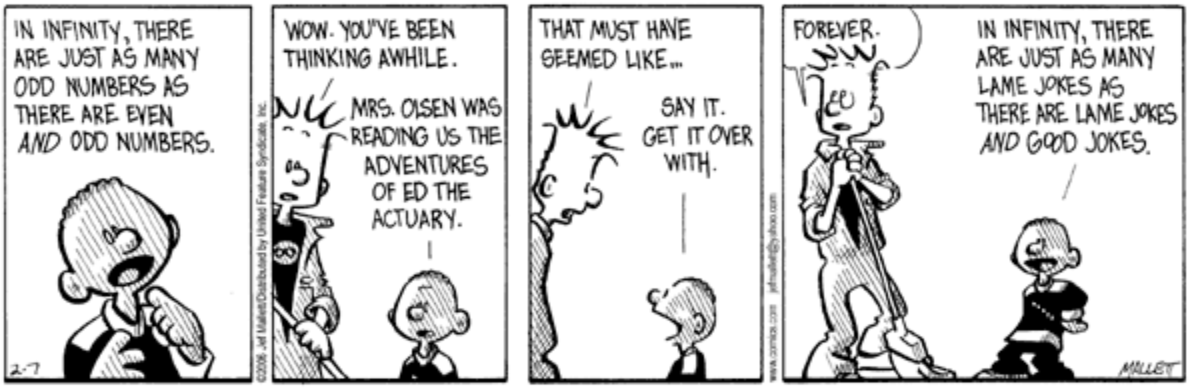
Functions & Applications 11

MCF3M

Course Notes

Unit 3and 4: Quadratic Functions



Homework

Contents with suggested problems from the Nelson Textbook. You are welcome to ask for help, from myself or your peers, with any of the following problems. They will be handed in on the day of the Unit Test as a homework check. Please post the work you do to OneNote, as you complete

HW 1- Section 3.2 – Page ~~142~~ 139-142
#3, 5, 10bcde, 11, 12, 13bcd, 14, 15

HW 2 – Day 1 : Section 4.1 – Page 203
#1, 3, 4, 6, 8, 9, 10

HW 2 – Day 2 : Section 4.2 – Page 214
#6, 7 (do not graph), 8-11

HW 3 - Section 3.4 – Page 162
#4ace, 5ace, 6ace, 7ace, 8-14

HW 4 - Section 4.3 – Page 222
#3, 6, 8, 9

*Section 3.6 – Page 177
#5-11, 14

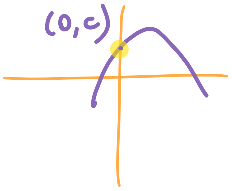
3.1 Properties of Parabolas and their Algebraic Forms

Learning Goal: We are learning about the three different algebraic forms of parabolas and their properties. We will learn to convert between the standard and factored forms of a quadratic function.

This lesson is a review of some of what we learned about quadratics in Grade 10. 😊
In Grade 10, we studied the three different forms of Quadratic Functions, and the information each gives.

1. Standard Form $f(x) = ax^2 + bx + c$

↑ y-intercept



Information:

$a > 0 \rightarrow$ Opens up

$a < 0 \rightarrow$ Opens Down

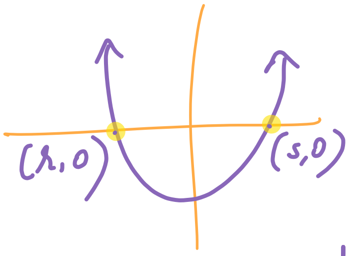
2. Zeros/Factored Form $f(x) = a(x - r)(x - s)$

Information:

$a \rightarrow$ Direction of Opening

$a > 0$

$a < 0$



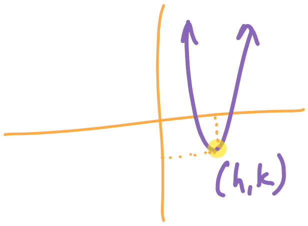
zeros/x-intercepts/roots = r and s

3. Vertex Form $f(x) = a(x - h)^2 + k$

Information:

$a > 0$

$a < 0$



Vertex (h, k)

The three different forms are equivalent, meaning that they generate the exact same information or graph. Let's see if we remember how to convert between forms using simple algebra. We'll do converting Standard to Vertex Form in a later lesson since it involves a process called "Completing the Squares". Today, we'll test our learning and the skills acquired from the first unit and try the following two conversions:

Question 1: Convert $f(x) = \frac{3x^2}{3} + \frac{18x}{3} - \frac{48}{3}$ to the Factored Form.

$f(x) = 3(x^2 + 6x - 16)$

$8 \quad -2$

$= 3(x+8)(x-2)$

Question 2: Covert $g(x) = -2(x-5)(x-3)$ to the Standard Form.

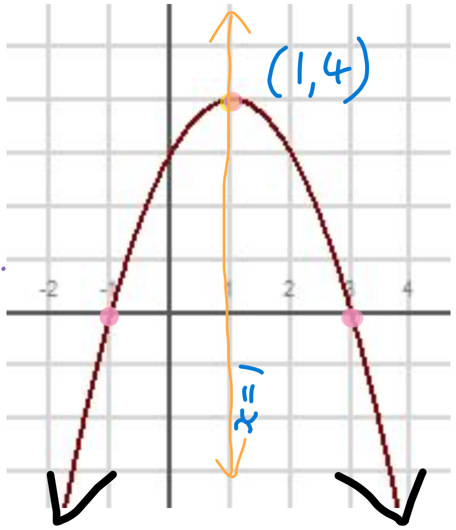
$= -2(x^2 - 8x + 15)$

$= -2x^2 + 16x - 30$

Do you recall the concept of the **Axis of Symmetry?**

1. The Axis of Symmetry (AoS) is

a line that passes through the vertex and divides the parabola into two identical halves.



2. We can, therefore, write the coordinates of the vertex as (1,4)

Determine the equation of the AoS and the Vertex for the given graph above.

