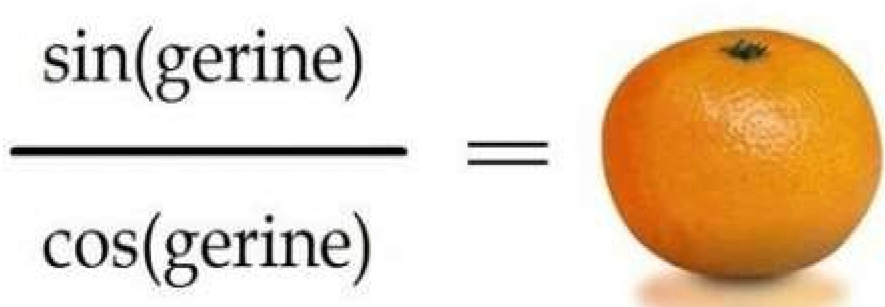
Functions & Applications 11

MCF3M

Course Notes

Chapter 5: Trigonometry & Acute Angles



Homework

Contents with suggested problems from the Nelson Textbook. You are welcome to ask for help, from myself or your peers, with any of the following problems.

Section 5.1 – Page 271
#1-4, 8, 10, 11, 14

Section 5.2 – Page 280
#1, 4, 7, 8, 16

Section 5.3 – Page 288
#2, 3, 5, 8a, 9a, 10, 13 (draw it!)

Section 5.4 – Page 299
#2, 3, 5ad, 7, 9

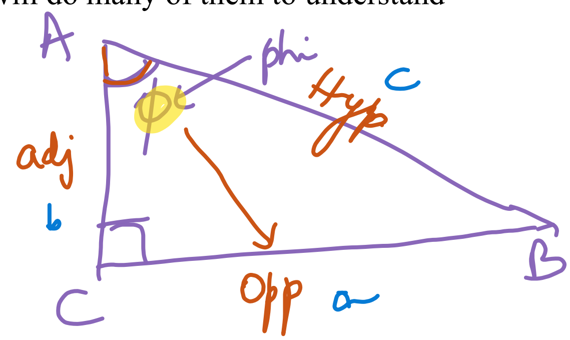
5.1 Applying the Primary Trigonometric Ratios

SOH CAH TOA

Learning Goal: We are learning the three primary trigonometric ratios and to use them to find missing side lengths or angles in right angled triangles.

$\sin \equiv \frac{opp}{Hyp}$; $\cos \equiv \frac{adj}{Hyp}$; $\tan \equiv \frac{opp}{adj}$

Trigonometry is a fascinating and wonderful branch of Mathematics. It is one of my favourite sections. The "math" of trigonometry is not too difficult, but it is the applying of it in word problems that makes is hard. There are a lot of word problems in this chapter, so we will do many of them to understand them.



First, what are the primary trigonometric ratios?

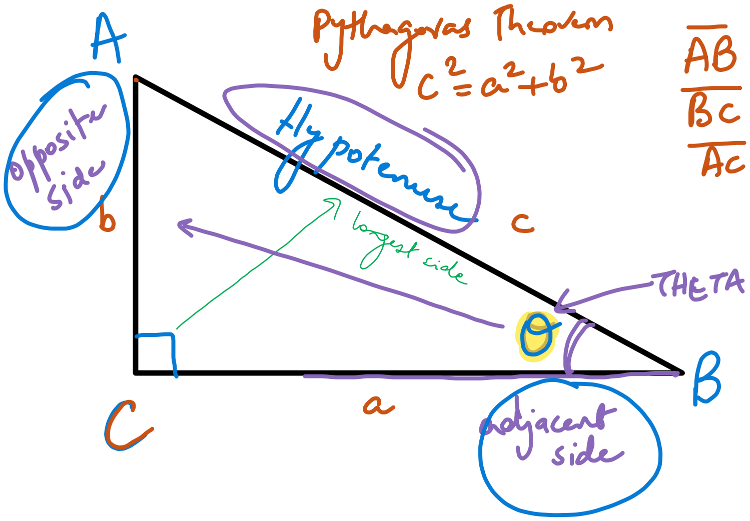
Pythagoras Theorem

$c^2 = a^2 + b^2$

$\overline{AB} = c$

$\overline{BC} = a$

$\overline{AC} = b$



$\sin\theta = \frac{b}{c}$

$\cos\theta = \frac{a}{c}$

$\tan\theta = \frac{b}{a}$

$\sin\phi = \frac{a}{c}$

$\cos\phi = \frac{b}{c}$

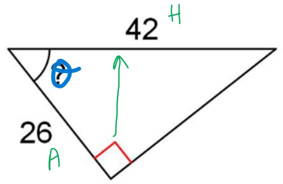
$\tan\phi = \frac{a}{b}$

Next, let's look at how we use the calculator with trigonometry:

