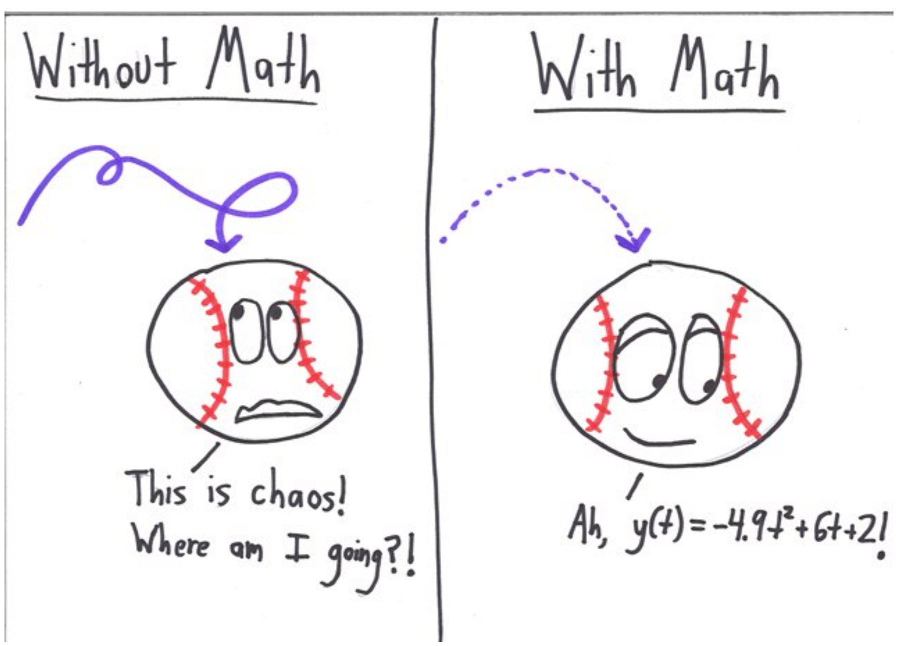
Functions & Applications 11

MCF3M

Course Notes

Unit 2: Introduction to Quadratics



Homework

Contents with suggested problems from the Nelson Textbook. You are welcome to ask for help, from myself or your peers, with any of the following problems.

Note: These problems are from Chapter 1 in the text.

Section 1.1 – Page 13
#1-5, 7, 11, 13

Section 1.3 – Page 33
#3, 4, 5, 6ab, 11a, 12

Section 1.7 – Page 64
#2-7 (your goal is finding the domain and range of a line)

Section 1.6 – Page 56
#1 (just state the transformations)
#3, 5abc, 7abc

2.1 Characteristics of a Function

Learning Goal: We are learning how to identify the difference between a function and a relation. Also, learning to recognize functions in various representations.

This course is called **FUNCTIONS**, so it seems rather important that you know what a function actually is. However, before we define and dive into the world of functions, it is important to be familiar with a few other commonly used terms in the language of functions.

In Grades 9 and 10, you learned about lines ($y = mx + b$) and parabolas ($y = ax^2 + bx + c$). Little did you know, these are called *functions*. Before we get into a formal definition of a function, let's first look at something more familiar, a *relation*.

A ***relation*** is an equation where there is a connection/relationship between two quantities represented by x and y

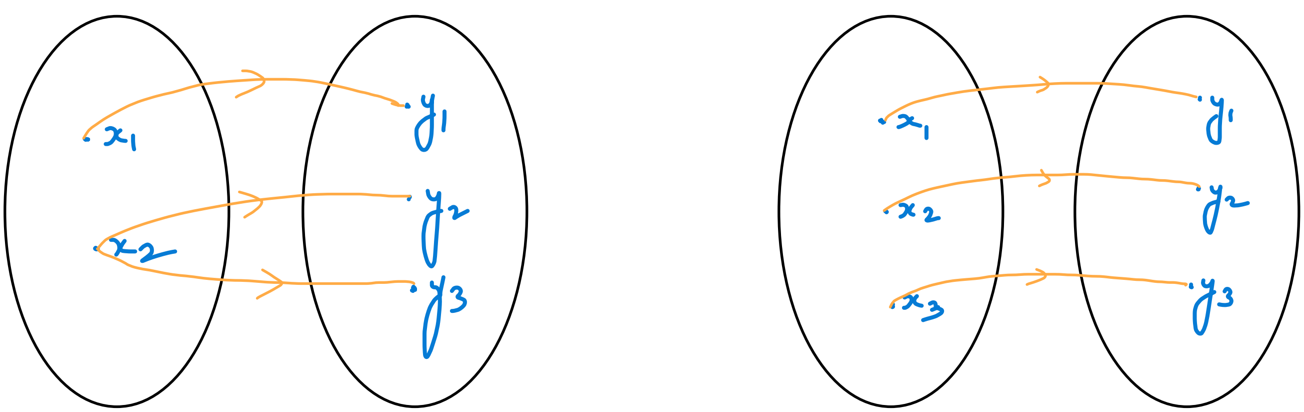
Ex: $y = 3x + 5$, $y = 5x^2$

Typically, x is the INDEPENDENT variable (INPUT)

