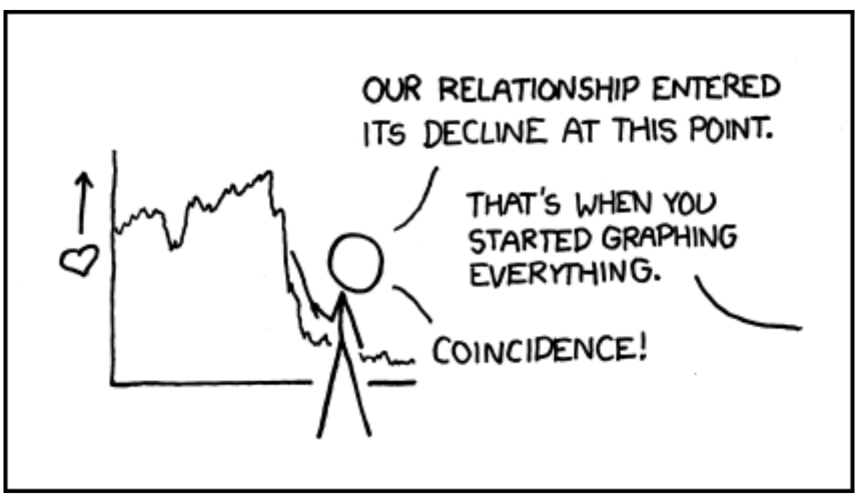
Functions 11

Course Notes

Chapter 1 – Introduction to Functions

We will learn

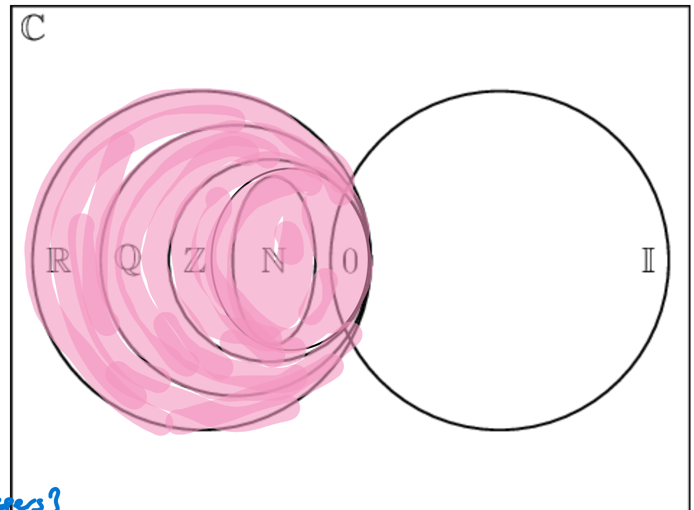
- *the meaning of the term Function and how to use function notation to calculate and represent functions*
- *the meanings of the terms domain and range, and how a function's structure affects domain and range*
- *how to use transformations to represent and sketch graphs*



1.1 Relations and Functions (*This is a KEY lesson!*)

Learning Goal: We are learning to recognize functions in various representations.

This course is called **FUNCTIONS**, so it seems rather important that you know what a function actually is. However, before we define and dive into the world of functions, it is important to be familiar with a few other commonly used terms in the language of functions.



Definition 0.a.

A **Set** is a well-defined collection of things or numbers.

Natural #s $\equiv \mathbb{N} = \{1, 2, 3, 4, \ldots\ldots\}$

Whole #s $\equiv \mathbb{W} = \{0, 1, 2, 3, 4, \ldots\}$

Integers $\equiv \mathbb{Z} = \{\ldots -2, -1, 0, 1, 2, \ldots\}$

Rational #s $\equiv \mathbb{Q} = \{\frac{p}{q};\ p \text{ and } q \text{ are integers}, q \neq 0\}$

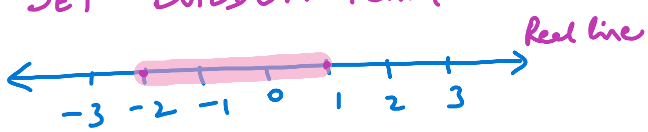
Irrational #s $\equiv \mathbb{Q}' = \{\text{All #s not rational}\}$

