Name: Mrs. Jacob

Math 10
Unit 1

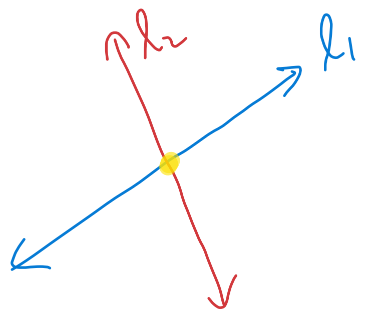
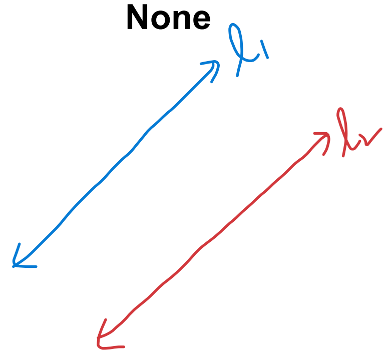
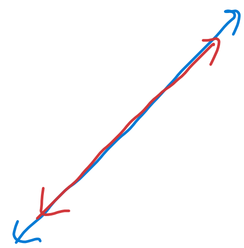
Systems of Linear Equations

Learning Goals. We are learning to:

- create the graphs which represent given linear equations;
- determine a solution to a linear system graphically;
- explain what the solution to a linear system means;
- determine a solution to a linear system algebraically by substitution;
- determine a solution to a linear system algebraically by elimination; and
- create and solve a linear system which models a given situation (word problem)

Solving Linear Systems

"Solving" a Linear System means finding the solution which is the point at which all the lines intersect. A solution will always satisfy each equation of the Linear System. A Linear System can intersect in 3 ways:

One	None	Infinite
		

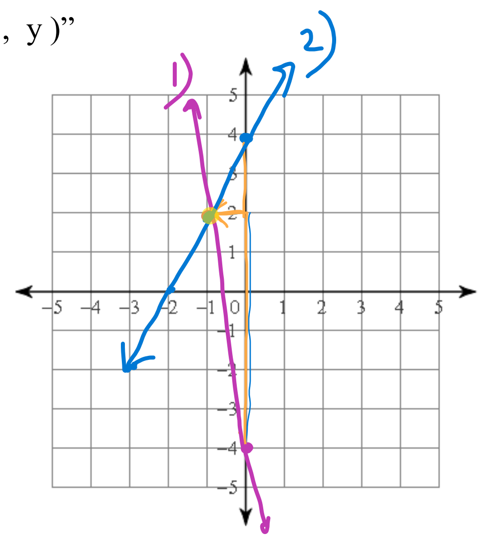
We will refer to the solution as the Point of Intersection POI = (x, y)

Method 1: Solve by Graphing:

Steps:

1. **Graph** the 2 linear equations
2. **State the POI** (Point of Intersection) by stating "POI = (x , y)"

Examples



1) $y=-6x-4$ $m=\frac{6}{-1}$; $b=-4$

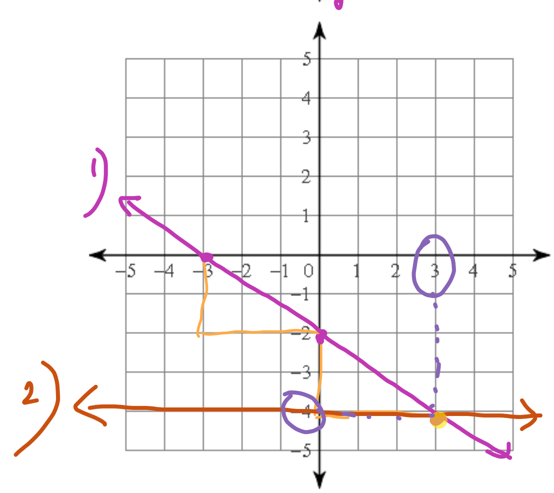
2) $y-2x=4$

$y=2x+4$

$m=\frac{2}{1}$; $b=4$

$m=\frac{-2}{-1}$

$\therefore$ POI = $(-1,2)$



1) $y=-\frac{2}{3}x-2$

2) $y=-4$

POI = $(3,-4)$

Method 2: Solve by Substitution:

