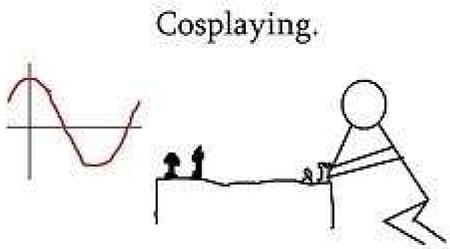
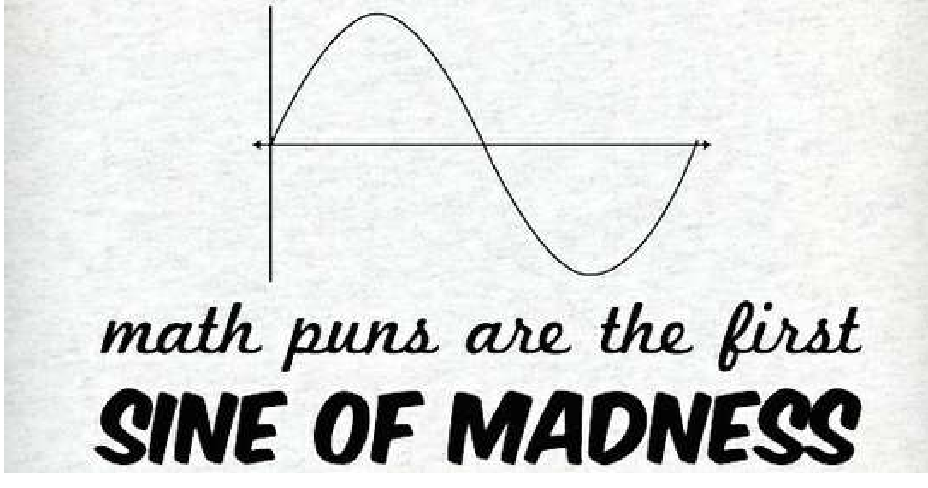
Functions & Applications 11

MCF3M

Course Notes

Chapter 6: Sinusoidal Functions





Homework

Contents with suggested problems from the Nelson Textbook. You are welcome to ask for help, from myself or your peers, with any of the following problems.

Section 6.2 – Page 330
#1, 5, 7, 8, 9

Section 6.3 – Page 339
#1, 3, 4, 6, 9

Section 6.4 – Page 348
#3ac, 4, 6, 7a-f (specify info for each graph)

Section 6.5 – Page 365
#6ac, 8, 9cf (*just graph yourself*), 10

Section 6.6 – Page 373
#4ace, 5, 7, 10, 13aef, 15

6.2: Periodic Behaviour

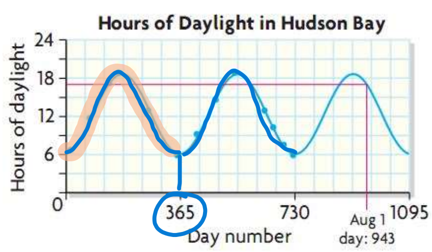
Learning Goal: We are learning to interpret and describe graphs that repeat at regular intervals.

A **PERIODIC FUNCTION** is unique in that it has a pattern in the *y-values* that repeats over time.

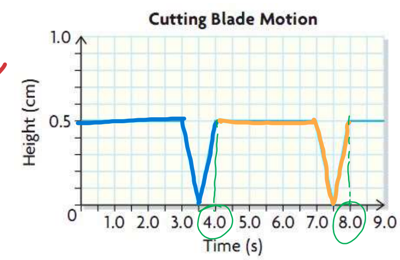
- There is a constant maximum height ← PEAK
- There is a constant minimum height ← TROUGH
- You can imagine a center line that goes through the middle ← CENTRAL AXIS
- The graph repeats in cycles that are the same size

e.g. Consider the following pictures: Determine which are periodic.