* Real #s $\equiv \mathbb{R} = \{\text{all rational and irrational #s}\}$

Definition 0.b. Set Notations: *Some fancy ways to represent sets of Numbers!*

① SET BUILDER FORM

eg



$\{x \mid x \in \mathbb{R};\ -2 \leq x \leq 1\}$

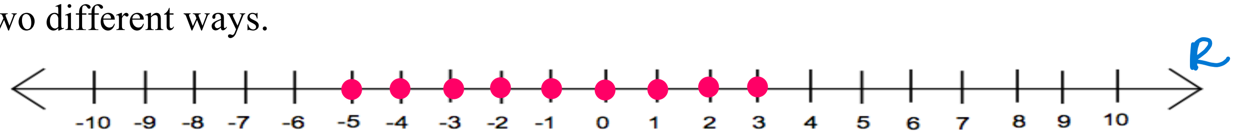
such that — belongs to — $x > -2;\ x = -2;\ x < 1;\ x = 1$

② ROSTER FORM

eg {☼, ☾, ☆}

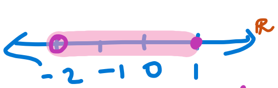
list out all #s within { }

- *Example:* Represent the given set of numbers highlighted on the number line in two different ways.



① ROSTER $= \{-5, -4, -3, -2, -1, 0, 1, 2, 3\}$

② SET-BUILDER $= \{x \in \mathbb{Z} \mid -5 \leq x \leq 3\}$



$\{x \mid x \in \mathbb{R};\ -2 < x \leq 1\}$

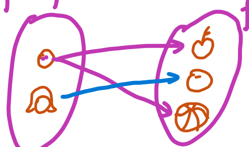
or $\{x \in \mathbb{R} \mid -2 < x \leq 1\}$

Definition 0.c.

A **Relation** is a mathematical relationship between elements of one set with the elements of another set.

Relations can be represented in different ways like an arrow diagram, an equation, a table of values, a set of ordered pairs, or a graph.

people fruits



(1) Arrow diagram

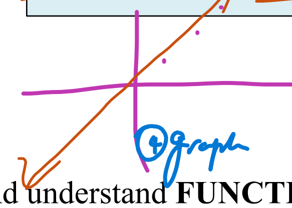
(2) Equation

$$\boxed{y = 5x}$$

Here, y related to x

(3) Table of values

x	y
1	1
2	2
3	3



(4) graph

$\{(5,7), (2,9)\}$

(5) ordered pairs

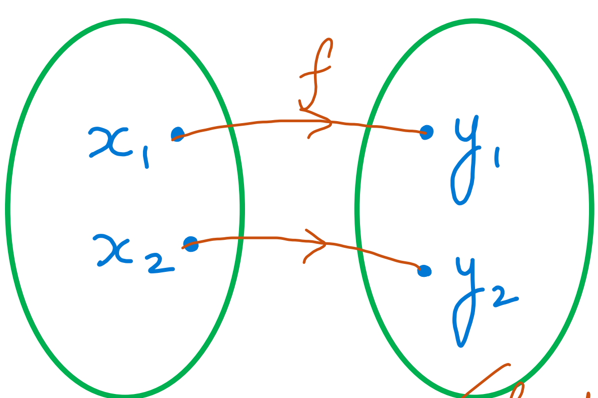
Now comes the moment finally!! We are ready to explore and understand **FUNCTIONS**! 😊

You need to know, very well, the following (algebraic) definition:

Definition 1.1.1

A **FUNCTION** is a very special relation where every element of one set (input) is related to one and only one element of the other set (output)

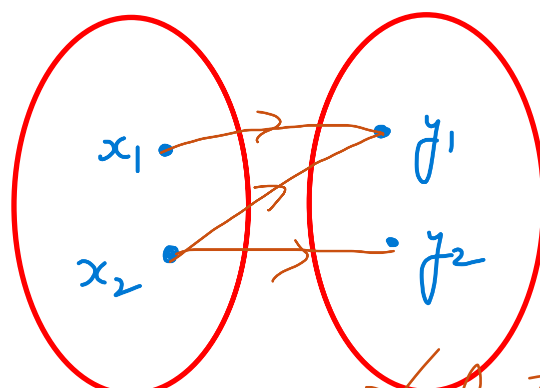
We can visualize what a function is (and **isn't**) by using so-called "**arrow diagrams**":



