Name: ______________________________

Solving Quadratic Equations

Unit Outline:

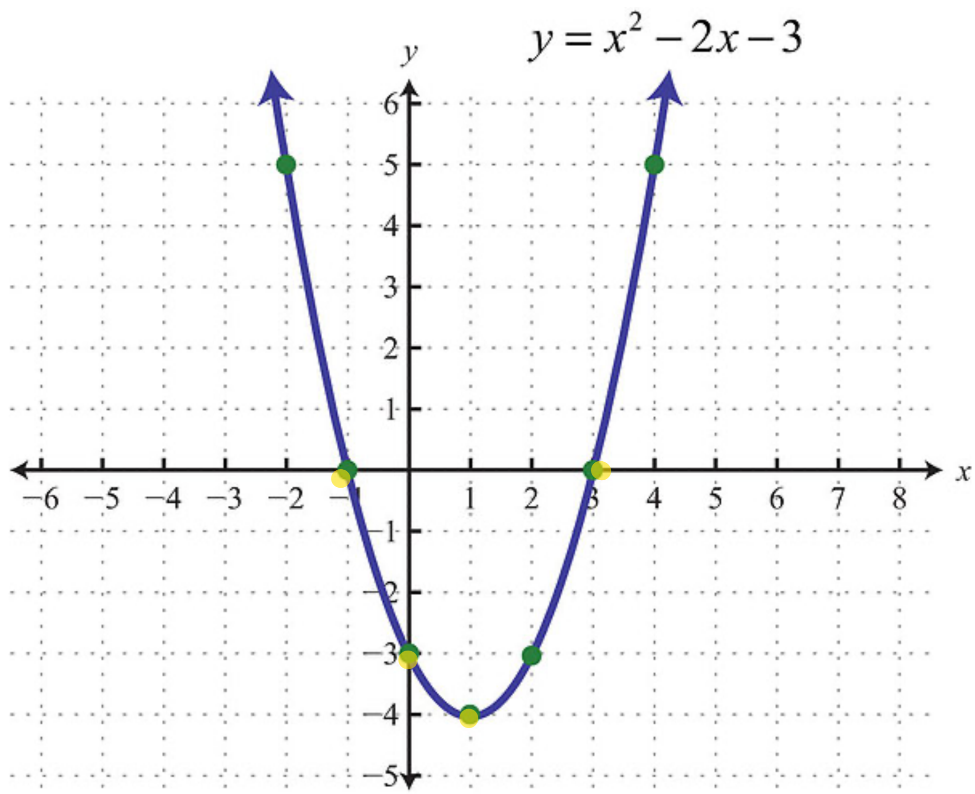
a. Solving by Factoring
b. Solving from Vertex Form
c. Solving using the Quadratic Formula
d. Real-Life Applications
e. Review

Standard Form	Zeros Form	Vertex Form
$y=ax^2+bx+c$	$y=a(x-r)(x-s)$	$y=a(x-h)^2+k$

$y=x^2-2x-3$ $\qquad$ $y=(x+1)(x-3)$ $\qquad$ $y=(x-1)^2-4$

x-int.

Vertex	Zeros	y-int	Vertical Stretch	Horizontal Shift	Vertical Shift	Axis of Symmetry AoS	Max/Min Value
(h,k)	r and s $(r,0)$ $(s,0)$	c	a	h	k	$x=h$	k
$(1,-4)$	$(-1,0)$ $(3,0)$	$(0,-3)$	1	1	-4	$x=1$	min. $=-4$



Solving Quadratic Equations

A quadratic relation, $y=ax^2+bx+c$, describes a relation between x and y.
A quadratic equation, $ax^2+bx+c=0$, needs to be solved for the value of x.

When we solve an equation like $4x+2=6$, we are finding the value of x.