And y is the DEPENDENT variable (OUTPUT)

A relation can be represented in a few ways:

1. **Mapping Diagram**



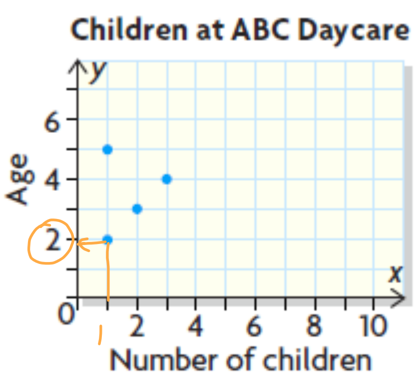
2. **Equation**

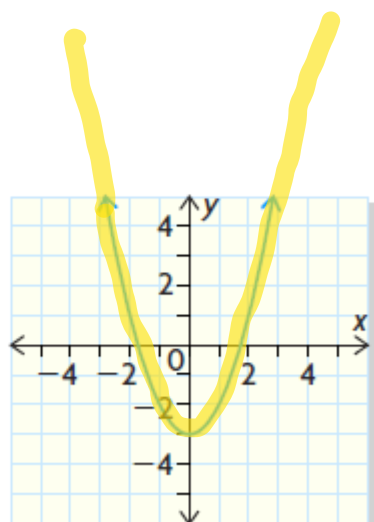
eg $y = 5x$; $y = 3x^2 + 2x + 5$

3. **Table**

km Driven x	Cost of Rental y
10	50
50	80
70	95
100	110

4. **Graph**





D = {all Real values of x}

R = {all Real values of y which are greater than -3}

5. **Set of Ordered Pairs**

{(-1,-3), (0,1), (1,1), (2,9)}

D = {-1, 0, 1, 2}

R = {-3, 1, 9}

{(1,4), (3,2), (0,5), (5,6), (3,0)}

D = {1, 3, 0, 5}

R = {4, 2, 5, 6, 0}

Before we define what a function is, we first need to define a few other things:

***Domain*:**

The set of all acceptable inputs (x-values)

x → $y = \frac{1}{x}$ → y

***Range*:**

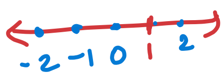
The set of all acceptable outputs (y-values)

Set Notations: *Some fancy ways to represent sets of Numbers!*

ROSTER FORM

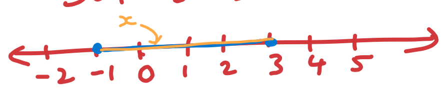
LISTABLE

$\{☼, ✱, ◁\}$



$\{-2, -1, 0, 2\}$

SET BUILDER FORM



$\{\underline{x \in \mathbb{R}} \mid -1 \le x \le 3\}$

x belongs to the set of REAL #s

(or) x is a real #

such that

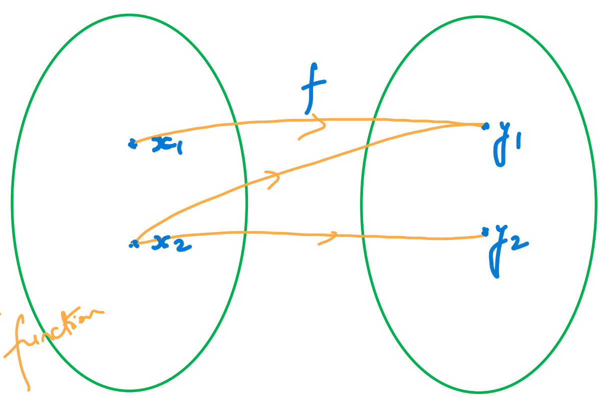
Now comes the moment finally!! We are ready to explore and understand **FUNCTIONS**! 😊

You need to know, very well, the following (algebraic) definition:

Definition 1.1.1

A FUNCTION is a very very special relation where each x-value (input value) has a unique (only one) y-value (output value) corresponding it.