Steps:

1. Isolate

2. Substitute

3. Solve

4. State the POI

Examples

$x = -6y - 11$
$x = (y + 3)$

$y + 3 = -6y - 11$
$y + 6y = -11 - 3$
$\frac{7y}{7} = \frac{-14}{7}$
$y = -2$

$\therefore x = -2 + 3$
$= 1$

PoI $(1, -2)$

$2x + y = -15 \rightarrow y = (-15 - 2x)$
$3x - y = -15$

$3x - (-15 - 2x) = -15$
$3x + 15 + 2x = -15$
$5x = -15 - 15$
$\frac{5x}{5} = \frac{-30}{5}$
$x = -6$

$\therefore y = -15 - 2(-6)$
$y = -15 + 12$
$y = -3$

$\therefore$ PoI $(-6, -3)$

$y - 3x = 5 \rightarrow y = (5 + 3x)$
$y + x = 3$

$5 + 3x + x = 3$
$+5 + 4x = 3$
$4x = 3 - 5$
$4x = -2$
$x = -\frac{2}{4} = -0.5$

$\therefore y = 5 + 3(-0.5)$
$y = 5 - 1.5$
$y = 3.5$

$\therefore$ PoI $(-0.5, 3.5)$

$4x + y = 11$
$x + 2y = 8 \rightarrow x = (8 - 2y)$

$4(8 - 2y) + y = 11$
$\Rightarrow 32 - 8y + y = 11$
$\Rightarrow +32 - 7y = 11$
$\Rightarrow -7y = 11 - 32$
$\Rightarrow \frac{-7y}{-7} = \frac{-21}{-7}$
$\Rightarrow \boxed{y = 3}$

$\therefore x = 8 - 2(3)$
$x = 8 - 6 = 2$

$\therefore$ PoI $(2, 3)$

Method 3: Solve by Elimination:

Steps:

1. Arrange
2. Same (But with Different Signs)
3. Eliminate
4. Solve
5. State the POI

Examples

$$\begin{aligned} 4x + 3y &= -15 \\ x - 3y &= 0 \end{aligned}$$

$+$

$\frac{5x}{5} = \frac{-15}{5}$

$\Rightarrow x = (-3)$

$x - 3y = 0$

$-3 - 3y = 0$

$\frac{-3y}{-3} = \frac{3}{-3}$

$y = (-1)$

$\therefore$ POI $(-3, -1)$

$$\begin{aligned} -1(-5x + 3y &= -1) \Rightarrow 5x - 3y = 1 \\ -7x + 3y &= -11 \Rightarrow -7x + 3y = -11 \end{aligned}$$

$+$

$\frac{-2x}{-2} = \frac{-10}{-2}$

$x = 5$

$-5x + 3y = -1$

$-5(5) + 3y = -1$

$-25 + 3y = -1$

$3y = -1 + 25$

$\frac{3y}{3} = \frac{24}{3}$

$y = 8$

$\therefore$ POI $(5, 8)$

$$\begin{aligned} 3(3x + 4y &= -6) \Rightarrow 9x + 12y = -18 \\ -2(5x + 6y &= -8) \Rightarrow -10x - 12y = 16 \end{aligned}$$

$+$

$\frac{-x}{-1} = \frac{-2}{-1}$

$\Rightarrow x = 2$

$5x + 6y = -8$

$5(2) + 6y = -8$

$+10 + 6y = -8$

$6y = -8 - 10$

$\frac{6y}{6} = \frac{-18}{6}$

$y = -3$

$\therefore$ POI $(2, -3)$

$$\begin{aligned} 4x &= -2y + 24 \Rightarrow 4x + 2y = 24 \Rightarrow 4x + 2y = 24 \\ -x - 3y &= -21 \Rightarrow 4(-x - 3y = -21) \Rightarrow -4x - 12y = -84 \end{aligned}$$