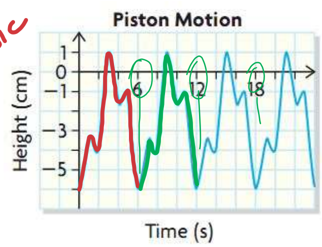
a) PERIODIC



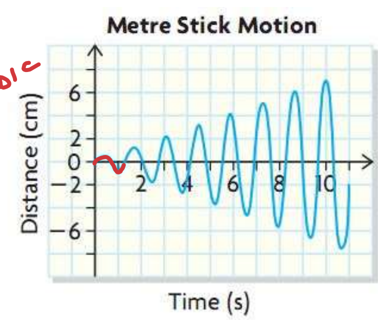
b) PERIODIC



c) PERIODIC



d) NOT PERIODIC



The **Period** of a periodic function is the amount of time that it takes one complete cycle to occur. This is determined by looking at the *x-values* of the function. In other words, looking at the domain.

Determine the periods of the above periodic functions:

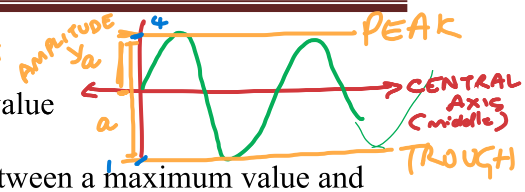
a) P = 365 days

c) P = 6 sec.

b) P = 4 sec.

d) P = not periodic

Other Unique Properties



PEAK TROUGH

a) Periodic functions have a **Maximum** value and a **Minimum** value

b) The **Amplitude** of a periodic function is half of the distance between a maximum value and the minimum value.

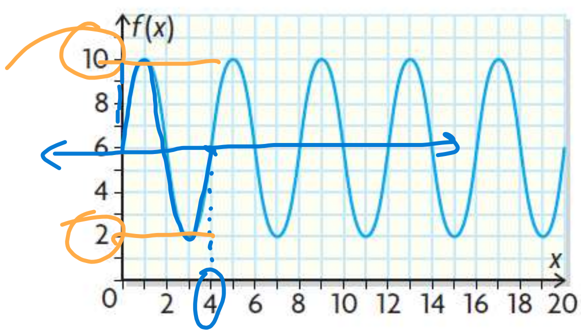
$$\boxed{\text{Amplitude} = \frac{\max - \min}{2}}$$

c) The **Central Axis** is half way between the maximum value and the minimum value. The equation of The Central Axis is given by $\boxed{y = \frac{\max + \min}{2}}$.

EoA

Example 1

Determine the period, maximum, minimum, equation of the central axis, and amplitude of the function shown.



i) P= 4 units

ii) max = 10, min = 2

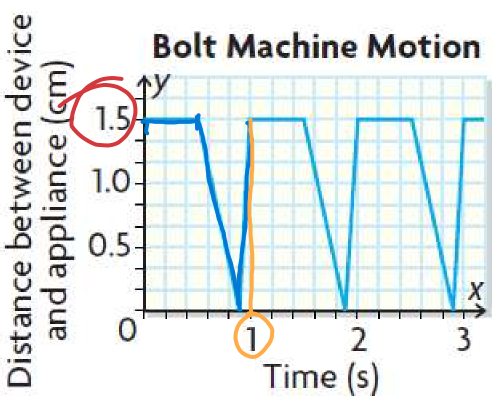
(iii) EoA: $y = 6$

(iv) $a = 4$

Example 2

The motion of an automated device for attaching bolts to a household appliance on an assembly line can be modelled by the graph shown below.

a) What is the period of one complete cycle?
b) What is the maximum distance between the device and the appliance?
c) How long does it take for the machine to complete five complete cycles?
d) Determine the equation of the central axis
e) Determine the amplitude.
f) There are several parts to each complete cycle of the graph. Explain what each part could mean in the context of "attaching the bolt".



a) $P_{1cycle} = 1$ sec.

b) max = 1.5

c) $P_{5cycles} = 5$ sec

d) $y = \frac{1.5+0}{2} = 0.75$

EoA: $y = 0.75$

e) $a = \frac{1.5-0}{2} = 0.75$

f) 1 The horizontal piece of the cycle represents the bolt machine's waiting time for the appliance to be bolted.