- Your calculator MUST BE IN DEGREE MODE. If not, your answers will be incorrect.
- Finding a ratio VS finding an angle

$\sin 30 = 0.5$

Given the triangles, solve for the indicated missing piece.

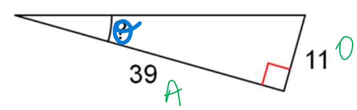


CAH

$\cos\theta = \frac{26}{42}$

$\theta = \cos^{-1}\left(\frac{26}{42}\right)$

$\theta \approx 52^\circ$

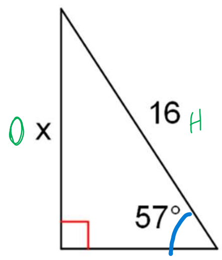


TOA

$\tan\theta = \frac{11}{39}$

$\theta = \tan^{-1}\left(\frac{11}{39}\right)$

$\theta \approx 16^\circ$

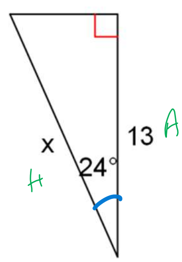


SOH

$\sin 57 = \frac{x}{16}$

$16 \sin 57 = x$

$x \approx 13.4$



CAH

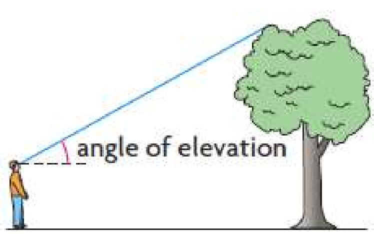
$\cos 24 = \frac{13}{x}$

$x \cos 24 = 13$

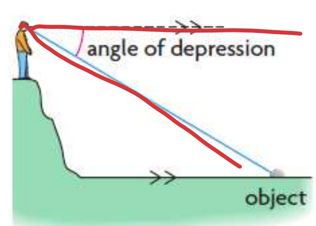
$x = \frac{13}{(\cos 24)} \approx 14.2$

Important things to note in word problems:

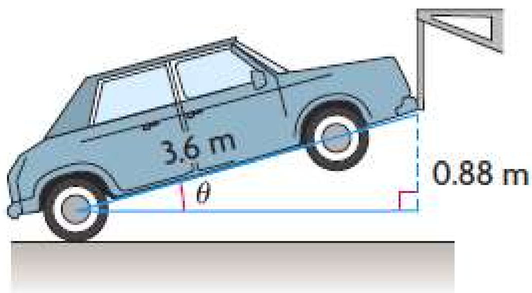
angle of elevation
the angle between the horizontal and the line of sight when looking up at an object

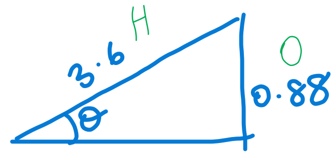


angle of depression
the angle between the horizontal and the line of sight when looking down at an object



Word Problem Example 1:
Eric's car alarm will sound if his car is disturbed, but it is designed to shut off if the car is being towed at an angle of elevation of more than 15°. Mike's tow truck can lift the bumper no more than 0.88m higher than the bumpers original height above the ground. The distance from the back wheel to the front bumper is 3.6m. Will the alarm sound?





SOH

$\sin\theta = \frac{0.88}{3.6}$

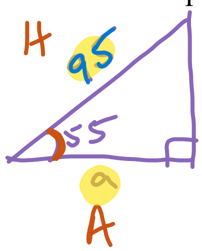
$\theta = \sin^{-1}\left(\frac{0.88}{3.6}\right)$

$\theta \approx 14° < 15°$

∴ Eric's car will sound the alarm.