AoS: $x = h$

VERTEX (AoS, f(AoS))

h , k

"f(h)

$\boxed{h = \frac{r+s}{2}}$

AoS: $x = 1$

VERTEX (1, 4)

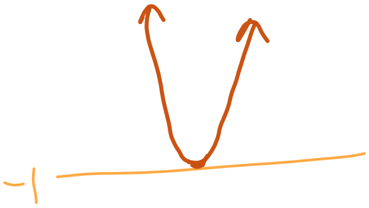
Example 3.1.1

$f(x) = a(x-h)^2 + k$

Given the quadratic function $f(x) = \frac{1}{2}(x+3)^2 - 1$, state:

a) The direction the parabola opens
b) The coordinates of the vertex
c) The equation of the axis of symmetry
d) Domain and Range

a) $a = \frac{1}{2} > 0$ ∴ opens up



b) $V(h,k) = (-3,-1)$

c) AoS: $x = h \Rightarrow x = -3$

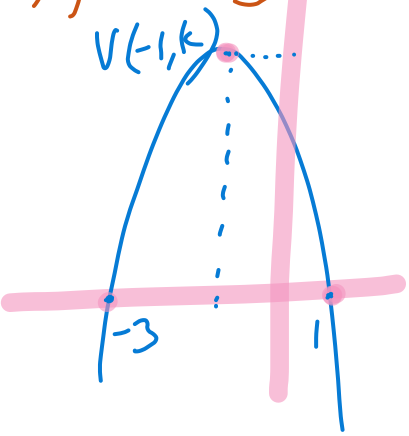
d) $D = \{x \in R\}$; $R = \{f(x) \in R \mid f(x) \geq -1\}$

Example 3.1.2

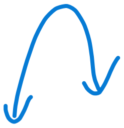
$g(x) = a(x-r)(x-s)$

Given the quadratic function $g(x) = -2(x+3)(x-1)$, state

a) The direction the parabola opens
b) The zeros of the quadratic
c) The equation of the axis of symmetry
d) The coordinates of the vertex



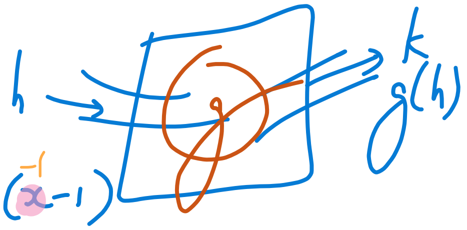
a) $a = -2$



b) $r = -3, \quad s = 1$

c) $x = h \qquad h = \frac{r+s}{2} = \frac{-3+1}{2} = \frac{-2}{2} = -1$

∴ AoS: $x = -1$



d) $V(h,k)$

$g(\overset{-1}{x}) = -2(\overset{-1}{x}+3)(\overset{-1}{x}-1)$

$k = g(-1) = -2(2)(-2) = 8$

$V = (-1, 8)$

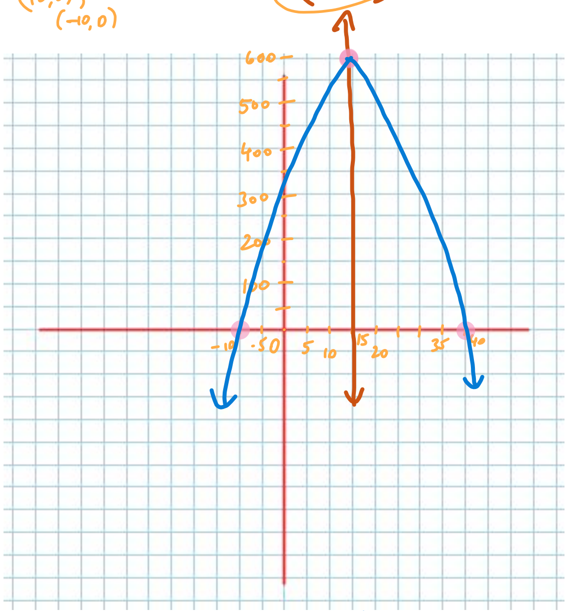
Complete the table, then graph the function.

	$f(x)=a(x-r)(x-s)$ **Factored Form**	$f(x)=ax^2+bx+c$ **Standard Form**	$x=h$ **Axis of Symmetry**	r and s **Zeros**	c ***y*-intercept**	(h,k) **Vertex**	k **Maximum or Minimum Value**
a)	$R(x) = (40 - x)(10 + x)$ $=-(x-40)(x+10)$	$R(x) = -x^2 + 30x + 400$	$x=15$	40 and −10 (40,0), (−10,0)	400	(15,625)	Max = 625

$$h=\frac{r+s}{2}=\frac{40-10}{2}=\frac{30}{2}=15$$

$$x=15 \rightarrow \boxed{R} \rightarrow k=R(15)$$

$$R(15)=(40-15)(10+15)=(25)(25)=625=k$$



#13 from your text. Complete the table.

	r and s **Zeros**	$x=h$ **Axis of Symmetry**	k **Maximum or Minimum Value**	(h,k) **Vertex**	$f(x)=a(x-r)(x-s)$ **Function in Factored Form**	$f(x)=ax^2+bx+c$ **Function in Standard Form**
a)	2 and 8	$x=5$	6	(5, 6)	$f(x)=-\frac{2}{3}(x-2)(x-8)$	$f(x)=-\frac{2}{3}x^2+\frac{20}{3}x-\frac{32}{3}$

$$h=\frac{r+s}{2}$$

$$h=\frac{2+8}{2}=\frac{10}{2}=5$$

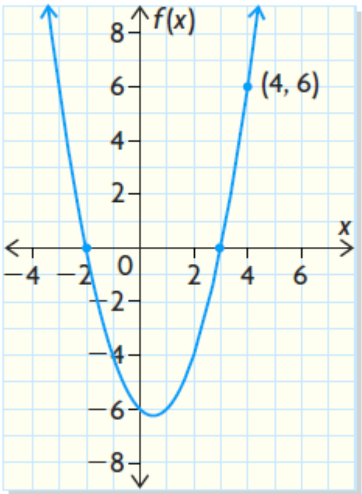
$(x, f(x))$

$$f(x)=a(x-2)(x-8)$$
$$6=a(5-2)(5-8)$$
$$6=a(3)(-3)$$
$$6=a(-9)$$
$$\frac{\cancel{6}^{2}}{-\cancel{9}_{3}}=a$$
$$\boxed{-\frac{2}{3}=a}$$