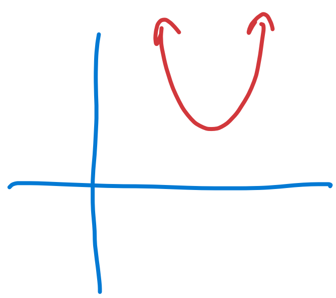
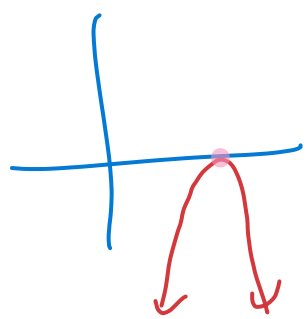
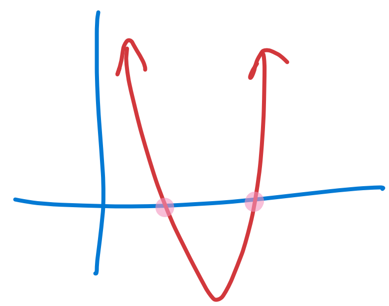
$4x = 6-2$
$\frac{4x}{4} = \frac{4}{4}$
$x = 1$

Try solving the equation: $\boxed{x^2 - 4x = 5}$

$x = -1$
$L.S = x^2 - 4x$
$= (-1)^2 - 4(-1)$
$= 1 + 4 = 5 = R.S.$

$x = 5$
$x^2 - 4x$
$= (5)^2 - 4(5)$
$= 25 - 20 = 5$

"Solving" a quadratic equation means finding the point at which the parabola intersects with the y-axis.

None	One	Two
		

In this unit, we will learn 3 ways to Solve a Quadratic Equation:

1. Solving by **Factoring**
2. Solving by **Vertex Form**
3. Solving by **Quadractic Formula**

Solving a Quadratic Equation can be compared to finding the **Zeros** in a quadratic relation. Why?

This is because solving for x in a quadratic equation is like finding x-values when $y=0$ i.e. x-intercepts or zeros of the quadratic relation.

Solving by Factoring

from section 6.1

A quadratic expression can be solved by ___FACTORING___. Another word for finding the "Solution" is finding the ___ZEROS.___.

Steps:

1. Put the equation into a form = 0 i.e. **$ax^2+bx+c=0$**
2. Factor the Quadratic Expression i.e. **$a(x-r)(x-s)=0$**
3. Set both parts of the product equal to ___0___ and then **`solve!!!`**

Examples:

$$0 = x^2 + x - 12$$

$4 \quad -3$

$0 = (x+4)(x-3)$

$x+4=0$

$x_1 = -4$

$x-3=0$

$x_2 = 3$

$$3x^2+9=-6x^2+10$$

$3x^2+9+6x^2-10=0$

$9x^2-1=0$

$(3x)^2-(1)^2=0$

$(3x+1)(3x-1)=0$

$3x+1=0$

$3x=-1$

$x_1 = \frac{-1}{3}$

$3x-1=0$

$3x=1$

$x_2=\frac{1}{3}$

$$0 = 2x^2 + 5x - 12$$

$M = -24$

$A = 5 = -3+8$

$0 = \underbrace{2x^2-3x}_{x} + \underbrace{8x-12}_{4}$

$0 = (x+4)(2x-3)$

$x+4=0$

$x_1=-4$

$2x-3=0$

$x_2 = \frac{3}{2} = 1.5$

Solving from Vertex Form

Let's solve from Vertex Form with some examples:

Remember SOLVE – SAMDEB

Calculate – BEDMAS

$$37 = 2(x - 3)^2 + 5$$

$$37 - 5 = 2(x-3)^2$$
$$32 = 2(x-3)^2$$
$$\frac{32}{2} = (x-3)^2$$
$$16 = (x-3)^2$$
$$\pm\sqrt{16} = x-3$$
$$\pm 4 = x-3$$

$$4 = x-3$$
$$4+3 = x$$
$$x_1 = 7$$

$$-4 = x-3$$
$$-4+3 = x$$
$$-1 = x_2$$

$$0 = \tfrac{1}{2}(x + 5)^2 - 2$$

$$2 = 0.5(x+5)^2$$
$$\frac{2}{0.5} = (x+5)^2$$
$$4 = (x+5)^2$$
$$\pm\sqrt{4} = x+5$$

$$2 = x+5$$
$$2-5 = x_1$$
$$x_1 = -3$$

$$-2 = x+5$$
$$-2-5 = x$$
$$x_2 = -7$$

More complicated examples:

$$0 = 3(x - 1)^2 - 5$$

$$5 = 3(x-1)^2$$
$$\frac{5}{3} = (x-1)^2$$
$$\sqrt{\frac{5}{3}} = (x-1)$$
$$\pm 1.3 \approx x-1$$

$$1.3 \approx x-1$$
$$x_1 \approx 1.3+1$$
$$x_1 \approx 2.3$$

$$-1.3 \approx x-1$$
$$x_2 \approx -1.3+1$$
$$x_2 \approx -0.3$$

$$0 = -2(x + 4)^2 + 5$$

$$-5 = -2(x+4)^2$$
$$\frac{-5}{-2} = (x+4)^2$$
$$\pm\sqrt{\frac{5}{2}} = x+4$$
$$\pm 1.6 \approx x+4$$

$$1.6 \approx x+4$$
$$x_1 \approx 1.6-4$$
$$x_1 \approx -2.4$$

$$-1.6 \approx x+4$$
$$x_2 \approx -1.6-4$$
$$x_2 \approx -5.6$$

Other methods of solving for x:

$2x^2 = 50$

METHOD 1:

$2x^2 = 50$

$x^2 = \frac{50}{2} = 25$

$x = \pm\sqrt{25}$

$x = \pm 5$

$x_1 = 5 \ ; \ x_2 = -5$

METHOD 2

$\frac{2x^2}{2} - \frac{50}{2} = 0$

$\Rightarrow 2(x^2 - 25) = 0$

$\Rightarrow 2(x+5)(x-5) = 0$

$x + 5 = 0$
$x = -5$

$x - 5 = 0$
$x = 5$

$x(x-2) = 36 - 2x$

$x^2 - 2x = 36 - 2x$

$x^2 - \cancel{2x} - 36 + \cancel{2x} = 0$

$x^2 - 36 = 0$

$(x+6)(x-6) = 0$

$x = \pm 6$

$(x+1)^2 - 16 = 0$

$(x+1)^2 = 16$

$x + 1 = \pm\sqrt{16}$

$x + 1 = \pm 4$

$x + 1 = 4$
$x_1 = 4 - 1$
$= 3$

$x + 1 = -4$
$x_2 = -4 - 1$
$= -5$

$0.25(x-4)^2 - 4 = 0$

$0.25(x-4)^2 = 4$

$(x-4)^2 = \frac{4}{0.25}$

$(x-4)^2 = 16$

$x - 4 = \pm\sqrt{16}$

$x - 4 = \pm 4$

$x - 4 = 4$
$x = 4 + 4$
$x_1 = 8$

$x - 4 = -4$
$x = -4 + 4$
$x_2 = 0$

Solving using the Quadratic Formula

from section 6.4

Today we will use the Quadratic Formula to solve *ANY* Quadratic Equation.

The Quadratic Formula gives the solution to any quadratic equation given in standard form. The Quadratic Formula is:

$$ax^2+bx+c=0 \qquad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Solve the following:

$3x^2-4x=2$

$3x^2-4x-2=0$

$a=3,\ b=-4,\ c=-2$

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(3)(-2)}}{2(3)}$$

$$x=\frac{4\pm\sqrt{16+24}}{6}=\frac{4\pm\sqrt{40}}{6}$$

$$x_1=\frac{4+\sqrt{40}}{6} \qquad x_2=\frac{4-\sqrt{40}}{6}$$

$$x_1\approx\frac{4+6.3}{6} \qquad x_2\approx\frac{4-6.3}{6}$$

$$x_1\approx 1.7 \qquad x_2\approx -0.4$$

$2x^2=3x^2-4(x+3)$

$0=3x^2-4x-12-2x^2$

$0=x^2-4x-12$

$a=1,\ b=-4,\ c=-12$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-12)}}{2(1)}$$

$$x=\frac{4\pm\sqrt{16+48}}{2}$$

$$x=\frac{4\pm 8}{2}$$

$$x_1=\frac{4+8}{2} \qquad x_2=\frac{4-8}{2}$$

$$x_1=6 \qquad x_2=-2$$

Note: b^2-4ac is referred to as the Discriminant. It can determine how many *real solutions* there are to the Quadratic Equation (two, one or no real solutions)

$b^2-4ac>0\rightarrow$ 2 solutions

$b^2-4ac=0\rightarrow$ 1 solution

$b^2-4ac<0\rightarrow$ No real solutions.