2 The piece with the downward slope is the bolt machine moving towards the appliance

3 The piece with the upward slope is the bolt machine going back to its original position after doing its job.

Success Criteria:

- I can find the period, maximum, minimum, central axis, and amplitude of a periodic function
- I can determine IF a function is periodic

6.3 The Sine Function

Learning Goal: We are learning how to graph the sine function and continuing our investigation into the properties of sinusoidal functions.

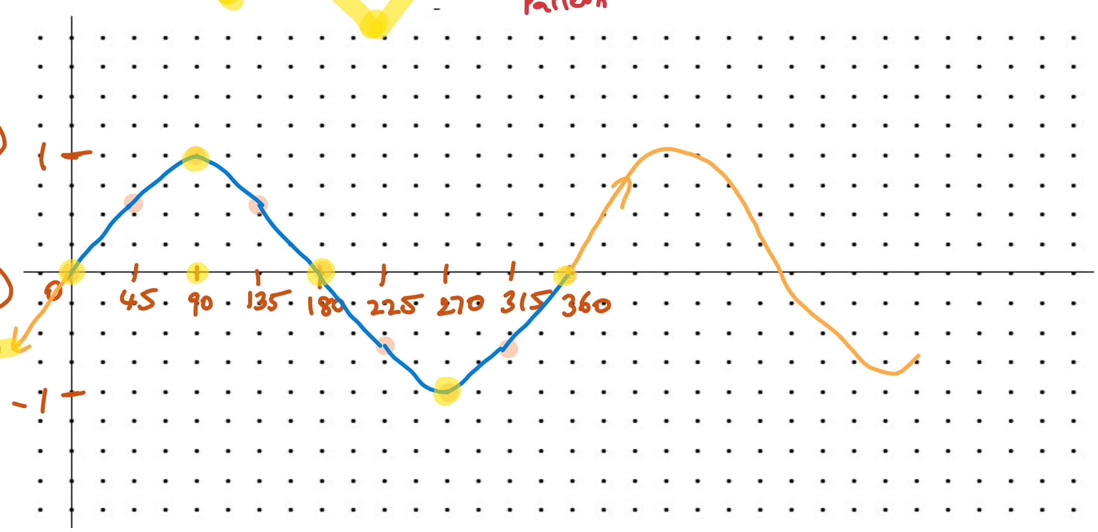
Sin, cos and tan can all be graphed! I know! Crazy! But we will be focusing on only one of these functions: $\sin x$

To graph it, lets generate a table of values for some angles $0° < x < 360°$

$f(x) = \sin x$

Sine Pattern: CA – max – CA – min – CA

x	$Y = f(x)$	$P(x, Y)$
0	sin 0	(0,0)
45	sin 45	(45,0.7)
90	sin 90	(90,1)
135	sin 135	(135,0.7)
180	sin 180	(180,0)
225	Sin225 = −0.7	
270	sin 270 = −1	
315	Sin315 = −0.7	
360	sin 360 = 0	



<u>Properties of the sine function:</u>

Maximum: 1

Minimum: −1

Equation of central axis: $y = 0$

Amplitude: 1

Period: 360°

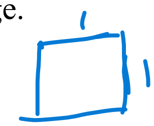
Domain: $\{x \in \mathbb{R}\}$

Range: $\{f(x) \in \mathbb{R} \mid -1 \le f(x) \le 1\}$

This is the "parent" sine function, and the properties associated with it. Since it meets all of the criteria from yesterday's lesson, it clearly models a sinusoidal function.

But let's deepen our understanding of the sine function. You already know that we use the sine ratio to solve angles and side lengths in right angle triangles (SOHCAHTOA). But it actually connects to circles too!

Let me illustrate, and this circle will also help explain the graph we got on the previous page.



