Chapter 2 – Polynomial and Rational Expressions

Extra Review

Write **very good** solutions to the following problems. Your mathematics needs to be neat, showing your thinking clearly.

1. Expand and Simplify

a) $(3a-4b)(2a-5ab+3b)$

$= 6a^2 - 15a^2b + 9ab - 8ab + 20ab^2 - 12b^2$

$= 6a^2 - 15a^2b + ab + 20ab^2 - 12b^2$

b) $(3x-5)^2$

$(3x-5)(3x-5)$

$= 9x^2 - 15x - 15x + 25$

$= 9x^2 - 30x + 25$

c) $(2x+3)^3$ *

* c) $(2x+3)(2x+3)(2x+3)$

Foil these two first

$= (2x+3)(4x^2+12x+9)$

$= 8x^3 + 36x^2 + 54x + 27$

* skipped steps but I hope you are comfortable collecting like terms

2. Factor by grouping

a) $3m^3 - 15m^2 - 2m + 10$

$3m^2(m-5) - 2(m-5)$

$= (m-5)(3m^2-2)$

* b) $x^2 - 10x + 25 - 4y^2$ (*hint – square minus square*)

* This one was not so obvious but it is a beautiful question!

$(x-5)^2 - (2y)^2$

$= (x-5+2y)(x-5-2y)$

3. Explain in your own words why we MUST state restrictions on Rational Expressions before doing any cancelling.

If you cancel before stating the restrictions, you might miss out a factor in the denominator that could potentially be zero. We can never divide by zero and all possibilities must be eliminated.

4. Simplify the following rational expressions:

a) $\dfrac{x^2-7x-60}{x^2-4}\times\dfrac{14-7x}{(x-12)^2}$

$\dfrac{(x-12)(x+5)}{(x-2)(x+2)} \times \dfrac{-7(x-2)}{(x-12)(x-12)}$

$= \dfrac{-7(x+5)}{(x+2)(x-12)}$

$x \neq 2, -2, 12$

b) $\dfrac{-3x^2+108}{3x^2+2x-5}\times\dfrac{3x^2+x-10}{x^2+8x+12}$

$\dfrac{-3(x^2-36)}{3x^2+5x-3x-5} \times \dfrac{3x^2+6x-5x-10}{(x+6)(x+2)}$

$x \neq -\frac{5}{3}, 1, -6, -2$

$= \dfrac{-3(x+6)(x-6)}{(3x+5)(x-1)} \times \dfrac{(x+2)(3x-5)}{(x+6)(x+2)}$

$= \dfrac{-3(x-6)(3x-5)}{(3x+5)(x-1)}$

c) $\dfrac{x^2-10x+25}{2x^2-11x+5}\div\dfrac{x^2+7x+12}{2x^2+5x-3}$

* requires restrictions from numerator too!!

$= \dfrac{(x-5)(x-5)}{(2x-1)(x-5)} \div \dfrac{(x+3)(x+4)}{(2x-1)(x+3)}$

$x \neq \frac{1}{2}, 5, -3, -4$

$= \dfrac{(x-5)(x-5)}{(2x-1)(x-5)} \times \dfrac{(2x-1)(x+3)}{(x+3)(x+4)}$

$= \dfrac{x-5}{x+4}$

d) $\dfrac{2x}{x^2+x-6}+\dfrac{x-4}{x^2+2x-3}$

$\dfrac{2x}{(x+3)(x-2)} + \dfrac{(x-4)}{(x+3)(x-1)}$

LCD $(x+3)(x-2)(x-1)$

$x \neq -3, 2, 1$

$= \dfrac{2x(x-1)+(x-4)(x-2)}{(x+3)(x-2)(x-1)}$

$= \dfrac{2x^2-2x+x^2-6x+8}{(x+3)(x-2)(x-1)} = \dfrac{3x^2-8x+8}{(x+3)(x-2)(x-1)}$

e) $\dfrac{2}{x^2+4x-60}-\dfrac{3}{x^2-36}$

$= \dfrac{2}{(x+10)(x-6)} - \dfrac{3}{(x-6)(x+6)}$

LCD $(x+10)(x-6)(x+6)$

$x \neq -10, 6, -6$

$= \dfrac{2(x+6)-3(x+10)}{(x+10)(x-6)(x+6)}$

$= \dfrac{2x+12-3x-30}{(x+10)(x-6)(x+6)} = \dfrac{-x-18}{(x+10)(x-6)(x+6)}$

f) $\dfrac{2x}{x^2-x-2}+\dfrac{x^2+3x-10}{x^2-5x+6}\times\dfrac{3x+12}{x^2+2x-8}$

BEDMAS!

