\pm 2 = x - 1$

$2 = x - 1$

$x_1 = 2 + 1 = 3$

$-2 = x - 1$

$-2 + 1 = x_2$

$-1 = x_2$

$\boxed{x_1 = 3}$

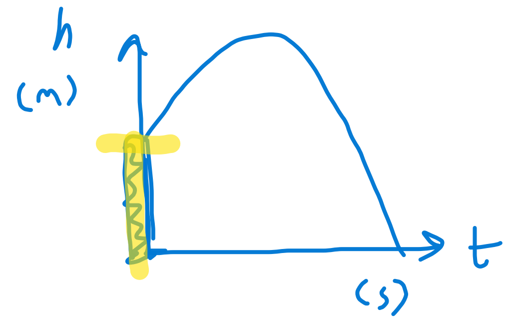
$\boxed{x_2 = -1}$

water will hit the ground 3dam away from the hose

2. A person throws a ball of a roof of a building. The relation $\boxed{h = -5t^2 + 20t + 12}$ models the height of the ball, in metres, and the time, in seconds.

a) What is the height of the building?

Height of the building = 12m.



b) How high will the ball be after one second?

$h = ?$ $\quad t = 1$

$\rightarrow h = -5t^2 + 20t + 12$

$(1) \quad (1)$

$h = -5 + 20 + 12 = 27m$

$\therefore$ The ball will be at a height of 27m at 1 sec.

c) When will the ball hit the ground?

$t = ?$ $\quad h = 0$

$$\boxed{\underset{t}{x} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$0 = -5t^2 + 20t + 12$

$$t = \frac{-20 \pm \sqrt{400 - 4(-5)(12)}}{2(-5)}$$

$$t = \frac{-20 \pm \sqrt{400 + 240}}{-10} = \frac{-20 \pm \sqrt{640}}{-10} = \frac{-20 \pm 25.3}{-10}$$

$$t_1 = \frac{-20 + 25.3}{-10} \qquad t_2 = \frac{-20 - 25.3}{-10}$$

$\boxed{t_1 = -0.53}$ ✗ $\qquad \boxed{t_2 = 4.53 \text{ sec.}}$ ✓

$\therefore$ Ball hits the ground at 4.53 sec.

3. The length of a rectangle is one more than two times the width. If the area of the rectangle is 136, what are the dimensions of the rectangle?

Let width $= w$

$\therefore$ length $= 2w+1$

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

w

$A = lw$

$136 = (2w+1)w$

$136 = 2w^2 + w$

$0 = 2w^2 + w - 136$

$a = 2,\ b = 1,\ c = -136$

$$w = \frac{-1 \pm \sqrt{1-4(2)(-136)}}{2(2)} = \frac{-1 \pm \sqrt{1+1088}}{4} = \frac{-1 \pm \sqrt{1089}}{4}$$

$$\boxed{w = \frac{-1 \pm 33}{4}}$$

$w_1 = \frac{-1-33}{4} = \frac{-34}{4}$ REJECTED

$w_2 = \frac{-1+33}{4} = \frac{32}{4} = 8$

$\therefore$ width $= 8$ units

$\therefore$ length $= 2(8)+1$
$= 17$ units

4. The product of two consecutive number is 156. Find the numbers (without guess and check!)

Let the two consecutive numbers be x and $x+1$

ATQ,

$x(x+1) = 156$

$x^2 + x = 156$

$x^2 + x - 156 = 0$

$$x = \frac{-1 \pm \sqrt{1-4(1)(-156)}}{2} = \frac{-1 \pm \sqrt{1+624}}{2}$$

$$x = \frac{-1 \pm \sqrt{625}}{2} = \frac{-1 \pm 25}{2}$$

$x_1 = \frac{-1+25}{2} = 12$

$x_2 = \frac{-1-25}{2} = -13$

$x_1 = 12 \Rightarrow x_1 + 1 = 13$

$x_2 = -13 \Rightarrow x_2 + 1 = -13 + 1 = -12$

$\therefore$ The two consecutive #s could be 12 and 13 or −12 and −13.