

Chapter 1 – Introduction to Functions

Review

You need to know: (read your notes!!)

From Section 1.1 – Introduction to Functions

- The definition of a function and the difference between a function and just a relation (e.g. the Vertical Line Test).
- Know what the graph of a function is (it's a set of points, not a picture!).
- Domain and Range of functions (and plain old relations too).

From 1.2 – Function Notation

- Know Function Notation (i.e. $f(x)$).
- How to find range values given domain values for any function (example 1.2.1).

From 1.3 – 1.4 – Parent Functions and Domain and Range

- Know the 5 basic parent functions, their basic shape, table of values, and their domains and ranges (See the chart on the first page of those notes).
- Know how to determine the domain and range of a function (relation) given the sketch of the graph.
- Know how to state domain and range in “Set Notation” (including how to deal with “restrictions” for domain and range).

From 1.6 – 1.8 (parts 1 and 2) – Transformations of Functions

- Know the three main types of transformations, and that those transformations can be applied both **horizontally (inside a parent function)** and **vertically (outside a parent function)**.
- Know how to state all transformations of a given function (including stating the parent function).
- Know how to find “**image points**” for transformed functions (see Example 1.8.5 from Part 2).
- Know how to sketch parent and transformed functions.
- **Remember, horizontal transformations are always “opposite”.**

1. What is a function? (use a formal definition or use your own words)

A function is a relation where every input value has one and only one output value.

2. For the following: state the domain, the range, and whether it is a function. If it is not, explain why it isn't.

a) $\{(2,4), (3,5), (4,6), (5,7), (8,9), (12,13)\}$

$D = \{2, 3, 4, 5, 8, 12\}$
 $R = \{4, 5, 6, 7, 9, 13\}$

✓ function

b) $\{(5,2), (8,1), (9,5), (3,7), (8,9), (1,0)\}$

$D = \{5, 8, 9, 3, 1\}$
 $R = \{2, 1, 5, 7, 9, 0\}$

✗ function because input 8 has two output values 1, 9.

3. Let $f(x) = 4x - 9$.

a) Evaluate $f(5)$



$x = 5$
 $\Rightarrow f(x) = 4(5) - 9 = 20 - 9 = 11$

b) Evaluate $f(10) - f(9)$

$f(10) = 31$
 $f(9) = 27$
 $f(10) - f(9) = 31 - 27 = 4$

c) Find the x so that $f(x) = -5$



$f(x) = 4x - 9$
 $-5 = 4x - 9$
 $-5 + 9 = 4x$
 $4 = 4x \Rightarrow x = 1$

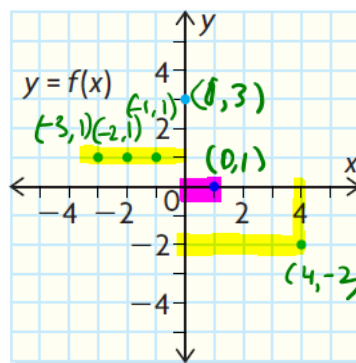
4. Use the graph to answer the following:

a) $f(4) = -2$ b) $f(x) = 0$

$x = 1$

c) $f(x) = 1$

$\Rightarrow x = -1, -2, -3$



d) Is this graph a function?

passes VLT \therefore It is a function.

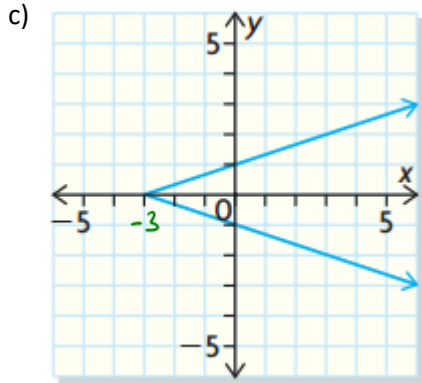
5. Find domain and range of each:

a) $f(x) = -3\sqrt{2x-8} + 5$

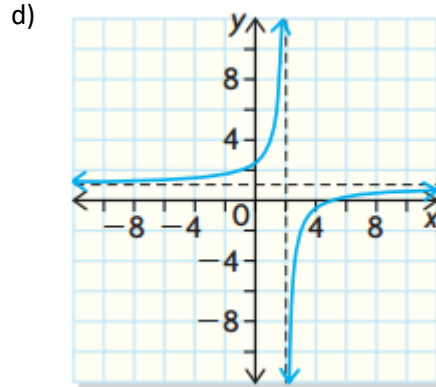
$2x - 8 \geq 0$
 $2x \geq 8$
 $x \geq 4$
 $D = \{x \in \mathbb{R} \mid x \geq 4\}$
 $R = \{f(x) \in \mathbb{R} \mid f(x) \leq 5\}$

b) $f(x) = 2(x+3)^2 - 7$

$D = \{x \in \mathbb{R}\}$
 $R = \{f(x) \in \mathbb{R} \mid f(x) \geq -7\}$



$D = \{x \in \mathbb{R} \mid x \geq -3\}$
 $R = \{f(x) \in \mathbb{R}\}$



$D = \{x \in \mathbb{R} \mid x \neq 2\}$
 $R = \{f(x) \in \mathbb{R} \mid f(x) \neq 1\}$

6. State the transformations and create the table of values you would use to graph these functions:

a) $f(x) = 5\sqrt{-\frac{1}{2}x - 3} + 8$

$= 5\sqrt{-\frac{1}{2}(x+6)} + 8$

$k = -\frac{1}{2}$

$\therefore \frac{1}{k} = \frac{2}{-1} = -2 = -2$

(x, y)

$(\frac{1}{k}x + d, ay + c)$

x	y
0	0
1	1
4	2
9	3
16	4

$y = \sqrt{x}$

$-2x-6$	$5y+8$
-6	8
-8	13
-14	18
-24	23
-38	28

H. transformations

$\frac{1}{k} = -2$
 $d = -6$
 H. Stretch by -2
 * H. Flip
 H. Shift 6 units left
 (bcz -6)

V. transformation

V. stretch by 5 $a=5$
 V. shift 8 units up.
 $c=8$

b) $g(x) = -7|4(x-6)| - 3$

H. Transformation	V. Transformation
$\frac{1}{k} = \frac{1}{4}$ H. stretch by $\frac{1}{4}$	$a = -7$ V. stretch by -7 * V. Flip
$d = 6$ H. shift 6 units right	$c = -3$ V. shift 3 units down. (bcz (-)ve)

$(x, y) \rightarrow (\frac{1}{k}x + d, ay + c)$

x	y	$\frac{1}{4}x + 6$	$-7y - 3$
-2	2	$0.25(-2) + 6$	-17
-1	1	5.5	-10
0	0	5.75	-3
1	1	6	-10
2	2	6.25	-17

$f(x) = \sqrt{x}$

7. A square root function has the following transformations applied to it:

- Vertical stretch of 4 $\rightarrow a = -4$
- Vertical Flip
- Vertical Shift of 8 up $\rightarrow c = 8$
- Horizontal Stretch of $\frac{1}{3}$ $\rightarrow \frac{1}{k} = \frac{1}{3} \Rightarrow k = 3$
- Horizontal Shift of 2 left $\rightarrow d = -2$

Write out the function.

General Transformed function:

$$g(x) = a f[k(x-d)] + c$$

$$\Rightarrow g(x) = -4 f(3(x - -2)) + 8$$

$$\Rightarrow g(x) = -4 f(3(x+2)) + 8$$

$$\Rightarrow g(x) = -4 \sqrt{3(x+2)} + 8$$

When studying, really know the parent functions and all their properties.