The Unit Circle

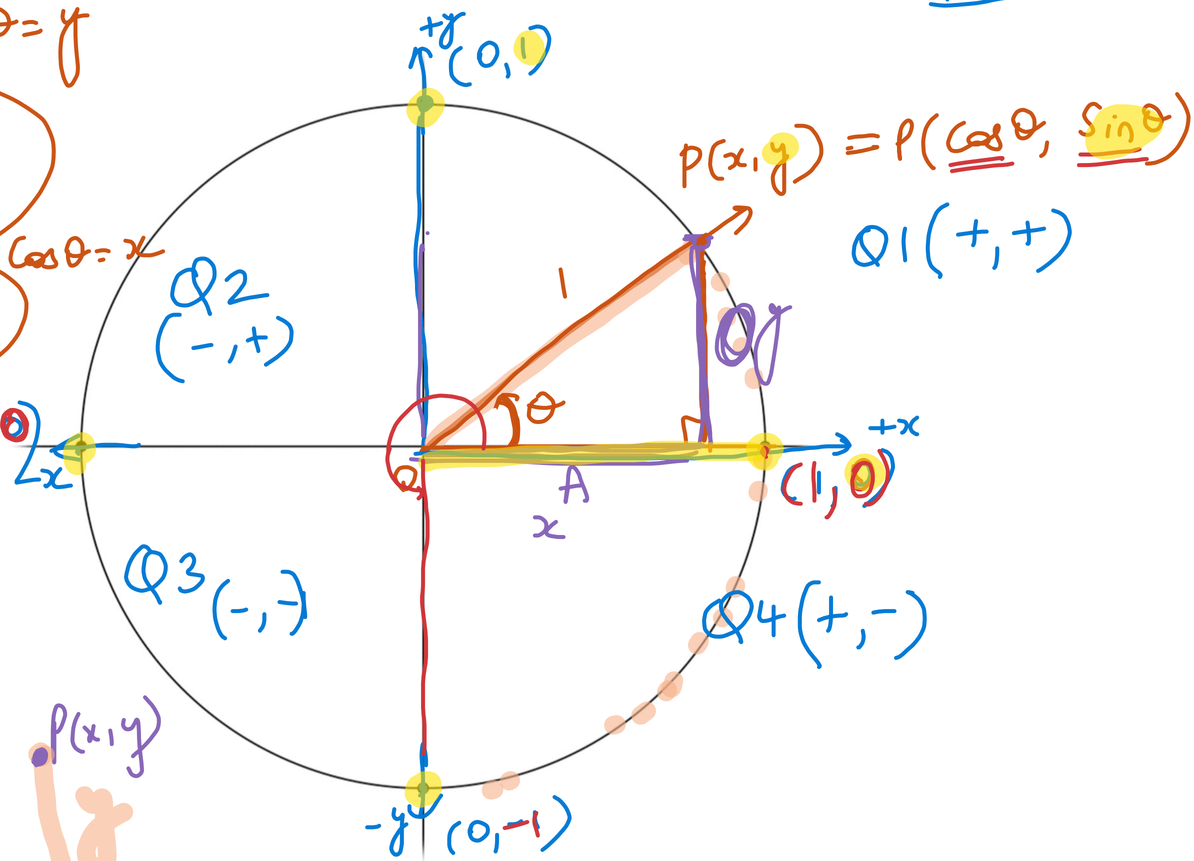
$\sin\theta = y$

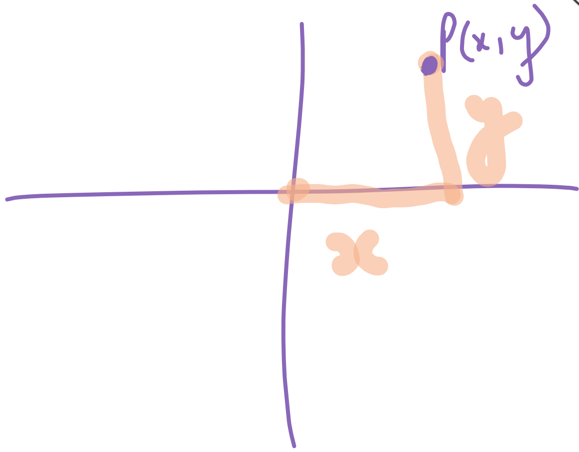
$\sin\theta = \frac{O}{H} = \frac{y}{1}$