$\frac{-10y}{-10} = \frac{-60}{-10}$

$y = 6$

$-x - 3y = -21$

$-x - 3(6) = -21$

$-x - 18 = -21$

$-x = -21 + 18$

$\frac{-x}{-1} = \frac{-3}{-1}$

$x = 3$

$\therefore$ POI $(3, 6)$

Unit 1 - Linear Systems: Success Criteria

1. I can determine the best method to use to solve the system.

a) I can solve by Graphing: Usually used for applications comparing two scenarios.
To be successful:
1. use an appropriate scale
2. use a ruler label: axes, equations scale, pencil .
3. Be able to interpret the graph to state the solution.

b) I can solve By Substitution:
To be successful:
1. Rewrite one equation to Isolate "x" or "y".
2. Substitute the new equation into the other equation.
3. Solve for one variable,
4. Substitute found value in #3 and solve for the other.

c) I can solve By Elimination:
To be successful:
1) Set up the equations so one of the variables have opposite coefficients.
2) Add the equations together to eliminate one variable and solve for the other.
3) Substitute found value in #2 and solve for the other variable.

2. I can clearly state the solution POI = (,).
3. I can check the solution.

To create a linear system: (Application Word problems)

1. I can Interpret the question to Identify the unknown variables (x and y)
2. I can Interpret the question to create 2 equations to represent the information in the question in one of 3 formats:
   a. y=mx+b
   b. x + y = total (sum)
   c. _____ x + _____ y = _____ total where ______ is a #, $ or %
      i. $4x + $2y = $ 250
      ii. 13% x + 35% y =25% total
3. I can Solve the system using graphing, substitution or elimination (see above criteria)
4. I can state the solution, ensuring that it answers the question.
5. I can check the solution.

Math 10D Name______________________________

Lesson: Creating Linear Systems - Word Problems (Applications).

1) The school that Shawna goes to is selling tickets to the annual talent show. On the first day of ticket sales the school sold 27 adult tickets and 26 student tickets for a total of \$579. The school took in \$412 on the second day by selling 46 adult tickets and 6 student tickets. Find the price of an adult ticket and the price of a student ticket.

Let x = cost of an adult ticket (\$)
y = cost of a student ticket (\$)

$-6(27x + 26y = 579) \Rightarrow -162x - \cancel{156y} = -3474$

$26(46x + 6y = 412) \Rightarrow 1196x + \cancel{156y} = 10712$

$\frac{1034x}{1034} = \frac{7238}{1034}$

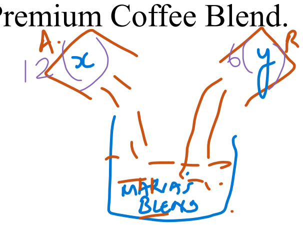
$\boxed{x = 7}$

PoI $(7, 15)$

$46x + 6y = 412$
$46(7) + 6y = 412$
$322 + 6y = 412$
$6y = 412 - 322$
$\frac{6y}{6} = \frac{90}{6}$
$y = 15$

Ans ∴ An adult ticket costs \$7 and student ticket costs \$15.

2) Maria's Premium Coffee Blend which costs \$7/lb is made by combining arabica coffee beans which cost \$12/lb with robusta coffee beans which cost \$6/lb. Find the number of lb of arabica coffee beans and robusta coffee beans required to make 6 lb of Maria's Premium Coffee Blend.



$7(6\text{lb}) = 7 \times 6 = 42$

Let x = lb of arabica beans in the blend
y = lb of robusta beans in the blend

$-6(x + y = 6) \Rightarrow -6x - \cancel{6y} = -36$

$12x + 6y = 42 \rightarrow 12x + \cancel{6y} = 42$

$6x = 6$

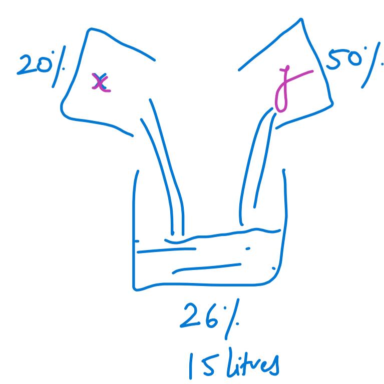
$x = 1$

$x + y = 6$
$y = 6 - x$
$y = 6 - 1$
$y = 5$

PoI $(1, 5)$

Ans ∴ Maria's coffee blend has 1 lb of arabica and 5 lb of Robusta beans.