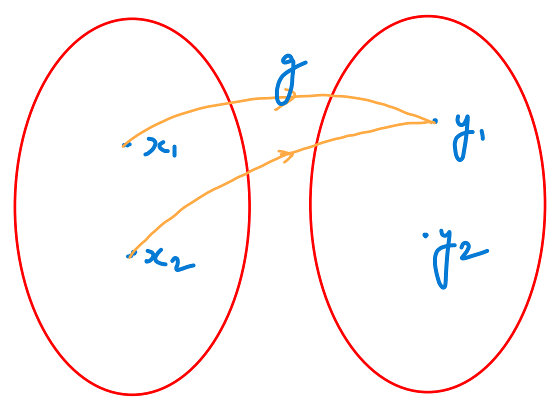
We can visualize what a function is (and **isn't**) by using so-called "**arrow diagrams**":



X Not function

D_f : $\{x_1, x_2\}$

R_f : $\{y_1, y_2\}$

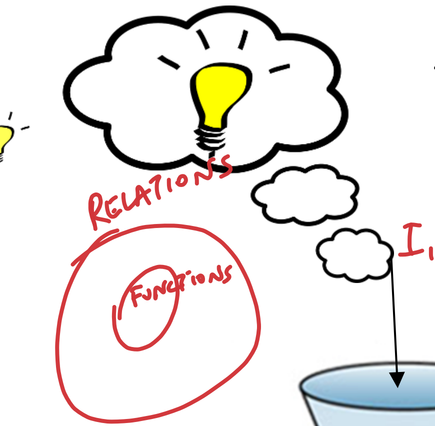


D_g : $\{x_1, x_2\}$

R_g : $\{y_1\}$

✓ function

We can also view ***Functions*** visually like a ***Vending Machine*** because they are ***PREDICTABLE*** just like our functions*!!*

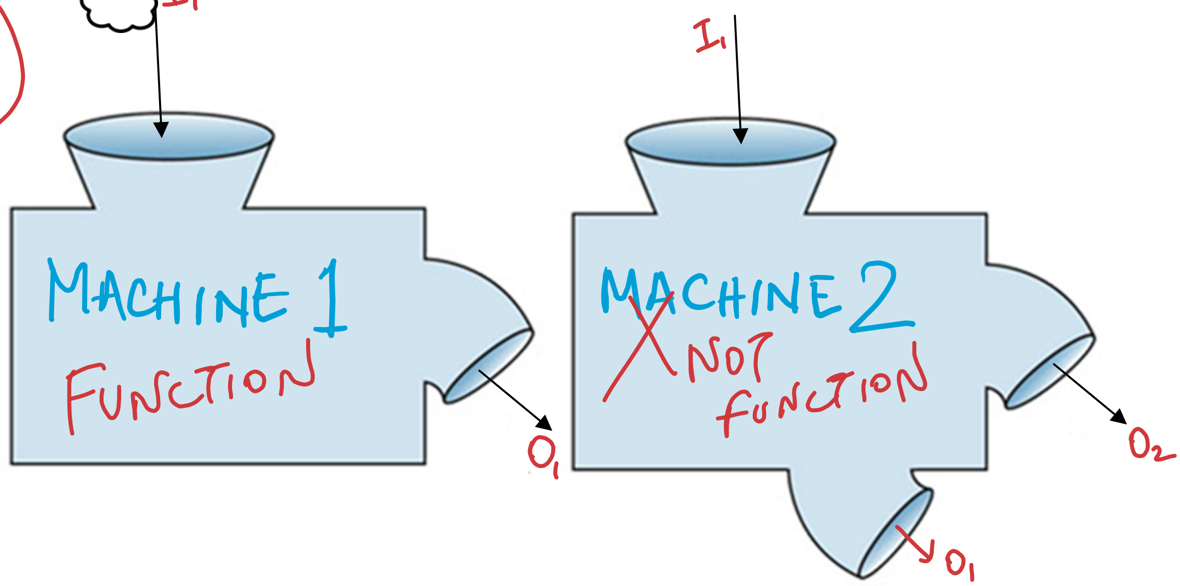


THINKING QUESTION:

i. *Are all Functions also Relations?* ....YES....

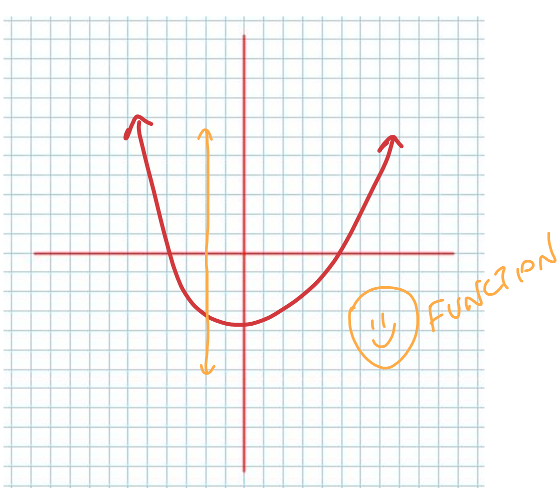
ii. *Are all Relations also Functions?* ....NO....