$\cos\theta = x$

$\cos\theta = \frac{A}{H} = \frac{x}{1}$

$\tan\theta = \frac{O}{A} = \frac{y}{x}$





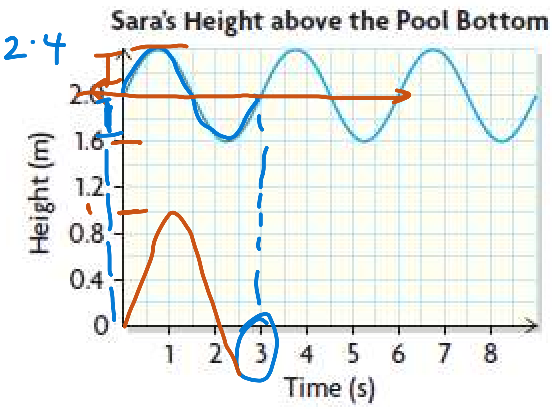
Key Points $(\theta, \sin\theta)$

$(0,0), (90,1), (180,0)$

$(270,-1), (360,0)$

We will be learning in a lesson or two, how to apply transformations to this function, just like we did to quadratics earlier in the course!

Example #1:
Sarah is sitting in an inner tube in a wave pool. The depth of the water below her in terms of time can be represented by the graph shown. What is the period, central axis, and amplitude of this graph? Can it be modelled by the sine function…if we used some transformations…?



PERIOD = 3 sec.

CENTRAL AXIS ⇒ $y = 2$

AMPLITUDE = $2 - 1.6 = 0.4$

Yes, the above can be modelled by the Sine graph with transformations like V. shift of 2 units up.

Success Criteria

- I understand that graphs that are periodic have the same shape and characteristics as the sine function
- I can recognize the properties of the sine function
  - Has an amplitude of 1
  - Has a period of 360˚
  - Its central axis is y = 0
  - Domain is $\{x \in \mathbb{R}\}$ and Range is $\{y \in \mathbb{R} | -1 \leq y \leq 1\}$
- I can recognize that the sine function passes through 5 key points

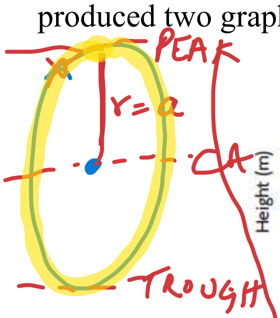
6.4 Comparing Sinusoidal Functions

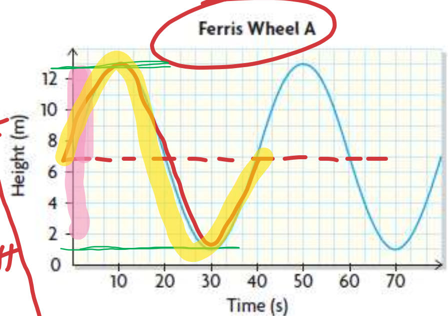
Learning Goal: We are learning to connect details of sinusoidal phenomena to their graphs.