(5,6) = Vertex

Zeros (2,0), (8,0)

$$f(x)=-\frac{2}{3}(x-2)(x-8)$$
$$=-\frac{2}{3}(x^2-8x-2x+16)$$
$$=-\frac{2}{3}(x^2-10x+16)$$
$$=-\frac{2}{3}x^2+\frac{20}{3}x-\frac{32}{3}$$



$a(x-r)(x-s)$

Find the equation of the parabola in both Factored form and Standard form.

ax^2+bx+c

$f(x)=a(x+2)(x-3)$ with $\underset{6}{f}$, $\underset{4}{x}$, $\underset{4}{x}$

$6=a(6)(1)$

$6=a(6)$

$a=\frac{6}{6}=1$

∴ factored form: $\boxed{f(x)=1(x+2)(x-3)}$

STANDARD FORM

$f(x)=(x+2)(x-3)$

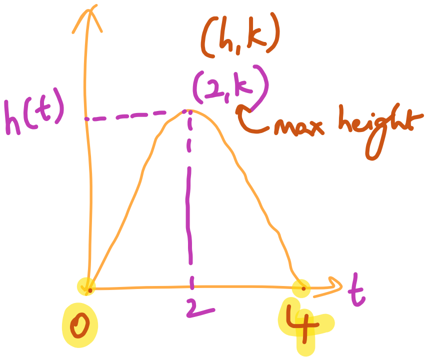
$f(x)=x^2+2x-3x-6$

$\boxed{f(x)=x^2-x-6}$

The last thing I want to do is word problems. **DON'T OVERTHINK OR OVER COMPLICATE WORD PROBLEMS.** First of all, you will most likely be given a function to work with. Then, understand the **CONTEXT** of the problem.

The height of a football kicked from the ground is given by the function $h(t) = -5t^2 + 20t$, where $h(t)$ is the height in metres and t is the time in seconds from its release.

a) Write the function in factored form.
b) When will the football hit the ground?
c) When will the football reach its maximum height?
d) What is the maximum height the football reaches?
e) Graph the height of the football in terms of time without using a table of values.



a) $h(t)=\underset{-5t}{-5t^2}+\underset{-5t}{20t}$

$=-5t(t-4)$

$=-5(t-0)(t-4)$

b) football hits the ground at 4 sec.

c) football reaches its max height at $\frac{0+4}{2}=2$ sec.

d) $h(t=2)=-5\underset{2}{t}^2+20\underset{2}{t}=-5(2)^2+20(2)=-20+40=20$m.

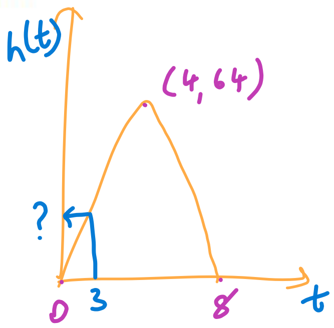
11. **A** The height of a rocket above the ground is modelled by the quadratic function $h(t) = -4t^2 + 32t$, where $h(t)$ is the height in metres t seconds after the rocket was launched.

a) Graph the quadratic function.

b) How long will the rocket be in the air? How do you know?

c) How high will the rocket be after 3 s?

d) What is the maximum height that the rocket will reach?



$h(t) = \underset{-4t}{-4t^2} + \underset{-4t}{32t}$

$= -4t(t-8)$

$= -4(t-0)(t-8)$

$V(H,K);\ H = \frac{0+8}{2} = 4$

$K = h(t=4) = -4\underset{4}{t^2} + 32\underset{4}{t}$

$= -4(4)^2 + 32(4)$

$= -64 + 128$

$= 64$

$\therefore V(H,K) = (4, 64)$

b) 8 sec.

c) $h(t=3)$

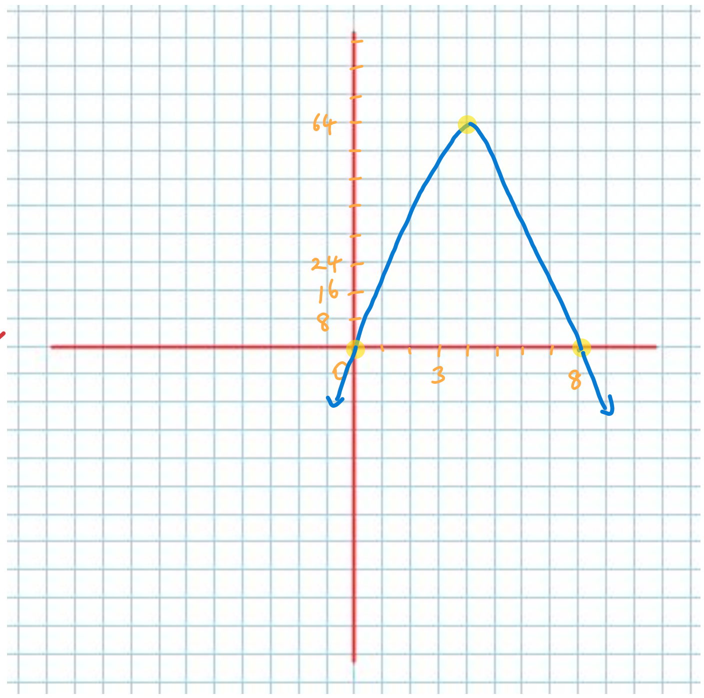
$= -4(3)^2 + 32(3)$

$= -4(9) + 96$

$= -36 + 96$

$= 60$ m high after 3 sec.

d) max height $= 64$ m.



Success Criteria

- I can convert standard form into factored form by factoring the function
- I can convert factored form into standard form by expanding the function
- I can identify the zeroes, vertex, max/min, axis of symmetry, and y-intercept of a quadratic function

3.2 Vertex Form-Maximum or Minimum

Learning Goal: We are learning to determine the maximum/minimum value of a quadratic function.