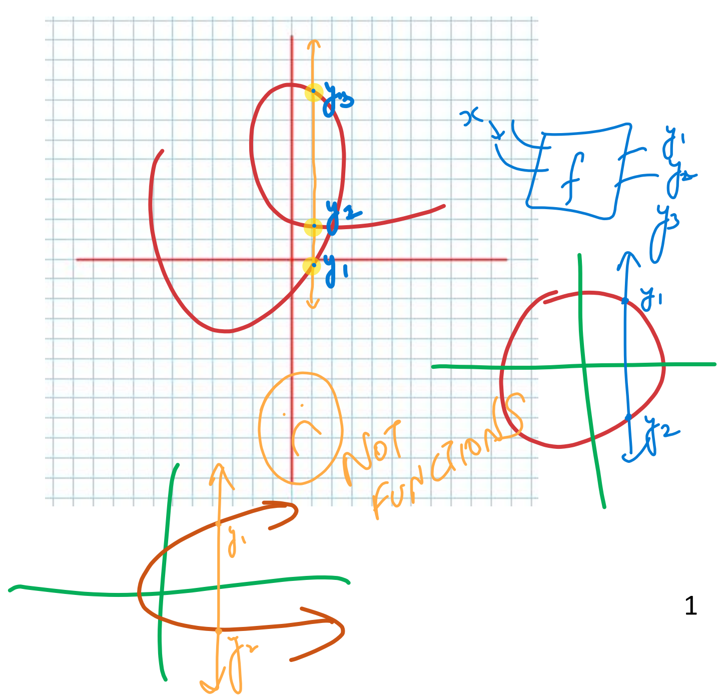
iii. *Which one of the two can be a function machine?*



KNOWING WHEN A RELATION IS, AND ISN'T, A FUNCTION

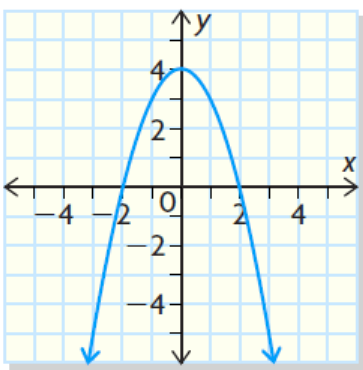
Graphically: *The Vertical Line Test* (VLT)

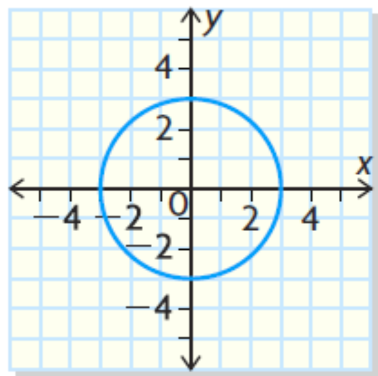




Let's go back and determine the domain and range and whether or not each relation is a function.

Domain and Range can be represented in just words ("x can be any number"), but Math is all about representing things in numbers and symbols. This is what makes math universal, because people in CoCoLoCo island may not understand "x can be any number", but they would understand the symbols used to represent that.





Domain: $\{x \in \mathbb{R}\}$

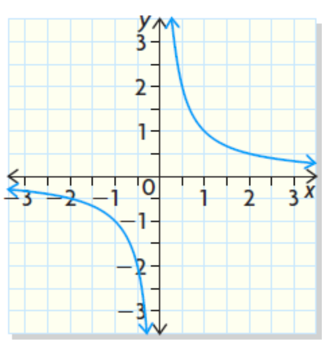
Domain: $\{x \in \mathbb{R} \mid -3 \le x \le 3\}$

Range: $\{\cancel{x}\ y \in \mathbb{R} \mid y \le 4\}$

Range: $\{y \in \mathbb{R} \mid -3 \le y \le 3\}$

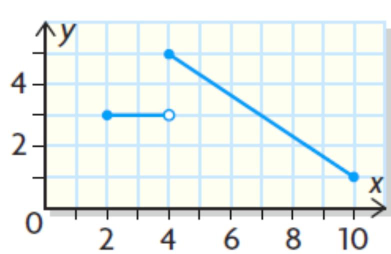
For the following, ~~determine the domain and range using set notation~~, and then state if it is a function.

a)



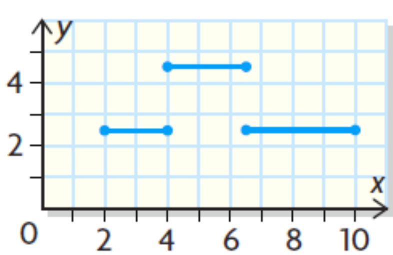
FUNCTION (passes VLT)

b)



FUNCTION (passes VLT)