Word Problem Example 2:
A hot-air balloon on the end of a taut 95m rope rises from its platform. Sarah, who is in the basket, estimates that the angle of depression to the rope is about 55°.
a) How far, to the nearest metre, did the balloon drift horizontally?
b) How high, to the nearest metre, is the balloon above the ground?
c) Viewed from the platform, what is the angle of elevation, to the nearest degree, to the balloon?

a)

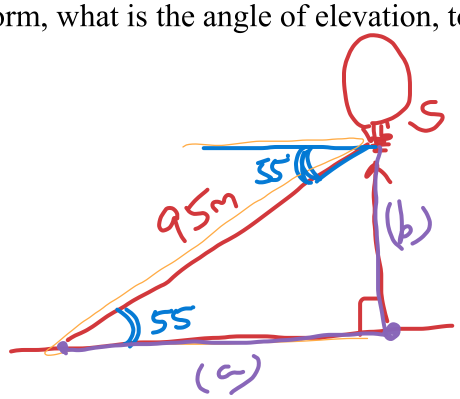


CAH

$\cos 55 = \frac{a}{95}$

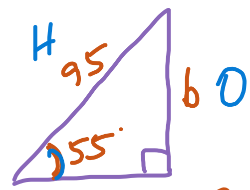
$95\cos 55 = a$

$a \approx 54\text{m}.$



c) Angle of elevation required = 55°

b)



SOH

$\sin 55 = \frac{b}{95}$

$95\sin 55 = b$

$b \approx 78\text{m}.$

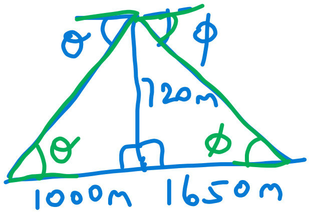
Success Criteria:
- I can use SohCahToa to determine the primary trigonometric ratios
- If given an angle and a side length, I can use the appropriate trig ratio to find another side length
- If given two side lengths, I can use the appropriate inverse trig ratio to find the angle
- I can identify the difference between an angle of elevation and an angle of depression

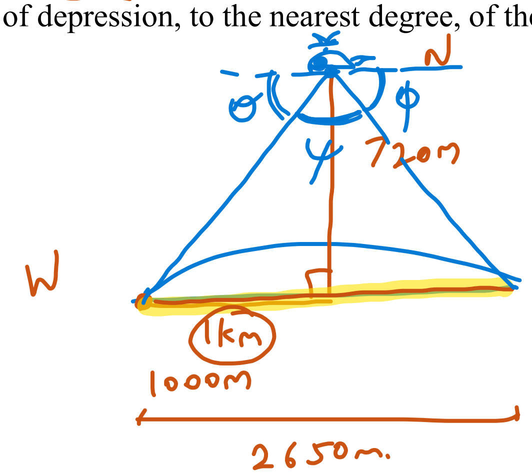
5.2 Applying the Primary Trigonometric Ratios

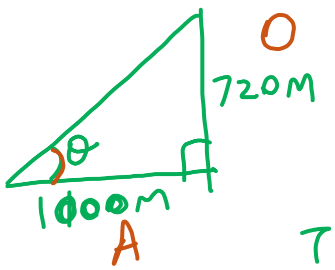
Learning Goal: We are learning to use the three primary trigonometric ratios to solve word problems involving right angles triangles.

Question 1: Karen is a photographer taking pictures of the Burlington Skyway Bridge. She is in a helicopter hovering 720m above the bridge, exactly 1 km horizontally from the west end of the bridge. The Skyway spans a distance of 2650m from east to west.

a) From Karen's position, what are the angles of depression, to the nearest degree, of the east and west ends of the bridge?





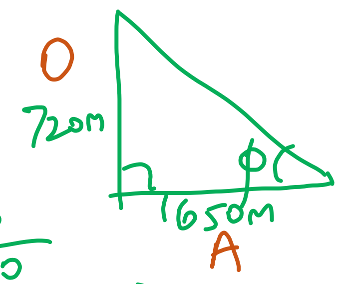


TOA

$\tan\theta = \frac{720}{1000}$

$\theta = \tan^{-1}\left(\frac{720}{1000}\right)$

$\boxed{\theta \approx 36^\circ}$



TOA

$\tan\phi = \frac{720}{1650}$

$\phi = \tan^{-1}\left(\frac{720}{1650}\right) \approx 24^\circ$

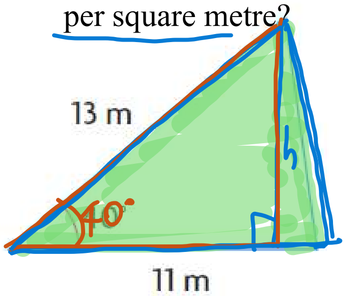
b) If Karen's camera has a wide-angle lens that can capture 150°, can she get the whole bridge in one shot?