One very important aspect of quadratic functions is that they all have either a maximum (if the associated parabola opens down) or a minimum (if the parabola opens up). **Max/Min's** have so many **applications** in the real world that it's **ridiculous**.

The **BIG QUESTION** we are faced with is this: **How do we find the Maximum or Minimum Value for some given Quadratic?**

But before that, let's do a quick review to reinforce our understanding of the vertex form.

Question 1: Convert $\boxed{f(x) = 3(x+4)^2 - 18}$ to the Standard Form.

$f(x) = 3(x+4)(x+4) - 18$

$x^2 + 4x + 4x + 16$

$f(x) = 3(x^2 + 8x + 16) - 18$

$f(x) = 3x^2 + 24x + 48 - 18$

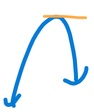
$\boxed{f(x) = 3x^2 + 24x + 30}$

Question 2: Given $\boxed{g(x) = -4(x+5)^2 + 3}$, state the vertex, axis of symmetry, direction of opening and range.

VERTEX: $(-5, 3)$

AoS: $x = -5$

DoO : $a = -4 < 0$



Has a max = 3

Range: $\{g(x) \in R \mid g(x) \leq 3\}$

Question 3: Given the vertex (-3, -8) and the coordinate (-6, 37), determine the equation of the parabola.

$V(h,k) = (-3,-8)$

$f(x) = a(x-h)^2 + k$

$\underset{37}{f(x)} = a(\underset{-6}{x}+3)^2 - 8$

$37 = a(-3)^2 - 8$

$37 = 9a - 8$

$37 + 8 = 9a$

$\frac{45}{9} = \frac{9a}{9}$

$\boxed{5 = a}$

$\therefore$ EQUATION: $f(x) = 5(x+3)^2 - 8$

Okay, Great!!! Now let's go back to the maximum and minimum.

In the Vertex Form, the *h* and *k* form together to give the coordinate of the vertex (h,k).

The maximum or minimum value of a quadratic function is the y-coordinate of the vertex.

So, clearly we do need to find the vertex. It's easy if the vertex form is given. If not, we need to find using algebraic techniques.

In order to find the vertex using algebra, we will consider the following techniques:

1) USING THE ZEROS, TO FIND THE AXIS OF SYMMETRY, and then the vertex (**this is the easiest technique**, ***assuming we can factor the quadratic***).
2) COMPLETING THE SQUARE to find the vertex (this is the toughest technique, but it's nice because you ***end up with the quadratic in vertex form***).

Note: We can also use graphing calculators to find the max/min of a quadratic!

Example

Determine the max or min value for the function $f(x) = -3x^2 - 12x + 15$ by finding **THE ZEROS** of the quadratic.

$f(x) = \frac{-3x^2}{-3} \frac{-12x}{-3} \frac{+15}{-3}$

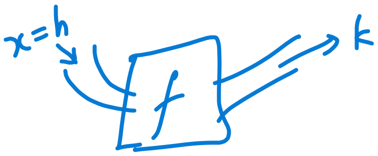
$= -3(x^2 + 4x - 5)$

$5 \quad -1$

$f(x) = -3(x+5)(x-1)$

$f(x) = a(x-r)(x-s)$

AoS: $h = \frac{r+s}{2} = \frac{-5+1}{2} = \frac{-4}{2} = -2$



$\therefore$ Max Value $= k$

$= f(x=-2)$

$= -3\underset{-2}{x}^2 - 12\underset{-2}{x} + 15$

$= -3(-2)^2 - 12(-2) + 15$

$= -12 + 24 + 15$

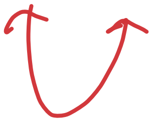
$= 27$

Example

COMPLETE THE SQUARE to find the vertex of the quadratic and state **where** the max (min) is and **what** the max (min) is.

$g(x)=\boxed{\frac{2x^2}{2}+\frac{8x}{2}}-5$

$$\boxed{g(x)=a(x-h)^2+k}$$



STEP 1: $= 2(x^2+4x)-5$

$\frac{4}{2}=2$

$\pm 2^2$

STEP 2: $= 2(\boxed{x^2+4x+2^2}-2^2)-5$

STEP 3: $= 2((x+2)^2-4)-5$

STEP 4: $= 2(x+2)^2-8-5$

$= 2(x+2)^2-13$

$V(h,k)=(-2,-13)$

Min $= -13$ at $x=-2$

Another Example

COMPLETE THE SQUARE to determine the axis of symmetry, find the vertex and state the min or max value.

$h(x)=\boxed{\frac{5x^2}{5}+\frac{15x}{5}}-3$

$= 5(x^2+3x)-3$

$\frac{3}{2}=1.5$

$\pm 1.5^2$

$= 5(\boxed{x^2+3x+1.5^2}-1.5^2)-3$

$= 5((x+1.5)^2-2.25)-3$

$= 5(x+1.5)^2-11.25-3$

$= 5(x+1.5)^2-14.25$

$a(x-h)^2+k$

AoS: $x=-1.5$

VERTEX $V(h,k)=(-1.5,-14.25)$

Min. $= -14.25$

SOME MORE PRACTICE

Example:

$$g(x) = \underset{-4}{\underline{-4x^2}} - \underset{-4}{\underline{40x}} - 7$$

$$= -4(x^2+10x)-7$$

$\frac{10}{2}=5$

$\pm 5^2$

$$= -4(x^2+10x+5^2-5^2)-7$$

$$= -4((x+5)^2-25)-7$$

$$= -4(x+5)^2+100-7$$

$$= -4(x+5)^2+93$$

$$V(h,k) = (-5,93)$$

Example:

$$f(x) = \frac{1}{2}x^2 + 7x + 6$$

$$f(x) = \underset{0.5}{\underline{0.5x^2}} + \underset{0.5}{\underline{7x}} + 6$$