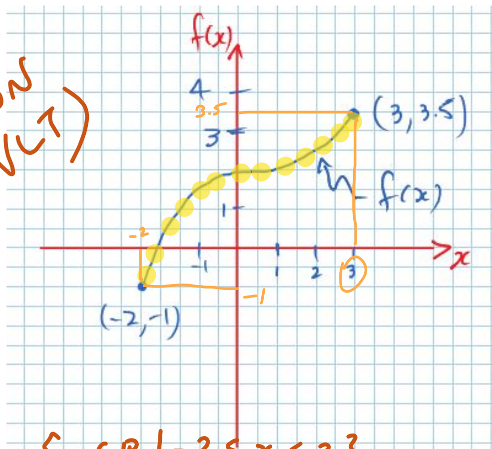
c)



X NOT FUNCTION (fails VLT)

Given the **SKETCH OF THE GRAPH** of the **RELATION** determine: the domain, the range of the relation, and whether the relation is, or is not, a function. *(Note that domain and range are sets of numbers and can be represented by the fancy **set notation**)*

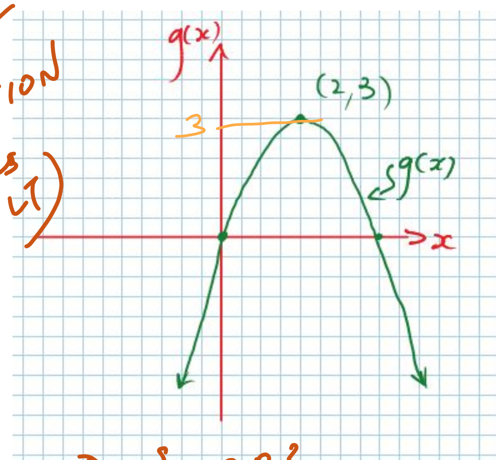
a) ✓ FUNCTION (passes VLT)



$D = \{x \in \mathbb{R} \mid -2 \le x \le 3\}$

$R = \{y \in \mathbb{R} \mid -1 \le y \le 3.5\}$

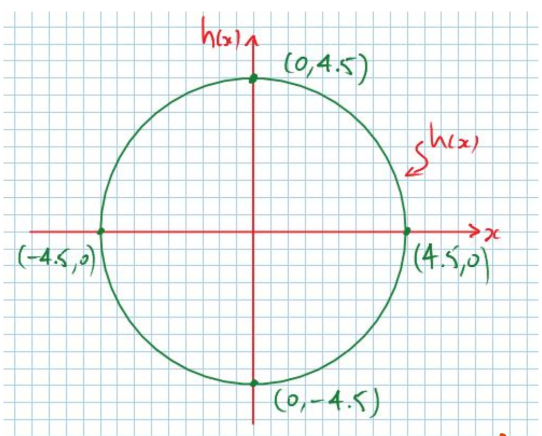
b) ✓ FUNCTION (passes VLT)



$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R} \mid y \le 3\}$

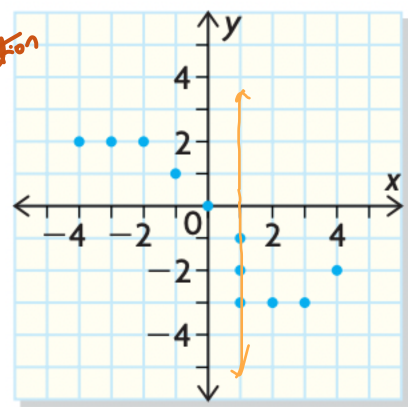
c). ✗ Not function (fails VLT)



$D: \{x \in \mathbb{R} \mid -4.5 \le x \le 4.5\}$

$R: \{y \in \mathbb{R} \mid -4.5 \le y \le 4.5\}$

d) ✗ Not function



$D = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$R = \{-3, -2, -1, 0, 1, 2\}$

Success Criteria:

- I can tell the difference between a function and a relation
  - I can use the vertical line test as a tool to tell if a relation is a function
- I can find the domain and range of a function or relation
- I can use set notation to state domain and range
- I can represent a function or relation using a table of values, set of ordered pairs, graph, mapping diagram, or equation

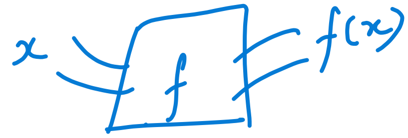
2.3 Working with Function Notation

Learning Goal: We are learning how to use Function Notation

Here we learn a NEW AND IMPROVED WAY for describing a function, algebraically.

When we determine that a relation is a function, such as $y = 3x + 4$, it is worthwhile to state that it is a function by giving it a name and indicating what the independent variable is.

$$y = 3x + 4 \rightarrow f(x) = 3x + 4$$



This much more useful way of writing y = f(x) is called the FUNCTION NOTATION.

Here, x is the independent variable,
which is used to determine the functional value (formerly known as y).

Let's look at how this works: Given $f(x) = 3x + 4$, evaluate $f(2)$.