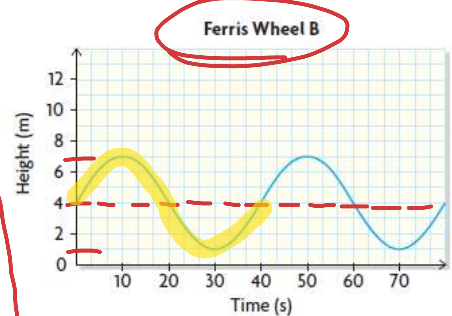
You can learn a great deal about sinusoidal functions by studying their graphs. In this lesson, we will deepen our understanding of the properties of sinusoidal functions by doing precisely that. We can compare different sinusoidal functions by looking at their:

Period (study peak-peak or trough-trough)	Amplitude
Central Axis	Min/Max

Example #1: At an amusement park, a math teacher had different students ride two Ferris wheels. Thomas rode on Ferris wheel A, and Ryan rode on Ferris wheel B. The teacher collected data and produced two graphs. Compare the two graphs by studying the four properties above.







PERIOD	40 sec.	40 sec.
E o A	$y = 7$ $\frac{13+1}{2} = \frac{14}{2}$	$y = 4$
AMPLITUDE	$a = 6$ $a = \frac{13-1}{2}$	3
MAX	13	7
MIN	1	1

Example 2: From the previous question, what is the circumference of each Ferris wheel, and what is the speed of each Ferris wheel?

$\boxed{C = 2\pi r}$

$$\boxed{\text{Speed} = \frac{\text{Distance}}{\text{Time}}}$$

$\boxed{\text{WHEEL A}}$

$r = 6$

$C = 2(3.14)(6)$

$C = 37.7$ units

$\text{Speed} = \frac{D}{t} = \frac{37.7\text{m}}{40\text{s}}$

$= 0.94$ m/s

WHEEL B

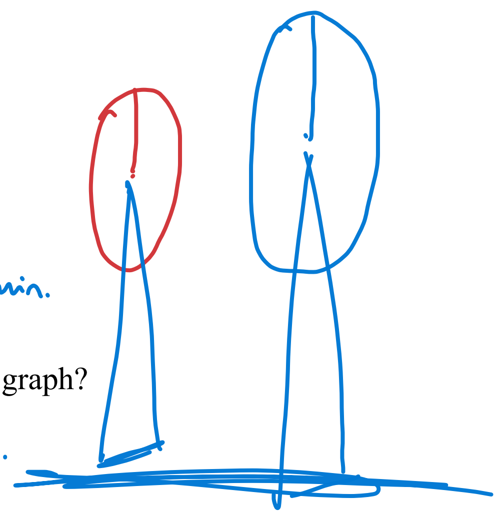
$r = 3$

$C = 2(3.14)(3)$

$C = 18.8$ units

$\text{Speed} = \frac{D}{t} = \frac{18.8\text{m}}{40\text{s}}$

$= 0.47$ m/s



Reflection Questions

How does changing the radius of the wheel affect the sinusoidal graph?

This changes the AMPLITUDE which changes the max and min.

How does changing the height of the axle of the wheel affect the sinusoidal graph?

This changes the CENTRAL AXIS which changes the max and min.

How does changing the speed of the wheel affect the sinusoidal graph?

This changes the PERIOD of the cycle.

Success Criteria:

- I can use a sinusoidal function to model or solve problems that involve circular or oscillating (back and forth) motion at a constant speed
  - I can recognize that one cycle of motion equals one period of the sinusoidal function.

6.5/6.6 Transformations of sin x

Learning Goal: We are learning how to sketch the graphs of sinusoidal functions using transformations.

The transformations of the sine function mimic the ones we learnt earlier in quadratics. The only difference is that we call them by different letters!

$f(x) = \sin x$

V. transformations

QUADRATICS

General Form of the Sine Function: $\boxed{f(x) = a\sin(x - d) + c}$

H. transformations

PARENT

$f(x) = x^2$

$g(x) = a(x-h)^2 + k$

$(x, y) \longrightarrow (x+h, ay+k)$

Transformation	Properties
$a =$ V. STRETCH when $a < 0$ $\longrightarrow$ V. FLIP (NEGATIVE) AMPLITUDE	$\boxed{a = \dfrac{\max - \min}{2}}$
$d =$ H. SHIFT (LEFT/RIGHT) [* always switch signs] PHASE SHIFT	eg $f(x) = -2\sin(x+15) - 2 \Rightarrow$ Phase Shift $= -15$ 15° left $f(x) = \sin(x - 30) + 1 \Rightarrow$ Phase Shift $=$ 30° right
(You can think of this as the starting point of the cycle.)	
$c =$ V. SHIFT. (UP/DOWN) CENTRAL AXIS	$\boxed{c = \dfrac{\max + \min}{2}}$