3) Mark wants to make 15 ml of a 26% saline solution by mixing together a 20% saline solution and a 50% saline solution. How much of each solution must he use?



Let x = ml of 20% solution in the 15 l.
y = ml of 50% solution in the 15 l.

$-50(x + y = 15) \Longrightarrow -50x - \cancel{50y} = -750$

$\frac{20}{100}x + \frac{50}{100}y = \frac{26}{100}(15) \Longrightarrow 20x + \cancel{50y} = 390$

$\frac{-30x}{-30} = \frac{-360}{-30}$

$x = 12$

$x + y = 15$
$12 + y = 15$
$y = 15 - 12 = 3$

PoI $(12, 3)$

∴ Mark uses 12 ml of 20% and 3 ml of 50% saline solution for his 15 ml of 26% solution.

Math 10D – Unit 1: Solving Systems of Linear Equations **Name:________________________**

Date:___________________

Word Problems- Review

All questions need to follow this format:

1. Establish the unknowns (usually x and y). These are called the "Let statements".
2. Create the two equations. Label/name the equations.
3. Solve. You may use elimination or substitution (or graphing).
4. Write the answer in a sentence statement.

Type 1: y=mx+b or "total cost = variable price + set/constant cost"

A house needs some plumbing work done, so two plumbers are called. Pipe Cleaners charges $120 service fee and then $60/h. Water Works charges $80 for showing up plus $75/h.

Thinking Questions: If you need them for 2 hours, how much will each plumber cost? If you need them for 3 hours, how much will each plumber cost? See the pattern? Now use it to create your equations!!!

x = # of hrs. taken for work.

y = cost for work ($)

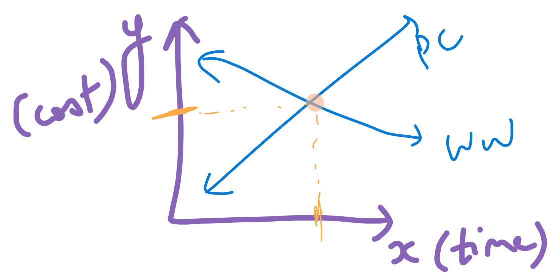
(PIPE CLEANERS): $y = 120 + 60x$

(WATER WORKS): $y = 80 + 75x$

$0 = 40 - 15x$

$15x = 40$

$x = \frac{40}{15} \approx 2.67$



Now the real question: After how many hours do the two plumbers charge the same amount? How much do they charge at that time?

POI

∴ After about 2.67 hrs, the two plumbers charge the same i.e. $280.25

$y = 80 + 75x$

$= 80 + 75(2.67)$

$= 80 + 200.25$

$= \$280.25$

∴ POI $(2.67, 280.25)$

Fixed $500/week + 6% commission

worth of cars

Another Type 1 Example:

Jasmyne is a car salesperson, and she earns $500/week plus 6% commission selling cars. She noticed a job posting at another company that offered $800/week plus 4% commission. How much does Jasmyne need to sell to earn the same at both jobs? Should she switch jobs?

$800/week + 4% commission. Fixed

Let x = Money made in sales (per week)

y = Total earning per week

A.T.Q.)

$$y = 500 + \frac{6}{100}x \Rightarrow y = 500 + 0.06x$$

$$y = 800 + \frac{4}{100}x \Rightarrow -y = -800 - 0.04x$$

$$0 = -300 + 0.02x$$

$$\Rightarrow 300 = 0.02x$$

$$\Rightarrow \frac{300}{0.02} = x$$

$$\Rightarrow x = \$15,000$$

$y = 500 + 0.06x$

$y = 500 + 0.06(15000)$

$y = 500 + 900$

$y = \$1400$

$\therefore$ Jasmyne needs to sell cars worth $15,000 in a week to earn $1400/week on both jobs.