Solutions

zeros

Use the quadratic formula to solve for the roots of the following equations:

1.) $y = 2x^2 + 8x - 5$

$0 = 2x^2 + 8x - 5$

$a = 2, \quad b = 8, \quad c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{64 - 4(2)(-5)}}{2(2)}$$

$$x = \frac{-8 \pm \sqrt{64 + 40}}{4}$$

$$x = \frac{-8 \pm \sqrt{104}}{4} \approx \frac{-8 \pm 10.2}{4}$$

$$\frac{-8 + 10.2}{4} = 0.55 = x_1$$

$$\frac{-8 - 10.2}{4} = -4.55 = x_2$$

2.) $y = 43x^2 - 1204x + 8428$

$$\frac{0}{43} = \frac{43x^2}{43} - \frac{1204x}{43} + \frac{8428}{43}$$

$0 = x^2 - 28x + 196$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{28 \pm \sqrt{784 - 4(1)(196)}}{2(1)}$$

$$x = \frac{28 \pm \sqrt{784 - 784}}{2} = \frac{28 \pm \sqrt{0}}{2}$$

$$x = \frac{28}{2} = 14 \Rightarrow x = 14$$

3.) $y = 6x^2 + 4x + 9$

$0 = 6x^2 + 4x + 9$

$a = 6, \quad b = 4, \quad c = 9$

$$x = \frac{-4 \pm \sqrt{16 - 4(6)(9)}}{2(6)}$$

$$x = \frac{-4 \pm \sqrt{16 - 216}}{12}$$

$$x = \frac{-4 \pm \sqrt{-200}}{12}$$

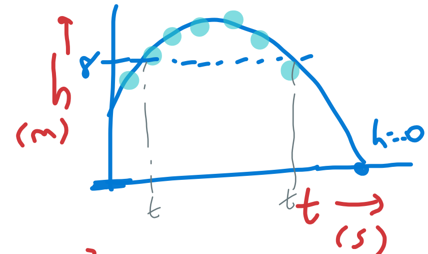
$\therefore$ No Real Solutions.

Real-Life Applications:

Let's start by applying our learning of solving quadratics with an example. Even if you are someone who doesn't enjoy Word Problems, I am sure you will find patterns that make these Word Problems super fun!!

A ball is thrown and follows the path of a parabola. It follows the path of $h = -5t^2 + 20t + 4$.

Where h is the height in metres and t is time in seconds.



a) Complete the Square to find the vertex.

$h = -5t^2 + 20t + 4$

-5

$h = -5(t^2 - 4t) + 4$

$\frac{4}{2} = 2$

$\pm 2^2$

$h = -5(t^2 - 4t + 2^2 - 2^2) + 4$

$h = -5((t-2)^2 - 4) + 4$

$h = -5(t-2)^2 + 20 + 4$

$h = -5(t-2)^2 + 24 \longrightarrow \therefore V = (2, 24)$

$y = a(x-h)^2 + k \quad V(h, k)$

b) When is the height of the ball above 8 m? Solve using " Quadratic Formula ".

$h = 8$

$h = -5t^2 + 20t + 4$

$8 = -5t^2 + 20t + 4$

$0 = -5t^2 + 20t + \cancel{4 - 8} - 4$

$a = -5, \; b = 20, \; c = -4$

$t = \dfrac{-20 \pm \sqrt{20^2 - 4(-5)(-4)}}{2(-5)} = \dfrac{-20 \pm \sqrt{400 - 80}}{-10}$

$t = \dfrac{-20 \pm 17.9}{-10}$

$t_1 = \dfrac{-20 + 17.9}{-10}$

$t_1 = 0.21$

$t_2 = \dfrac{-20 - 17.9}{-10}$

$t_2 = 3.79$

$\therefore$ The ball is above 8m between 0.21sec and 3.79sec.

c) When does the ball hit the ground? Solve using "Solving from the Vertex Form".

$h = 0$

$h = -5(t-2)^2 + 24$

$0 = -5(t-2)^2 + 24$

$\dfrac{-24}{-5} = \dfrac{-5}{-5}(t-2)^2$

$\dfrac{24}{5} = (t-2)^2$

$\pm\sqrt{\dfrac{24}{5}} = t - 2$

$\pm 2.2 \approx t - 2$

$2.2 \approx t - 2$

$\Rightarrow 2.2 + 2 \approx t_1$

$\Rightarrow 4.2 \approx t_1$

$-2.2 \approx t - 2$

$\Rightarrow -2.2 + 2 \approx t_2$

$\Rightarrow -0.2 \approx t_2$

TIME CANNOT BE NEGATIVE

$\therefore$ In about 4.2 sec. the ball hits the ground.