Example #1: Determine the amplitude, period, phase shift, and equation of the central axis for:

$f(x) = a\sin(x - d) + c$

a) $f(\theta) = \underset{a}{2}\sin\left(\theta + \underset{d}{60^\circ}\right) + \underset{c}{1}$

AMPLITUDE $= 2$

PHASE SHIFT $=$ 60° LEFT

EoA: $y = 1$

PERIOD: $P_{1\,cycle} = 360°$

b) $g(\theta) = -3\sin(\theta - 45°) + 0$

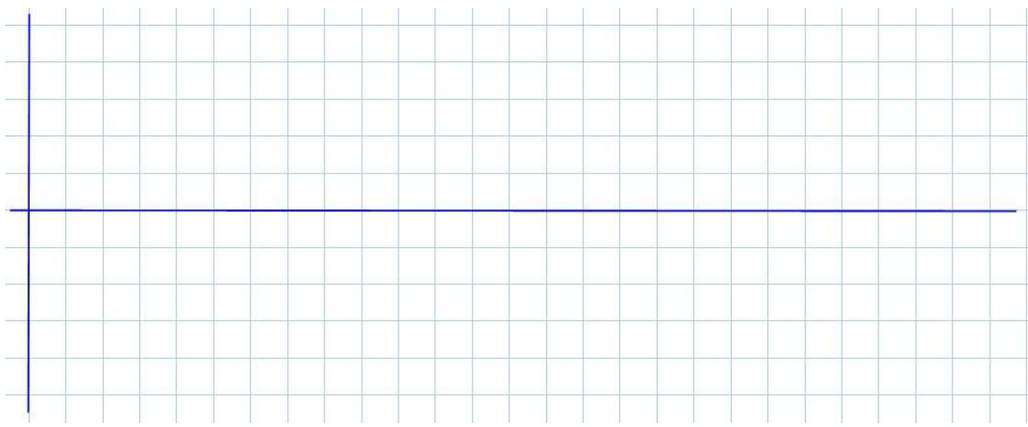
AMPLITUDE $= 3$

PHASE SHIFT $=$ 45° RIGHT

EoA: $y = 0$

PERIOD: $P_{1\,cycle} = 360°$

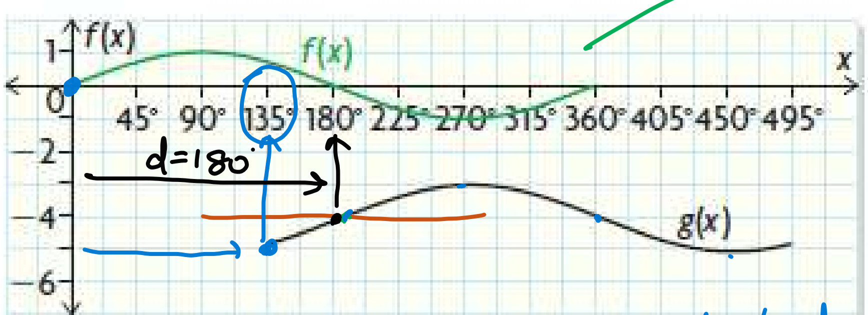
Example #2: Sketch $f(x) = \sin(x)$ and $g(x) = \sin(2x)$ for $0^\circ \le x \le 360^\circ$ on the same set of axes. Identify which properties of the sine function change with the transformation.



Example #3: Explain what transformations were applied to $f(x) = \sin x$ (green curve) to create the black curve, *g(x)*. What is the new equation of *g(x)*?

PATTERN

CA-max-CA-min-CA.