* Check what y-value is for different values of x to know if she can make more money by the switch.

Type 2: Selling or buying two items for a total cost. Prices of items are unknown.

Marcus and his friends are having a Final Four party. Marcus buys 5 burgers and 9 drinks for $28.75 at the Beefy Burger joint. When more of his friends show up, he heads back to the Beefy Burger joint for 10 more burgers and 10 more drinks, spending $47.50. How much is the cost of one burger and one drink?

Let x = cost of one burger

y = cost of one drink.

A.T.Q.,

$$-2(5x + 9y = 28.75) \Rightarrow -10x - 18y = -57.5$$

$$10x + 10y = 47.50 \Rightarrow 10x + 10y = 47.5$$

$$-8y = -10$$

$$\Rightarrow y = \frac{-10}{-8} = \frac{5}{4} = 1.25$$

$\therefore$ One burger costs $3.50 and a drink costs $1.25

$$\therefore 5x + 9(1.25) = 28.75$$

$$\Rightarrow 5x + 11.25 = 28.75$$

$$\Rightarrow 5x = 28.75 - 11.25$$

$$\Rightarrow 5x = 17.5$$

$$\Rightarrow x = \frac{17.5}{5} = 3.5$$

$\therefore$ PoI $(3.5, 1.25)$

Another Type 2 Example:

The school that Chelsea goes to is selling tickets to the annual talent show. On the first day of ticket sales the school sold 11 student tickets and 4 adult tickets for a total of $166. The school took in $124 on the second day by selling 11 student tickets and 1 adult ticket. Find the price of an adult ticket and the price of a student ticket.

Let x = price of an adult ticket

y = price of a student ticket

A.T.Q.,

$$4x + 11y = 166 \Longrightarrow 4x + \cancel{11y} = 166$$

$$-1(x + 11y = 124) \Longrightarrow -x - \cancel{11y} = -124$$

$$3x = 42$$

$$x = \frac{42}{3} = 14$$

$\therefore$ The price of an adult ticket is $14 and the price of a student ticket is $10.

$$\therefore x + 11y = 124$$

$$14 + 11y = 124$$

$$11y = 124 - 14$$

$$11y = 110$$

$$y = 10$$

$\therefore$ PoI $(14, 10)$

<u>**Type 3: Sum and Difference (easy!)**</u>

Find the value of two numbers if their sum is 135 and their difference is 33.

Let $x = 1^{st}$ #

$y = 2^{nd}$ #

ATQ,

$$x + \cancel{y} = 135$$

$$+\ x - \cancel{y} = 33$$

$$2x = 168$$

$$\Rightarrow x = \frac{168}{2}$$

$$\Rightarrow \boxed{x = 84}$$

$$\therefore y = 135 - x = 135 - 84 = 51$$

$\therefore$ The two numbers are 84 and 51.

The difference of two numbers is 10.56. Their sum is 167.42. Find the numbers.

Let $x = 1^{st}$ #

$y = 2^{nd}$ #

A.T.Q.,

$$x + \cancel{y} = 167.42$$

$$x - \cancel{y} = 10.56$$

$$2x = 177.98$$

$$x = \frac{177.98}{2} = 88.99$$

$$\therefore x + y = 167.42$$

$$88.99 + y = 167.42$$

$$y = 167.42 - 88.99$$

$$y = 78.43$$

Type 4: Sum of two unknowns coupled with other information.

A money jar contains 87 coins, made up of only dimes and quarters. The total value is $10.90. How many of each coin is in the jar?

* value of 1 dime = $0.10 ∴ x dimes = $0.10x
value of 1 quarter = $0.25 ∴ y quarters = $0.25y

Let x = # dimes
y = # quarters

ATQ, $x + y = 87$
$0.10x + 0.25y = 10.90$

Solve!! PoI = (72, 15)
(approx) (approx)

∴ There are about 72 dimes and 15 quarters.