$\frac{14}{2}=7$

$\pm 7^2$

$$= 0.5(x^2+14x+7^2-7^2)+6$$

$$= 0.5((x+7)^2-49)+6$$

$$= 0.5(x+7)^2-24.5+6$$

$$\boxed{f(x) = 0.5(x+7)^2-18.5}$$

$$V(h,k) = (-7,-18.5)$$

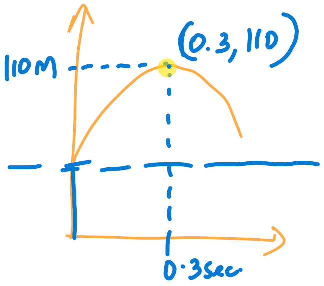
Example:

The height above the ground of a bungee jumper is modelled by the quadratic function $\boxed{h(t) = -5(t-0.3)^2 + 110}$, where height, *h(t)*, is in metres and time, *t*, is in seconds.

a) When does the bungee jumper reach a maximum height? Why is it a maximum?

b) What is the maximum height reached by the jumper?

c) Determine the height of the platform from which the bungee jumper jumps.



a) Max height at 0·3sec. because Vertex = (0·3, 110)

b) Max height = 110m

c) $h(t) = -5(t-0.3)^2 + 110$

$(t-0.3)(t-0.3)$

$(t^2-0.3t-0.3t+0.09)$

$h(t) = -5(t^2-0.6t+0.09)+110$

$h(t) = -5t^2+3t-0.45+110$

$$\boxed{h(t) = -5t^2+3t+109.55}$$

c

∴ Height of platform = 109.55m.

Success Criteria:

- I can recognize when a function has a maximum or minimum value (based on "*a*")
- I can find the max/min (vertex) value using various methods (*partial factoring* ☺)

3.3 Solve Quadratic Equations

Learning Goal: We are learning to solve quadratic functions in different ways.

1. **Solving by Factoring**

Steps

1) Make one side of your equation equal to zero (stuff = zero)
2) **Factor**. The solutions will be the x values when each factor is set equal to zero

Example 1:

Solve $x^2 - 9x + 12 = -8$

$x^2 - 9x + 12 + 8 = 0$

$x^2 - 9x + 20 = 0$

$-4 \quad -5$

$(x-4)(x-5) = 0$

$x - 4 = 0 \qquad x - 5 = 0$

$\boxed{x_1 = 4} \qquad \boxed{x_2 = 5}$

Example 2:

Solve $16x^2 - 25 = 0$

$(4x)^2 - (5)^2$ ← Difference of Squares.

$(4x-5)(4x+5) = 0.$

$4x - 5 = 0 \qquad 4x + 5 = 0$

$4x = 5 \qquad 4x = -5$

$\boxed{x_1 = \frac{5}{4} = 1.25} \qquad \boxed{x_2 = \frac{-5}{4} = -1.25}$

Example 3:

Solve $2(x+3)^2 = 5(x+3)$

$2(x+3)^2 - 5(x+3) = 0$

$(x+3)(2(x+3) - 5) = 0$

$(x+3)(2x + 6 - 5) = 0$

$(x+3)(2x+1) = 0$

$x + 3 = 0 \qquad 2x + 1 = 0$

$\boxed{x_1 = -3} \qquad 2x = -1$

$\boxed{x_2 = \frac{-1}{2} = -0.5}$

Example 4:

The profit of a skateboard company can be modelled by the function $\boxed{P(x) = -63 + 133x - 14x^2}$, where *P(x)* is the profit in thousands of dollars and *x* is the number of skateboards sold, also in thousands ~~of dollars~~. When will the company break even, and when will it be profitable?

$P(x) = 0$

$0 = \frac{-14x^2}{-7} + \frac{133x}{-7} - \frac{63}{-7}$

$0 = -7(2x^2 - 19x + 9)$

$0 = -7(\underbrace{2x^2 - 18x}_{2x} \underbrace{- x + 9}_{-1})$

$0 = -7(2x-1)(x-9)$

$2x - 1 = 0$

$2x = 1$

$\boxed{x_1 = \frac{1}{2} = 0.5}$ → # of skateboards $0.5 \times 1000 = \boxed{500}$ ✓ → BREAK EVEN

$x - 9 = 0$

$\boxed{x_2 = 9}$ → # of skateboards $9 \times 1000 = \boxed{9000}$

Example 5:

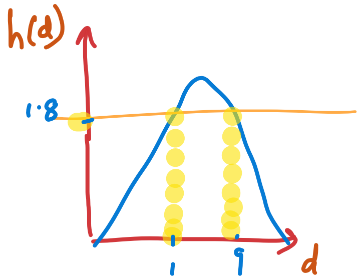
The path a dolphin travels when it rises above the ocean's surface can be modelled by the function $\boxed{h(d) = -0.2d^2 + 2d}$, where $h(d)$ is the height of the dolphin above the water's surface and *d* is the horizontal distance from the point where the dolphin broke the water's surface, both in feet. When will the dolphin reach a height of 1.8 feet?

$h(d) = -0.2d^2 + 2d$

$1.8 = -0.2d^2 + 2d$

$0 = -0.2d^2 + 2d - 1.8$

$a = -0.2$ $b = 2$ $c = -1.8$



QUADRATIC FORMULA $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $ax^2 + bx + c = 0$

When factoring looks difficult, we use QUADRATIC FORMULA

$d = \frac{-2 \pm \sqrt{2^2 - 4(-0.2)(-1.8)}}{2(-0.2)}$

$d = \frac{-2 \pm \sqrt{4 - 1.44}}{-0.4}$

$d = \frac{-2 \pm 1.6}{-0.4}$

$d_1 = \frac{-2 + 1.6}{-0.4} = \frac{-0.4}{-0.4} = 1$

$\boxed{d_1 = 1}$

$d_2 = \frac{-2 - 1.6}{-0.4} = \frac{-3.6}{-0.4} = 9$

$\boxed{d_2 = 9}$

2. **Solving by using Quadratic Formula** (especially useful when not factorable)

The quadratic formula is the one formula to solve them all. With great power, comes great responsibility. It is a tricky formula, but once you learn how to properly use it, you will be that much happier. The key is to write and communicate your math carefully. $\boxed{D = b^2 - 4ac}$

Given $\boxed{ax^2 + bx + c = 0}$, then $\boxed{x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$ ← DISCRIMINANT