$= \dfrac{2x}{(x-2)(x+1)} + \dfrac{(x+5)(x-2)}{(x-3)(x-2)} \times \dfrac{3(x+4)}{(x+4)(x-2)}$

$= \dfrac{2x}{(x-2)(x+1)} + \dfrac{3(x+5)}{(x-3)(x-2)}$

LCD: $(x-2)(x+1)(x-3)$

$x \neq 2, -1, 3$

$= \dfrac{2x(x-3)+3(x+5)(x+1)}{(x-2)(x+1)(x-3)}$

$= \dfrac{2x^2-6x+3x^2+18x+15}{(x-2)(x+1)(x-3)} = \dfrac{5x^2+12x+15}{(x-2)(x+1)(x-3)}$

Rational Expressions

Name_____

Practice Test

Date_____

Simplify each expression. State the restrictions.

1) $\frac{x-7}{x^2-12x+35} \cdot \frac{x^2-6x-7}{x-7}$

$= \frac{(x-7)}{(x-7)(x-5)} \times \frac{(x-7)(x+1)}{(x-7)}$

$x \neq 7, 5$

$= \frac{(x+1)}{(x-5)}$; $x \neq 7, 5$

2) $\frac{7v+49}{8v^2} \div \frac{v^2+3v-28}{8v^2}$

$= \frac{7(v+7)}{8v^2} \div \frac{(v+7)(v-4)}{8v^2}$

$= \frac{7(v+7)}{8v^2} \times \frac{8v^2}{(v+7)(v-4)}$

$v \neq 0, -7, 4$

$= \frac{7}{v-4}$; $v \neq 0, -7, 4$

3) $\frac{1}{n-5} \div \frac{n-7}{8n-56}$

$= \frac{1}{n-5} \times \frac{8(n-7)}{n-7}$; $n \neq 7, 5$

$= \frac{1 \times 8}{n-5}$

$= \frac{8}{n-5}$

$n \neq 5$

$n \neq 7$ ∴

4) $\frac{5 \times 6}{5(n-1)} - \frac{8(n-1)}{5(n-1)}$

$= \frac{30-8n+8}{5(n-1)}$

$= \frac{38-8n}{5(n-1)}$; $n \neq 1$

5) $\dfrac{1}{x+3} \div \dfrac{7}{7x-35}$

$\dfrac{1}{x+3} \div \dfrac{7}{7(x-5)}$

$= \dfrac{1}{(x+3)} \times \dfrac{7(x-5)}{7}$

$= \dfrac{(x-5)}{(x+3)}$; $x \neq -3, 5$

6) $\dfrac{v+3}{v-7} + \dfrac{7}{6v}$

$\dfrac{6v(v+3)}{6v(v-7)} + \dfrac{7(v-7)}{6v(v-7)}$

$= \dfrac{6v(v+3)+7(v-7)}{6v(v-7)}$

$= \dfrac{6v^2+18v+7v-49}{6v(v-7)}$

$= \dfrac{6v^2+25v-49}{6v(v-7)}$; $v \neq 0, 7$.

7) $\dfrac{7}{3} - \dfrac{r+6}{8r^2+4r}$

$= \dfrac{7}{4r(2r+1)\times 3} - \dfrac{(r+6)\times 3}{4r(2r+1)\times 3}$

$= \dfrac{28r(2r+1) - 3(r+6)}{12r(2r+1)}$

$= \dfrac{56r^2+28r-3r-18}{12r(2r+1)}$

$= \dfrac{56r^2+25r-18}{12r(2r+1)}$; $r \neq 0, -\frac{1}{2}$

8) $\dfrac{7r}{4r+24} + \dfrac{5r}{2r}$

$= \dfrac{7r}{4(r+6)} + \dfrac{5r \times 2(r+6)}{2r \times 2(r+6)}$

$= \dfrac{7r+10(r+6)}{4(r+6)}$

$= \dfrac{7r+10r+60}{4(r+6)}$

$= \dfrac{17r+60}{4(r+6)}$; $r \neq -6$

9) $\dfrac{x+7}{9x^2-18x} \div \dfrac{4x^2}{9x^2-18x}$

$\dfrac{(x+7)}{9x(x-2)} \div \dfrac{4x^2}{9x(x-2)}$

$\dfrac{(x+7)}{9x(x-2)} \times \dfrac{9x(x-2)}{4x^2}$; $x \neq 0, 2$

$\dfrac{x+7}{4x^2}$

10) $\dfrac{x-3}{6x-48} - 5$

$\dfrac{x-3}{6(x-8)} - 5$

$= \dfrac{x-3-30(x-8)}{6(x-8)}$

$x \neq 8$ $= \dfrac{x-3-30x+240}{6(x-8)}$

$= \dfrac{-29x+237}{6(x-8)}$