CHECK
$0.10x = 0.10(72) = 7.20$
$0.25y = 0.25(15) = 3.75$
+
$10.95 \approx 10.90.$

There are 23 animals in the field. Some are pigs and some are chickens. There are 76 legs in all. How many of each animal are in the field?

* 1 pig legs = 4 ∴ x pigs = (4x) legs
1 chicken legs = 2 ∴ y chicken = (2y) legs

Let x = # pigs
y = # chickens

ATQ, $x + y = 23$
$4x + 2y = 76$

Solved!! ∴ PoI = (15, 8)

∴ There are 15 pigs and 8 chickens on the field.

There are 17 vehicles lining up to get gas. Each car gets 8 gallons of gasoline. Each truck gets 19 gallons of gasoline. The station sells 169 gallons of gasoline. How many of each vehicle pumped gas?

∴ x cars = (8x) gallons of gasoline

∴ y trucks = (19y) gallons of gasoline.

Let x = # cars
y = # trucks

ATQ, $x + y = 17$
$8x + 19y = 169$

Solve!!!
∴ PoI = (14, 3)

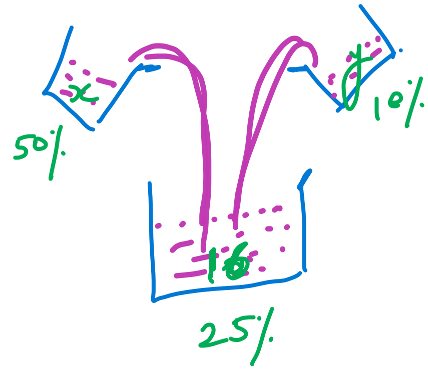
∴ There are 14 cars and 3 trucks lined up to get gas.

Type 5: Mixture

Kali is working on a chemistry experiment. She needs 16 litres of a 25% acid solution. Unfortunately, she doesn't have any. The good news is that she does have a 50% acid solution and a 10% acid solution. She can use some 50% acid solution and dilute it with the 10% solution. How many litres of each solution does she need?

Let x = litres of 50% acid solution used

y = litres of 10% acid solution used



ATQ, $x + y = 16$

$50x + 10y = 25(16) = 400$

Solve!!

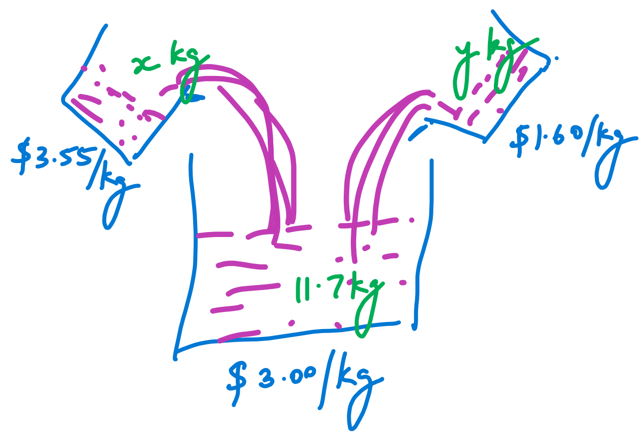
$\therefore$ POI = $(6, 10)$

$\therefore$ 6 litres of 50% solution and 10 litres of 10% acid solution were used to make 16 litres of 25% solution.

Jacob's Red-Hot Peanuts, which cost \$3.00/kg, are made by combining peanuts, which cost \$3.55/kg, with spices, which cost \$1.60/kg. Find the number of kg of peanuts and spices required to make 11.7kg of Jacob's Red-Hot Peanuts.

Let x = kg of peanuts used

y = kg of spices used



ATQ,

$x + y = 11.7$

$3.55x + 1.60y = 3(11.7) = 35.1$

Solve! POI = $(8.4, 3.3)$

$\therefore$ 8.4 kg of peanuts and 3.3 kg of spices were used to make Jacob's red hot peanuts.

Note: This is not an exhaustive list. There could be others types. Use these as a guide.