$\boxed{\text{PATTERN}}$

CA-max-CA-min-CA

AMPLITUDE = 1 = a = V. stretch but No flip.

d = 180° Right

EoA: y = -4 = c

max = -3

min = -5

$g(x) = a\ \text{Sin}(x - d) + c$

$g(x) = 1\ \text{Sin}(x - 180) - 4$

$\boxed{g(x) = \text{Sin}(x - 180) - 4}$

Example #4: Sketch $f(\theta) = -3\sin(\theta - 90) - 1$. State transformations, create tables, and state domain and range of the function.

$f(\theta) = a\sin(\theta - d) + c$

$a = -3$ V. stretch, V. flip   AMPLITUDE = 3

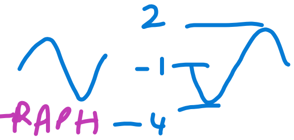
$d = 90°$ H. shift $90°$ RIGHT = PHASE SHIFT

$c = -1$ V. shift Down 1 unit   Central Axis: $y = -1$

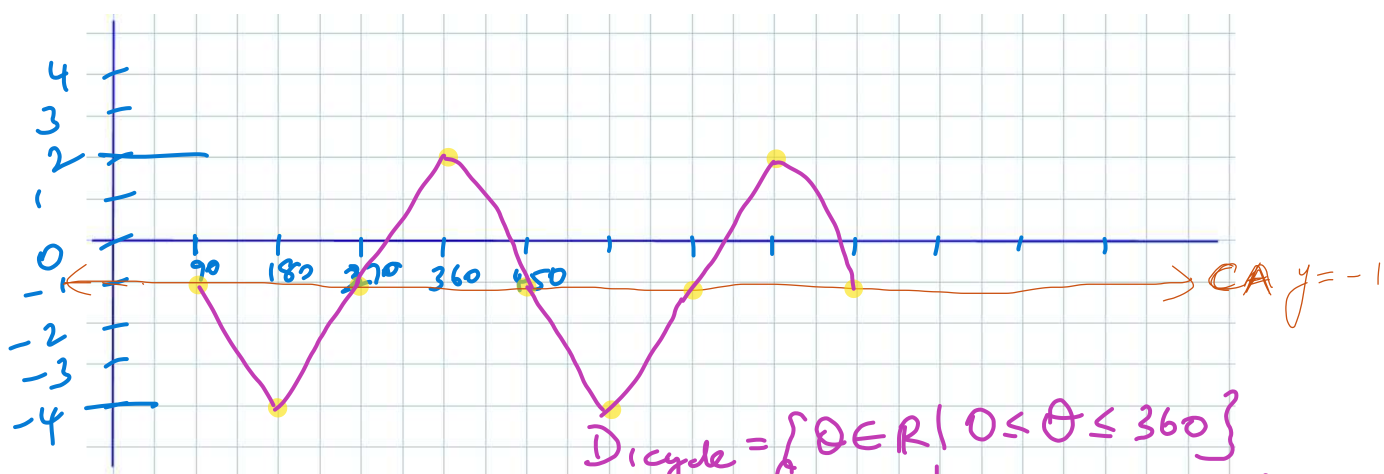
θ	y = Sinθ
0	0
90	1
180	0
270	-1
360	0

θ+d   ay+c

θ+90	-3y-1
90	-1
180	-4
270	-1
360	2
450	-1



* ALWAYS DRAW TWO CYCLES ON THE GRAPH



$D_{1cycle} = \{\theta \in R \mid 0 \le \theta \le 360\}$

$\rightarrow D_{2cycles} = \{\theta \in R \mid 0 \le \theta \le 720\}$

$R = \{f(\theta) \in R \mid -4 \le f(\theta) \le 2\}$

Success Criteria:

- I can transform the graph of the sine function in three ways
  - Vertical stretch (a)
  - Horizontal shift (b) – the opposite occurs)
  - A vertical shift (c)
- I can identify transformations by studying the equation of a sinusoidal function