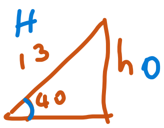
$\theta + \phi = 36 + 24 = 60^\circ$

$\psi = 180 - (\theta + \phi) = 180 - 60 = 120^\circ < 150^\circ$

∴ She can get the whole bridge in one shot.

5.1.5**Question 2:** Sue's parents have a house with a triangular front lawn as shown. They want to cover the lawn with sod rather than plant grass seed. How much would it cost to put sod in if it costs $13.75 per square metre?



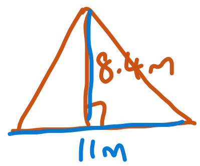


SOH

$\sin 40 = \frac{h}{13}$

$13 \sin 40 = h$

$h \approx 8.4m$



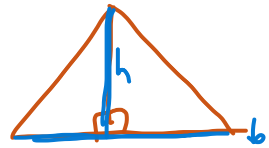
$\text{Area} = \frac{11 \times 8.4}{2}$

$= 46.2 m^2$

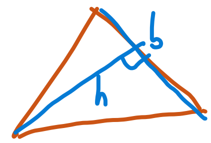
$\therefore$ Cost @ $13.75/m²

$= 46.2 \times 13.75$

$= \boxed{\$635.25}$

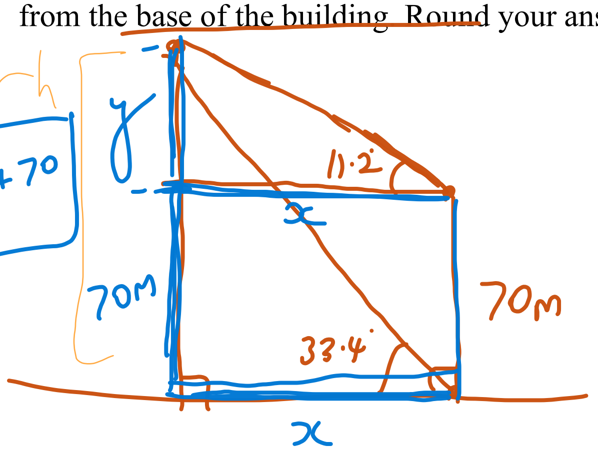


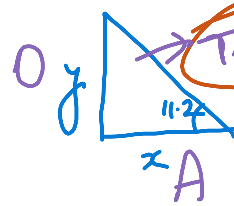
$\text{area}(\Delta) = \frac{bh}{2}$



Question 3: A communications tower is some distance from the base of a 70m high building. From the roof of the building, the angle of elevation to the top of the tower is 11.2°. From the base of the building, the angle of elevation to the top of the tower is 33.4°. Determine the height of the tower and how far it is from the base of the building. Round your answer to the nearest metre.

$\boxed{h = y + 70}$

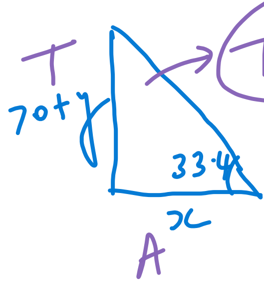




TOA

$\boxed{\tan 11.2 = \frac{y}{x}}$

$\Rightarrow x \tan 11.2 = y$



TOA

$\tan 33.4 = \frac{70+y}{x}$

$\tan 33.4 = \frac{70 + x\tan 11.2}{x}$

$\Rightarrow x \tan 33.4 = 70 + x \tan 11.2$

0.66 0.20

$\Rightarrow 0.66x = 70 + 0.20x$

$\Rightarrow 0.66x - 0.20x = 70$

$\Rightarrow 0.46x = 70$

$\Rightarrow x = \frac{70}{0.46} = 152.17m$

$\therefore y = x \tan 11.2$

$= (152.17)(0.20)$

$= 152.37m.$

$\therefore h = 152.37 + 70$

$= 222.37m.$

Success Criteria:

- I can draw a diagram which illustrates the information contained in a word problem
- I can use an/the appropriate trigonometric ratio(s) to solve a word problem

∴ Height of tower is 222.37m and the distance between tower & building is 152.17m.