STANDARD FORM

(i) $D > 0 \Rightarrow$ TWO SOLUTIONS

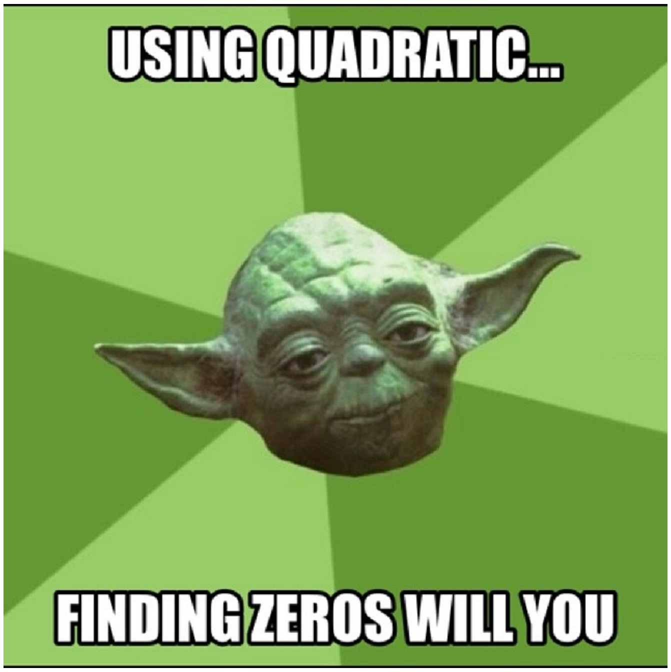
(ii) $D = 0 \Rightarrow$ ONE SOLUTION

(iii) $D < 0 \Rightarrow$ NO REAL SOLUTIONS

Two notes:

1. Inside the square root, you start with b^2. No matter what you plug in, you get a positive number. If $b = 9, \ b^2 = 81$. If $b = -5, \ b^2 = 25$. Why do I make note of this? I have seen many people do it wrong, so don't be one of those people.

2. Since we are calculating a square root, we have three options. If the number inside the square root is positive, there are two solutions. If the number is zero, there is only one solution. If the number is negative, there are no solutions since you cannot square root a negative. This is a fact. Don't try to square root a negative as it absolutely 100% cannot be done. Please don't try to do it. It doesn't work. No solution is an answer, so do not fret. If you don't like the negative, double check your work.



Example 1:

$$\frac{3x^2}{3} - \frac{24x}{3} + \frac{45}{3} = \frac{0}{3}$$

$$1x^2 - 8x + 15 = 0$$

$a = 1$, $b = -8$, $c = 15$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{64 - 60}}{2} = \frac{8 \pm 2}{2}$$

$$x_1 = \frac{8+2}{2} = \frac{10}{2} = \boxed{5}$$

$$x_2 = \frac{8-2}{2} = \frac{6}{2} = \boxed{3}$$

Example 2:

$$3x^2 + 2x + 15 = 0$$

$a = 3$, $b = 2$, $c = 15$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(15)}}{2(3)}$$

$$x = \frac{-2 \pm \sqrt{4 - 180}}{6}$$

$$x = \frac{-2 \pm \sqrt{-176}}{6} \leftarrow D < 0$$

$\therefore x$ has no real solutions

Example 3:

$$4x^2 - 8x + 10 = 2x + 7$$

$$4x^2 - 8x + 10 - 2x - 7 = 0$$

$$4x^2 - 10x + 3 = 0$$

$a = 4$, $b = -10$, $c = 3$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(4)(3)}}{2(4)}$$

$$= \frac{10 \pm \sqrt{100 - 48}}{8}$$

$$= \frac{10 \pm \sqrt{52}}{8}$$

$$\approx \frac{10 \pm 7.2}{8}$$

$$x_1 \approx \frac{10 + 7.2}{8}$$

$$\boxed{x_1 \approx 2.15}$$

$$x_2 \approx \frac{10 - 7.2}{8}$$

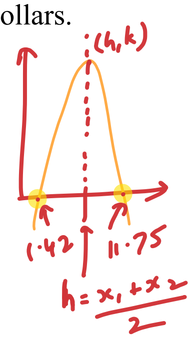
$$\boxed{x_2 \approx 0.35}$$

Example 4: The profit on a school drama production is modelled by the quadratic equation $P(x) = -60x^2 + 790x - 1000$, where $P(x)$ is the profit in dollars and x is the price of the ticket, also in dollars.

a) Use the quadratic formula to determine the break-even price for the tickets. $P = 0$

b) At what price should the drama department set the tickets to maximize their profit?

VERTEX (h, k) $h = ?$



$h = \frac{x_1 + x_2}{2}$

a) $\overset{0}{\cancel{P(x)}} = \frac{-60x^2}{10} + \frac{790x}{10} - \frac{1000}{10}$ $\frac{}{10}$

$0 = 6x^2 - 79x + 100$

$a = 6,\ b = -79,\ c = 100$

$$x = \frac{-(-79) \pm \sqrt{(-79)^2 - 4(6)(100)}}{2(6)}$$

$$x = \frac{79 \pm \sqrt{6241 - 2400}}{12} = \frac{79 \pm \sqrt{3841}}{12} \approx \frac{79 \pm 62}{12}$$

$x_1 \approx \frac{79+62}{12}$

$\boxed{x_1 \approx 11.75}$

$x_2 \approx \frac{79-62}{12}$

$\boxed{x_2 \approx 1.42}$

∴ BREAK EVEN price = \$1.42

b) Ticket price for max profit $= \frac{11.75 + 1.42}{2}$

$= \$6.59$

Another Example:

8. The population of a region can be modelled by the function $P(t) = 0.4t^2 + 10t + 50$, where $P(t)$ is the population in thousands and t is the time in years since the year 1995.

a) What was the population in 1995?

b) What will be the population in 2010?

c) In what year will the population be at least 450 000? Explain your answer.