$f(2) = 3(2) + 4$
$f(2) = 6 + 4$
$\boxed{f(2) = 10}$

Let's do some examples

1. Given $f(x) = 2x^2 + 3x - 1$, evaluate

a) $f(3)$ b) $f\left(\frac{1}{2}\right) = f(0.5)$ c) $f(5-3) = f(2)$ d) $f(5) - f(4)$

a) $f(3) = 2(3)^2 + 3(3) - 1$
$f(3) = 18 + 9 - 1$
$y = \boxed{f(3) = 26}$

b) $f(0.5) = 2(0.5)^2 + 3(0.5) - 1$
$f(0.5) = 0.5 + 1.5 - 1$
$\boxed{f(0.5) = 1}$

c) $\Rightarrow f(2) = 2(2)^2 + 3(2) - 1$
$f(2) = 8 + 6 - 1$
$\boxed{f(2) = 13}$

d) $f(5) = 2(5)^2 + 3(5) - 1$
$= 50 + 15 - 1$
$\boxed{f(5) = 64}$

$f(4) = 2(4)^2 + 3(4) - 1$
$= 32 + 12 - 1$
$\boxed{f(4) = 43}$

$\therefore f(5) - f(4) = 64 - 43$
$= 21$

2. Given $\boxed{g(x) = 5x - 8}$, determine the x so that $g(x) = 18$.

$\Rightarrow 18 = 5x - 8$
$\Rightarrow 18 + 8 = 5x$
$\Rightarrow \frac{26}{5} = \frac{5x}{5}$
$\Rightarrow \boxed{5.2 = x}$

$g(5.2) = 18$

$(x, g(x))$
y

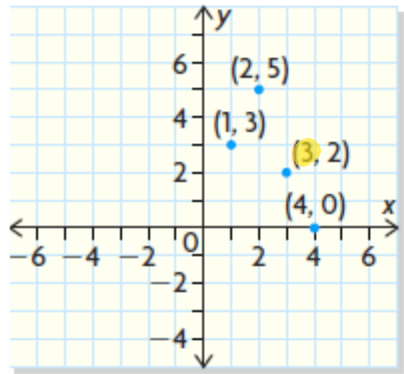
3. Evaluate $f(3)$ for each of the following.

a) $\{(1, 2), (2, 0), (3, 1), (4, 2)\}$

b)

x	1	2	3	4
y	2	3	4	5

c)



a) $f(3) = 1$

b) $f(3) = 4$

c) $f(3) = 2$

4. Lastly, something a little strange: given $h(x) = 2x^2 - 3x + 4$, evaluate $h(a)$ and $h(x-2)$.

(i) $h(a) = 2a^2 - 3a + 4$

(ii) $h(x-2) = 2(x-2)^2 - 3(x-2) + 4$ $\Longrightarrow h(x) = 2x^2 - 11x + 18$

$= 2(x^2 - 4x - 4) - 3(x-2) + 4$

$= 2x^2 - 8x - 8 - 3x + 6 + 4$

One more Example- $= 2x^2 - 11x + 18$

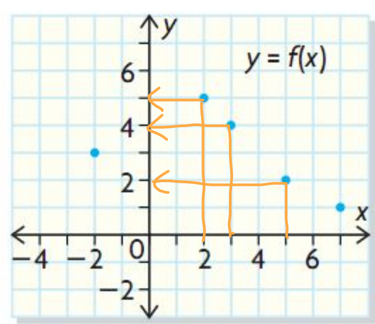
6. The graph of $y = f(x)$ is shown at the right.

a) State the domain and range of f.

b) Evaluate.

i) $f(3)$ iii) $f(5-3) = f(2)$

ii) $f(5)$ iv) $f(5) - f(3)$



(i) $f(3) = 4$

(ii) $f(5) = 2$

(iii) $f(2) = 5$

(iv) $2 - 4 = -2$

Success Criteria:

- I can recognize that "x" represents the domain value in the function notation "$f(x)$"
- I can represent that $f(x)$ is the range value (y-value) that corresponds to the x input
  - $y = f(x)$

2.7 Domain and Range of Quadratic Functions

Learning Goal: We are reviewing the graphs and equations of quadratic functions; and using their tables, graphs, or equations to determine the domain and range of quadratic functions.