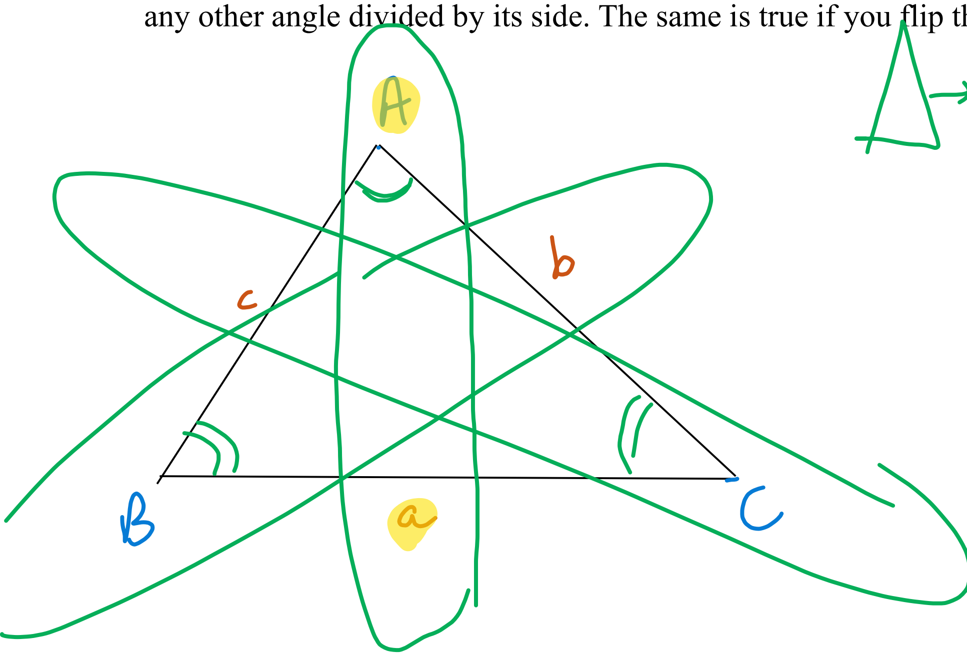
5.3 Applying the Sine Law in Acute Triangles

Learning Goal: We are learning to use the Sine law to solve non-right angle triangles.

What happens when we have a **non-right triangle**? There are two laws which we can use to solve for missing sides and pieces. The law that we use depends on the information that we have. Today, we will look at the Sine Law.

$$\boxed{\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}} \Longleftrightarrow \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

Essentially, the sine of an angle divided by its side is equal to a ratio. That is the same ratio as the sine of any other angle divided by its side. The same is true if you flip the fractions.

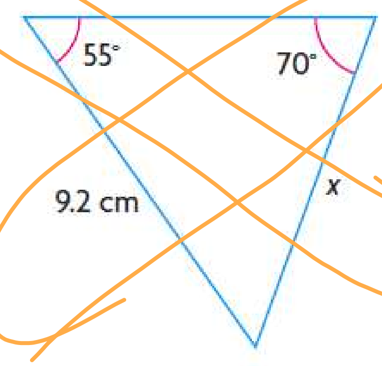


→ * When 90° Δ
Then SOH
CAH
TOA

→ When not 90° Δ
Then
CORRESPONDING PAIR
of Angle and Side

To use Sine Law, you need a full pair, plus anything else. Let's try it out:

Determine x to the nearest tenth of a centimetre.