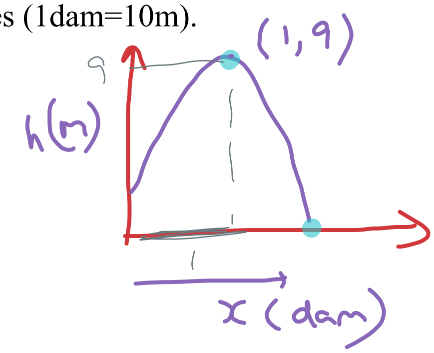
Students really dislike word problems, no matter what type they are.

Things to Remember: There are two types of word problems in Quadratics: equations given and equations not given. However, no matter the situation, once you have an equation, there are 4 things you can do:

1. You are given an x value. Plug it in and work it out.
2. You are asked, in some way, to find the zeros. Solve it by whichever method works.
3. You are given a y value. Plug it into the y, then bring it over to the other side, then solve it using whichever method works.
4. Find the maximum or minimum. Complete the Square, or if you have the zeros, find the axis of symmetry, then plug that into the original equation.

None of this is new! You just need to apply it the skills you have acquired over the past few weeks!
Let's do 4 examples. Two with equations given, 2 without.

1. An automated hose on a tower sprays water on a forest fire. The height of the water, h, in metres, can be modelled by the relation $h = -2.25x^2 + 4.5x + 6.75$, where x is the horizontal distance of the water from the hose, in decametres (1dam=10m).



k-value of vertex

a) What is the maximum height of the water?

$h = -2.25x^2 + 4.5x + 6.75$

$\frac{2}{2} = 1$ $\quad \pm 1^2$

$h = -2.25(x^2 - 2x + 1^2 - 1^2) + 6.75$

$h = -2.25((x-1)^2 - 1) + 6.75$

$h = -2.25(x-1)^2 + 2.25 + 6.75$

$h = -2.25(x-1)^2 + 9$

$\therefore$ max height of water = 9m.

b) When will the water hit the ground?

h=0

$0 = -2.25x^2 + 4.5x + 6.75$

$a = -2.25, \quad b = 4.5, \quad c = 6.75$

$$x = \frac{-(4.5) \pm \sqrt{(4.5)^2 - 4(-2.25)(6.75)}}{2(-2.25)}$$

$$= \frac{-4.5 \pm \sqrt{20.25 + 60.75}}{-4.5}$$

$$= \frac{-4.5 \pm 9}{-4.5}$$

$x_1 = \frac{-4.5 + 9}{-4.5} = -1$ ✗

$x_2 = \frac{-4.5 - 9}{-4.5} = 3$ ✓

$\therefore$ water hits the ground 30m away from the hose.

2. A person throws a ball from a roof of a building. The relation $h = -5t^2 + 20t + 12$ models the height of the ball, in metres, and the time, in seconds.

a) What is the height of the building?

b) How high will the ball be after one second?

c) When will the ball hit the ground?

3. The length of a rectangle is one more than two times the width. If the area of the rectangle is 136, what are the dimensions of the rectangle?

Let the width = w

$\therefore \ l = (2w+1)$

$A = 136$

$w(2w+1) = 136$

$2w^2 + w = 136$

$2w^2 + w - 136 = 0$

$$w = \frac{-1 \pm \sqrt{1 - 4(2)(-136)}}{2(2)}$$

$$w = \frac{-1 \pm \sqrt{1 + 1088}}{4}$$

$$w = \frac{-1 \pm 33}{4}$$

$\frac{-1+33}{4} = 8 = w_1$ ✓

$\frac{-1-33}{4} = -8.5 = w_2$ ✗

$\therefore \ l: 2(8)+1 = 17$

$\therefore$ The dimensions are:

$l = 17$ units

$w = 8$ units

4. The product of two consecutive number is 156. Find the numbers (without guess and check!)

Let the two consecutive #s be x and $(x+1)$

$\therefore \ x(x+1) = 156$

$x^2 + x = 156$

$x^2 + x - 156 = 0$

13 $\quad$ −12

$(x+13)(x-12) = 0$

$x_1 + 13 = 0$	$x_2 - 12 = 0$
$x_1 = -13$	$x_2 = 12$
	$x_2 + 1 = 12 + 1$
	$= 13$

$\therefore \ x_1 + 1 = -13 + 1$

$= -12$

$\therefore$ The consecutive #s for the given situation:

Possibility 1: 12 and 13

Possibility 2: −12 and −13