THE PARENT QUADRATIC FUNCTION *(for Grade 11UC)*

Together we will explore (graphically) basic properties of the *parent* function:

a) Quadratic $y = x^2$

$g(x) = x^2$

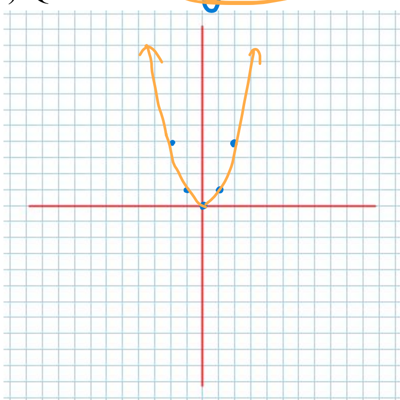


TABLE OF VALUES

x	$g(x)$	$(x, g(x))$
-2	4	$(-2, 4)$
-1	1	$(-1, 1)$
0	0	$(0, 0)$
1	1	$(1, 1)$
2	4	$(2, 4)$

The graph is a PARABOLA

So, we see that the **GRAPH OF A FUNCTION**, $f(x)$, is given by:

$f(x) = 5(x-2)^2 + 3$

$h = 2$ H. shift 2 Right

$$f(x) = \{(x, f(x)) \mid x \in D_f\}$$

In Grade 10, we learned about transformation of quadratic functions. To **TRANSFORM** something is to change the form.

Transformations are values which change the shape, direction, and position of the function. In a quadratic function,

Vertex $= (h, k)$

$$\boxed{f(x) = x^2} \rightarrow f(x) = a(x-h)^2 + k$$

where $a \rightarrow$ Direction of Opening ($a > 0 \rightarrow$ UP ; $a < 0 \rightarrow$ DOWN)
$h \rightarrow$ Horizontal Shift (Left $\leftrightarrow$ Right)
$k \rightarrow$ Vertical Shift (Up $\leftrightarrow$ Down)

Recall that **$y = a(x-h)^2 + k$** is the **Vertex Form** and is considered the ***strongest form*** for its ability to tell us about all the ***transformations***.

So, for quadratic functions(and functions in general) we have two things (NUMBERS!) to "transform". We can apply transformations to

1) **Domain** values (which we call **HORIZONTAL TRANSFORMATIONS**)
2) **Range** values (which we call **VERTICAL TRANSFORMATIONS**)

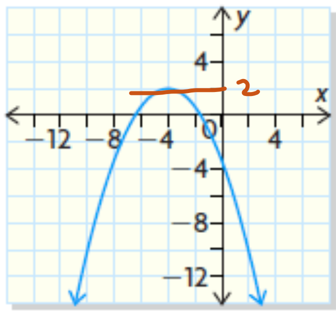
$(x, y) \rightarrow (x + h, ay + k)$

Also, there are **THREE BASIC FUNCTIONAL TRANSFORMATIONS**

1) Flips (*Reflections "across" an axis*)
2) Stretches (*Dilations*)
3) Shifts (*Translations*)

Example: Given *f(x)* and its graph, state the vertex, domain and range

Always for a Quadratic
$D=\{x\in R\}$
$R=\{f(x)\in R \mid f(x)\_k\}$



$a(x-h)^2+k$

$f(x)=-\frac{1}{3}(x+4)^2+2$

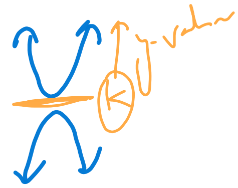
Vertex: $V(h,k)=(-4,2)$

$D=\{x\in R\}$

$R=\{f(x)\in R \mid f(x)\leq 2\}$

In general, given $f(x)=a(x-h)^2+k$, the domain is ***ALWAYS*** $\{x\in R\}$

The range, however, depends on the vertical stretch, or "*a*":



If $a>0$, $R=\{f(x)\in R \mid f(x)\geq k\}$

If $a<0$, $R=\{f(x)\in R \mid f(x)\leq k\}$

Determine the domain and range of each quadratic function:

$f(x)=3(x-4)^2-8$ −8

$D=\{x\in R\}$

$R=\{f(x)\in R \mid f(x)\geq -8\}$

$g(x)=-23(x+365)^2+4303$ 4303

$D=\{x\in R\}$

$R=\{g(x)\in R \mid g(x)\leq 4303\}$

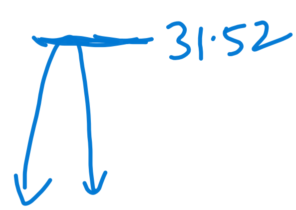
Note that sometimes the domain needs to be *restricted*. This means that instead of $\{x\in R\}$, there will be some limitations to both the domain and the range.

Example: A baseball thrown from the top of a building falls to the ground below. The path of the ball is modelled by the function $h(t)=-5t^2+5t+30$, where $h(t)$ is the height of the ball above ground, in metres, and t is the elapsed time in seconds. What are the domain and range of this function?