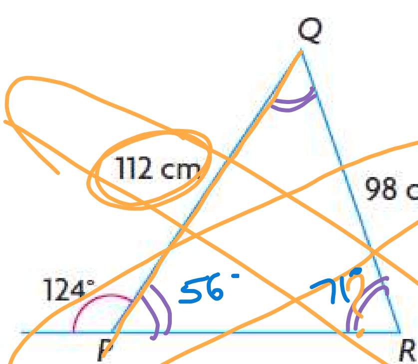


$$\frac{x}{\sin 55} = \frac{9.2}{\sin 70}$$

$$\Rightarrow x = \frac{9.2 \sin 55}{\sin 70}$$

$$\Rightarrow x = 8.019 \approx 8.02\text{ cm} \approx 8\text{ cm}$$

Determine all the interior angles in $\triangle PQR$. Round your answers to the nearest degree.



$$\angle P = 180 - 124 = 56°$$

$$\frac{\sin R}{112} = \frac{\sin 56}{98}$$

$$\Rightarrow \sin R = \boxed{\frac{112 \sin 56}{98}}$$

$$\sin R = 0.9474\ldots$$

$$R = \sin^{-1}(0.9474\ldots)$$

$$\angle R \approx 71°$$

$$\therefore \angle Q = 180 - 71 - 56$$

$$\angle Q = 53°$$

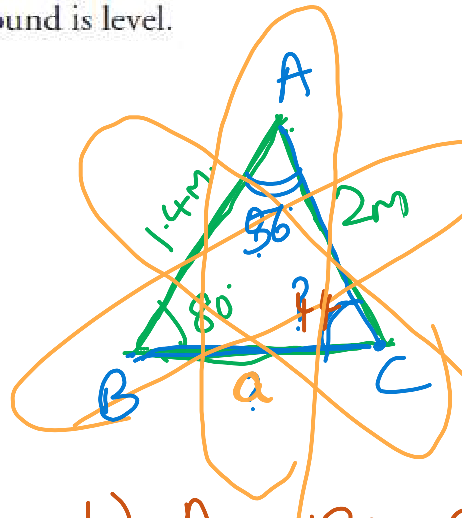
P	56°
Q	53°
R	71°

A wall that is 1.4 m long has started to lean and now makes an angle of 80° with the ground. A 2.0 m board is jammed between the top of the wall and the ground to prop the wall up. Assume that the ground is level.

a) What angle, to the nearest degree, does the board make with the ground?

b) What angle, to the nearest degree, does the board make with the wall?

c) How far, to the nearest tenth of a metre, is the board from the base of the wall?



a) $\frac{\sin C}{1.4} = \frac{\sin 80}{2}$

$\sin C = \frac{1.4 \sin 80}{2}$

$C = \sin^{-1}(0.68936\ldots)$

$\boxed{C \approx 44^\circ}$

∴ The board makes 44° with the ground

b) $A = 180 - 80 - 44$

$\boxed{A \approx 56^\circ}$

∴ The board makes 56° with the wall

c) $\frac{a}{\sin 56} = \frac{2}{\sin 80}$

$a = \frac{2 \sin 56}{\sin 80}$

$\boxed{a \approx 1.7\text{m.}}$

∴ The board and base of wall are 1.7m apart.

Success Criteria

- I can use the sine law to solve for an unknown value in an acute triangle that **DOES NOT** have a right angle!!!!
  - *If you have a right triangle, please, please, please just use SohCahTo*
- I can use $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ to find an unknown angle
- I can use $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ to find an unknown side length

5.4 Applying the Cosine Law in Acute Triangles

Learning Goal: We are learning to use the Cosine law to solve non-right angle triangles.

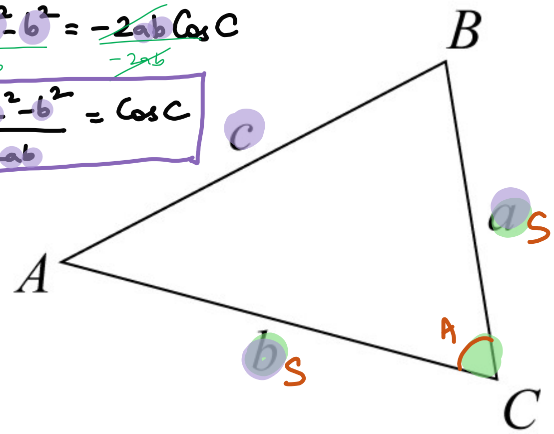
The Sine Law only works when you have a "pair" (a side and its opposite angle). This means that the Sine Law will not solve all triangles, and we need another law to help us out. Enter the Cosine Law!

Given ΔABC,

$$\boxed{c^2 = a^2 + b^2 - 2ab\,\text{Cos}\,C}$$

$$c^2 - a^2 - b^2 = -2ab\,\text{Cos}\,C$$
$$\frac{}{-2ab} \qquad \frac{}{-2ab}$$

$$\boxed{\frac{c^2 - a^2 - b^2}{-2ab} = \text{Cos}\,C}$$



To find a different letter (side), **do not rearrange**. Just swap the letters around.