$t = ?$

a) $P(t=0) = 0.4(0)^2 + 10(0) + 50 = 50$

∴ In 1995, there were 50,000 people

b) $1995 \rightarrow 2010 \Rightarrow t = 15$

$P(t=15) = 0.4(15)^2 + 10(15) + 50 = 90 + 150 + 50 = 290$

∴ In 2010, population = 290,000.

c) $P = 450,\ t = ?$ ∴ $450 = 0.4t^2 + 10t + 50$

$0 = 0.4t^2 + 10t + 50 - 450$

$\boxed{0 = 0.4t^2 + 10t - 400}$

$$t = \frac{-10 \pm \sqrt{10^2 - 4(0.4)(-400)}}{2(0.4)}$$

$$t = \frac{-10 \pm 27.2}{0.8}$$

$t_1 = 21.5$ $\cancel{t_2 = -46.5}$

$t_1 \approx 22$ years

∴ $1995 + 22 = 2017$

The pop. is 450,000 in 2017

Success Criteria

- I can solve quadratic functions by factoring, then setting each factor equal to zero
- I can solve quadratic functions by using the quadratic formula

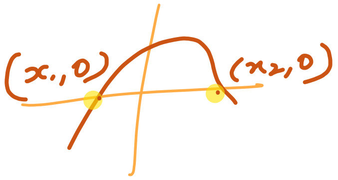
3.3 cont… Zeros of Quadratic Functions

Learning Goal: We are learning to determine the number of zeros of a quadratic function.

Before beginning we should look at the difference between a Quadratic **FUNCTION** and a Quadratic **EQUATION**. A function such as $\boxed{f(x)=3x^2-5x+1}$ ⇒ 0 has a graph with **infinitely** many points. On the other hand, a quadratic equation (in standard form) looks like:

$$\boxed{3x^2-5x+1=0}$$

Solve for x



(*What is the difference between the function and the equation?)*

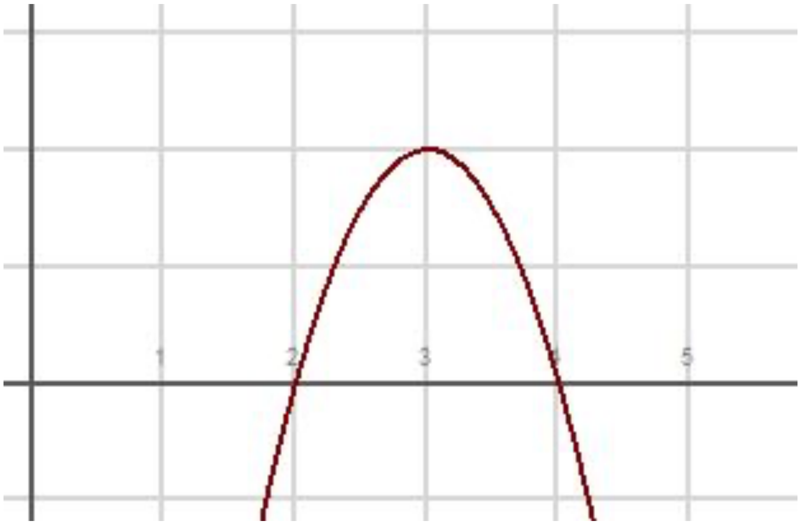
As it turns out, solving a quadratic equation is **Exactly the Same as finding the zeros of quadratic functions.**

Quadratic functions, therefore can have __0__, __1__, or __2__ **ZEROS**.

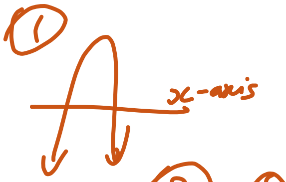
Remember – **FUNCTIONS CAN BE DESCRIBED AS A SET OF ORDERED PAIRS**, where the "ordered pair" is a pair of numbers: a **domain value** and a **range value** which can look like $(x, f(x))$. We have talked about the vertex of a parabola. Consider a parabola opening down (which means it will have a maximum value.

The vertex of that parabola is NOT the maximum. Instead, the vertex is a POINT which is made up of two special numbers. The domain value is WHERE the max occurs and the functional value (the "y" value) is the maximum.

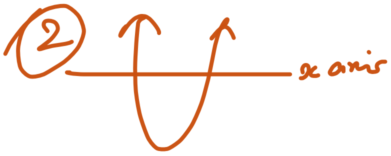
When we talk about the ZEROS of a quadratic we need to understand what we mean by that. Consider the sketch of the graph of the quadratic function $f(x)=-2(x-3)^2+2$



CASE 1 $\boxed{\text{2 zeros}}$

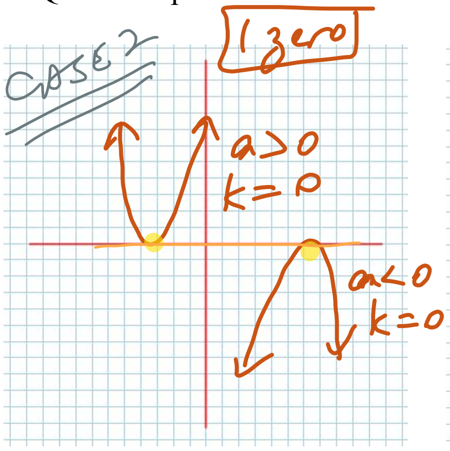


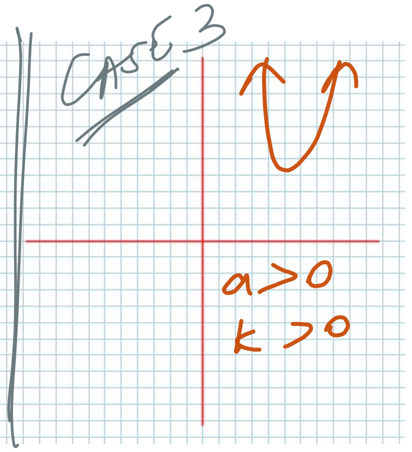
a < 0
k > 0

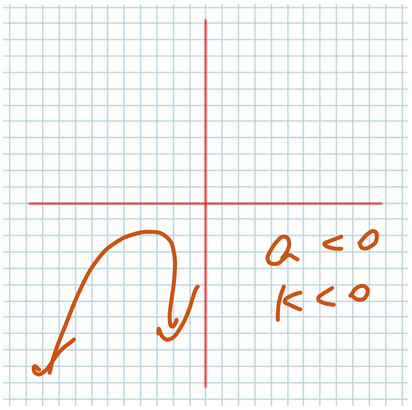


a > 0
k < 0

Q. Do all quadratics have 2 zeros? NO!!!!!!!







Finding the Zeros of a Quadratic

We can find those pesky zeros in a number of ways:

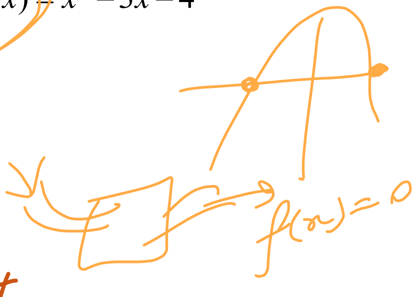
1) Writing the quadratic in zeros form (by factoring)
2) Using the quadratic formula (but the quadratic ***MUST BE IN STANDARD FORM*** - $f(x)=ax^2+bx+c$)
3) Using graphing technology (lame, but legit)

Example

Determine the zeros by factoring:

a) $f(x)=x^2-3x-4$

$0 =$



$0 = x^2 - 3x - 4$

$0 = (x-4)(x+1)$

$x_1 - 4 = 0$

$x_1 = 4$

$x_2 + 1 = 0$

$x_2 = -1$

∴ zeros are $(4, 0)$ and $(-1, 0)$

b) $g(x)=2x^2+x-1$

$0 = 2x^2 + 2x - x - 1$

$0 = (2x-1)(x+1)$

$2x_1 - 1 = 0$

$2x_1 = 1$

$x_1 = \frac{1}{2}$

$\boxed{x_1 = 0.5}$

$x + 1 = 0$

$\boxed{x_2 = -1}$

∴ zeros are $(0.5, 0)$ and $(-1, 0)$

Example

Using the quadratic formula, determine the zeros of the quadratic:

In case you've forgotten, the quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

a) $f(x) = 2x^2 + 3x - 7$

$0 = 2x^2 + 3x - 7$

$a = 2, \ b = 3, \ c = -7$

$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-7)}}{2(2)}$

$x = \frac{-3 \pm \sqrt{9+56}}{4}$

$x = \frac{-3 \pm \sqrt{65}}{4}$

$x = \frac{-3 \pm 8.06}{4}$

$x_1 = \frac{-3 + 8.06}{4}$ $x_2 = \frac{-3 - 8.06}{4}$

$x_1 = 1.265$ $x_2 = -2.765$

$(1.265, 0)$ and $(-2.765, 0)$

b) $g(x) = 3x^2 - 2x + 4$

$0 = 3x^2 - 2x + 4$

$a = 3, \ b = -2, \ c = 4$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(4)}}{2(3)}$

$x = \frac{2 \pm \sqrt{4 - 48}}{6}$

$x = \frac{2 \pm \sqrt{\boxed{-44}}}{6} \rightarrow D = b^2 - 4ac < 0$

$\therefore$ No Zeros.

The Discriminant

The Discriminant of the quadratic formula is called the **DISCRIMINANT** because

The Discriminant is $b^2 - 4ac$ $= D$ $\sqrt{\bigcirc}$

1) If $b^2 - 4ac > 0$, then the quadratic has 2 zeros

2) If $b^2 - 4ac = 0$, then the quadratic has 1 zeros

3) If $b^2 - 4ac < 0$, then the quadratic has No zeros

Example

Determine the number of zeros using the discriminant:

a) $f(x) = 2x^2 + 3x - 2$

$D = b^2 - 4ac$

$D = 3^2 - 4(2)(-2)$

$= 9 + 16$

$= 25$

$D > 0 \therefore$ 2 zeros

b) $g(x) = -x^2 + 4x - 4$

$a = -1$, $b = 4$, $c = -4$

$D = b^2 - 4ac$

$= 4^2 - 4(-1)(-4)$

$= 16 - 16$

$= 0$

$D = 0 \therefore$ 1 zero.

c) $h(x) = 3x^2 + 5x + 6$

$D = b^2 - 4ac$

$= 5^2 - 4(3)(6)$

$= 25 - 72$

$= -47 < 0$

$D < 0 \therefore$ No zeros.

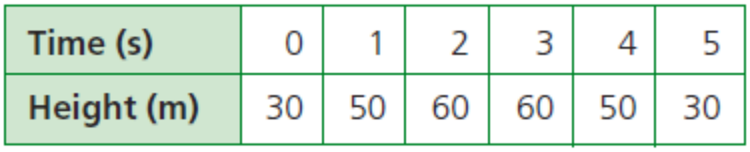
Success Criteria:

- I can recognize that a quadratic function may have 0, 1, or 2 zeros
- I can use the discriminant of the quadratic formula to determine the number of zeros

3.6 Creating a Quadratic Model from Data

In this section, we get to use a fantastic piece of software which will allow us to take some data, and compute a quadratic regression model. Yes, it is as exciting as it sounds. A regression model is a very important thing in mathematics. You input data into a spreadsheet or other software, and you create a line or curve of best fit (a best estimate). With this line or curve, you also get an equation which allows you to make predictions or fill in the gaps.

Question: A ball is thrown into the air from the top of a building. The table of values gives the height of the ball at different times during the flight. What is a function that will model the data?



Time (s)	0	1	2	3	4	5
Height (m)	30	50	60	60	50	30

Step 1: Turn on computer and open GeoGebra (that's a combo of Geometry and Algebra)
Step 2: Click "View—Spreadsheet"
Step 3: Input the *Time* values in one column, the *Height* values in the adjacent column. The first column must always represent the "x" axis, or the independent variable.
Step 4: Highlight the data, right-click it, then click "Create—List of Points". The points should show on the graph.
Step 5: In the input box, type FitPoly[list1,2]. This will create the parabola, and you should now see the equation of said parabola.

Now we can figure out the zeros, vertex, or anything else of significance!