(For this unit, let's use Desmos/GeoGebra to find the vertex form of the quadratic function)

$D=\{t\in R \mid t\geq 0\}$

$R=\{h(t)\in R \mid h(t)\leq 31.25\}$



Success Criteria:

- I can state the domain and range of a quadratic function using set notation
- I can apply restrictions to the domain and/or range to model real-life scenarios

2.6 Graphing Quadratics with Transformations

Learning Goal: We are learning to use transformations to sketch the graphs of quadratic functions

Graphing a Quadratic function (and other functions) requires an understanding of *transformations*. Transformations are values which change the shape, direction, and position of the function. In a quadratic function,

$$\boxed{f(x) = x^2} \rightarrow f(x) = a(x-h)^2 + k$$

In general: $f(x) = a(x-h)^2 + k$

VERTICAL TRANSFORMATIONS:

$a \rightarrow$ V. STRETCH
V. FLIP

$a > 0$ (POSITIVE) $\Rightarrow$ PARABOLA OPENS UP

$a < 0$ (NEGATIVE) $\Rightarrow$ PARABOLA OPENS DOWN

$k \rightarrow$ V. SHIFT (UP or DOWN)

HORIZONTAL TRANSFORMATION

$h \rightarrow$ H. SHIFT (LEFT or RIGHT)

eg $y = 2(x+3)^2 + 5$
$h = -3$

$y = -0.5(x-2)^2 + 1$
$h = 2$

$(x, y) \rightarrow (x+h, ay+k)$ IMAGE POINT

$y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

$x+h$	$ay+k$

The process to graphing is straight-forward.
1. Identify the transformations
2. Create starting points from the base "parent" function
3. Transform the starting points
4. Graph the transformed points

Example: $f(x) = -2(x+4)^2 + 6$

$a = -2 \rightarrow$ V. stretch & flip

$h = -4 \rightarrow$ H. shift to left

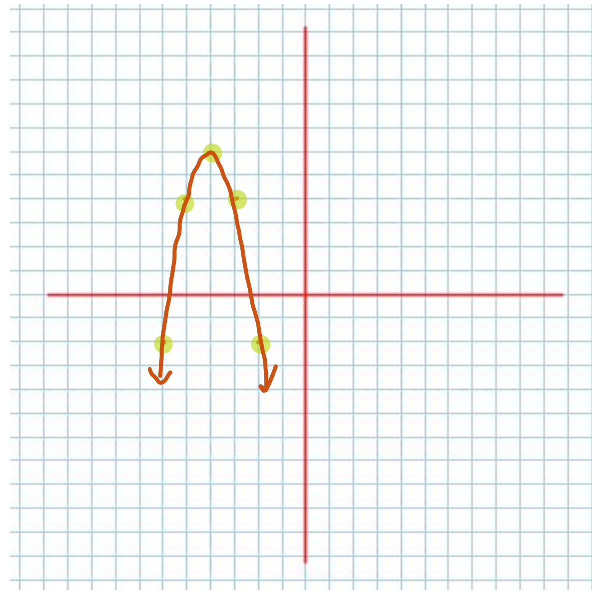
$k = 6 \rightarrow$ V. shift up

x^2

x	y
-2	4
-1	1
0	0
1	1
2	4

$x+h$ $ay+k$

$x-4$	$-2y+6$
-6	-2
-5	4
-4	6
-3	4
-2	-2



Example: $g(x) = \frac{1}{3}(x-2)^2 - 5$

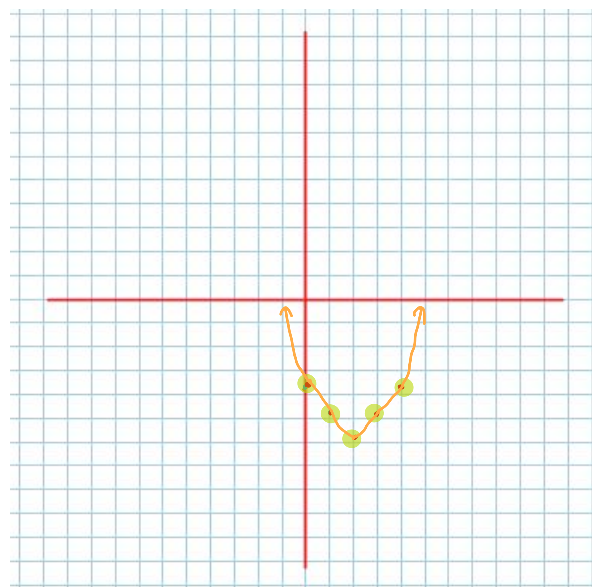
$a = \frac{1}{3} \rightarrow$ V. stretch

$h = 2 \rightarrow$ H. shift right

$k = -5 \rightarrow$ V. shift down

x	y
-2	4
-1	1
0	0
1	1
2	4

$x+2$	$\frac{1}{3}y - 5$
0	-3.7
1	-4.7
2	-5.7
3	-4.7
4	-3.7



Success Criteria:

- I can identify the transformations *a, h, and k*, in the equation of a quadratic function
- I can develop a table of values for the base/parent function $f(x) = x^2$
- I can apply the transformations to the x and y values of the base/parent function in order to draw the transformed quadratic function