If you want to find a,

$$a^2 = b^2 + c^2 - 2bc\,\text{Cos}\,A$$

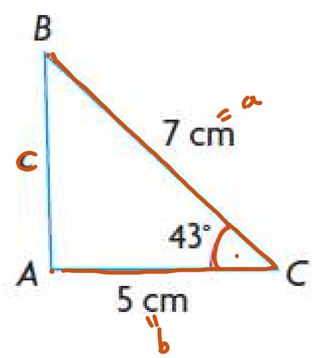
$$b^2 = a^2 + c^2 - 2ac\,\text{Cos}\,B$$

CASE 1: SAS

CASE 2: SSS

This formula requires 2 sides and the angle that connects those two sides.

Determine the length of c to the nearest centimetre.



SAS

$c^2 = a^2 + b^2 - 2ab\,\text{Cos}\,C$

$c^2 = 7^2 + 5^2 - 2(7)(5)\,\text{Cos}\,43$

$c^2 = 49 + 25 - 70\,\text{Cos}\,43$

$c = \sqrt{22.80524...}$

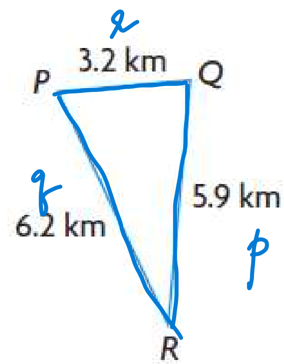
$c \approx 4.8 \approx 5\text{ cm}$

You can also use it to find angles by rearranging the cosine law if you have all three sides.

$$\boxed{\operatorname{Cos} C = \frac{c^2 - a^2 - b^2}{-2ab}}$$

Once again, if looking for a different letter, simply swap the letters to get what you need.

Determine the measure of $\angle R$ to the nearest degree.



SSS

$$\operatorname{Cos} R = \frac{r^2 - p^2 - q^2}{-2pq}$$

$$\operatorname{Cos} R = \frac{3.2^2 - 5.9^2 - 6.2^2}{-2(5.9)(6.2)}$$

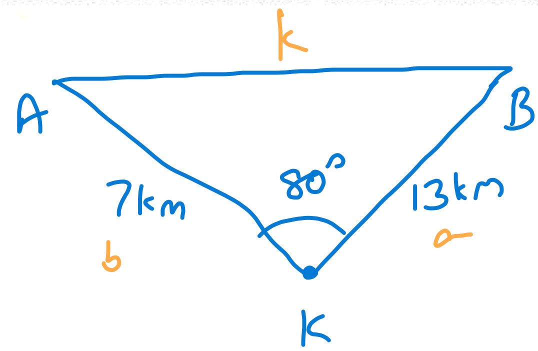
$$\operatorname{Cos} R = \frac{10.24 - 34.81 - 38.44}{-73.16}$$

$$\operatorname{Cos} R = \frac{+63.01}{+73.16}$$

$$R = \operatorname{Cos}^{-1}(0.86126\ldots)$$

$$\boxed{R \approx 31^\circ}$$

Ken's cell phone detects two transmission antennas, one 7 km away and the other 13 km away. From his position, the two antennas appear to be separated by an angle of 80°. How far apart, to the nearest kilometre, are the two antennas?



SAS

$k^2 = a^2 + b^2 - 2ab \cos K$

$k^2 = 13^2 + 7^2 - 2(13)(7)\cos 80$

$k^2 = 169 + 49 - 182 \cos 80$

$k = \sqrt{186.396031....}$

$k \approx 14\text{km}$

∴ The two antennas are 14km apart.

Success Criteria:

- I can use the sine law to solve for an unknown value in an acute triangle that **DOES NOT** have a right angle!!!!
    - *If you have a right triangle, please, please, please just use SohCahToa*
- I can use $c^2 = a^2 + b^2 - 2ab\operatorname{Cos} C$ to find an unknown side length
- I can find a different letter by simply swapping the letters around
- I can use $\operatorname{Cos} C = \dfrac{c^2 - a^2 - b^2}{-2ab}$ to find an unknown angle