function

D_f : $\{x_1, x_2\}$

R_f : $\{y_1, y_2\}$



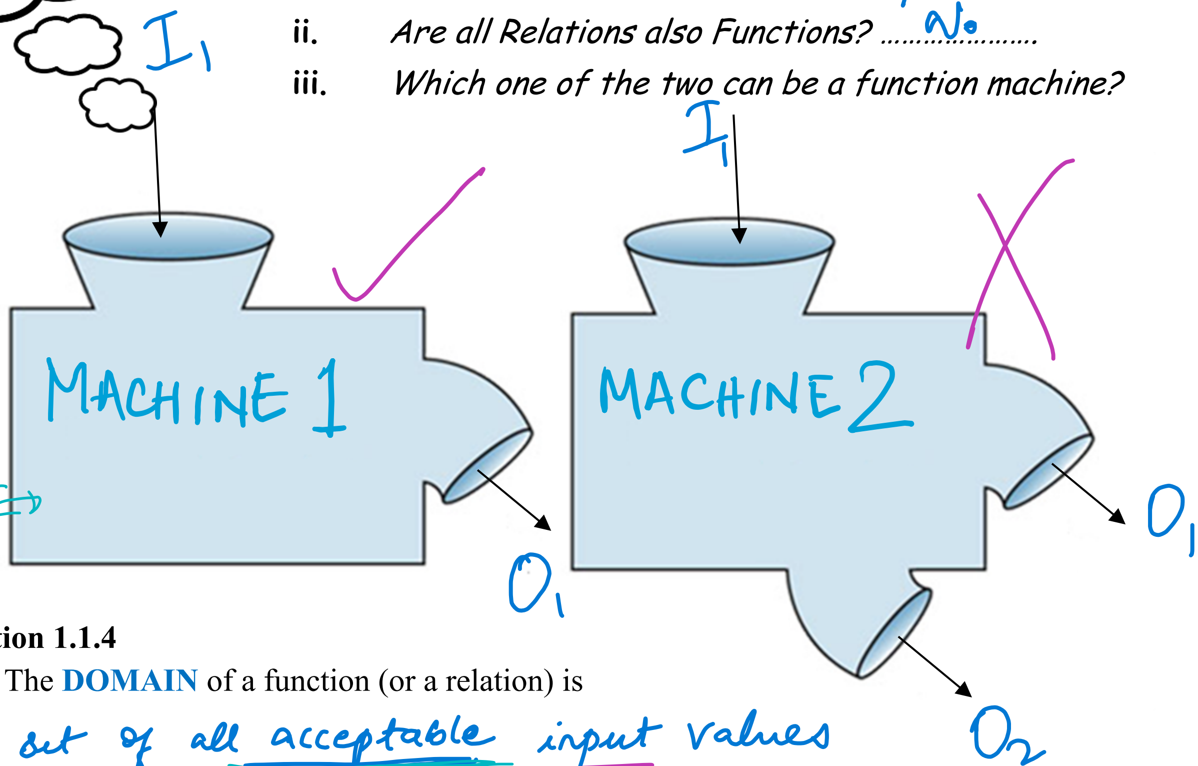
function

D_g : $\{x_1, x_2\}$

R_g : $\{y_1, y_2\}$

We can also view ***Functions*** visually like a ***Vending Machine*** because they are ***PREDICTABLE*** just like our functions***!!***

- **THINKING QUESTION:**
  - i. *Are all Functions also Relations?* ......Yes......
  - ii. *Are all Relations also Functions?* ......No......
  - iii. *Which one of the two can be a function machine?*



1^3 $\sqrt[3]{-27}$

x $(-3)^3 = -27$

$-3 = \sqrt[3]{-27}$

$\sqrt[3]{x}$

Definition 1.1.4

The **DOMAIN** of a function (or a relation) is

the set of all acceptable input values

$f(x) = \frac{1}{x}$ $\quad 5 \to \frac{1}{5}$ $\quad 9 \to \frac{1}{9}$ $\quad 0 \to \frac{1}{0}$ × Not defined ∴ Not acceptable

(x-values)

*independent variable

Definition 1.1.5

The **RANGE** of a function (or a relation) is

the set of all acceptable output values.

(y-values)

*dependent

R

$y = 10^x$

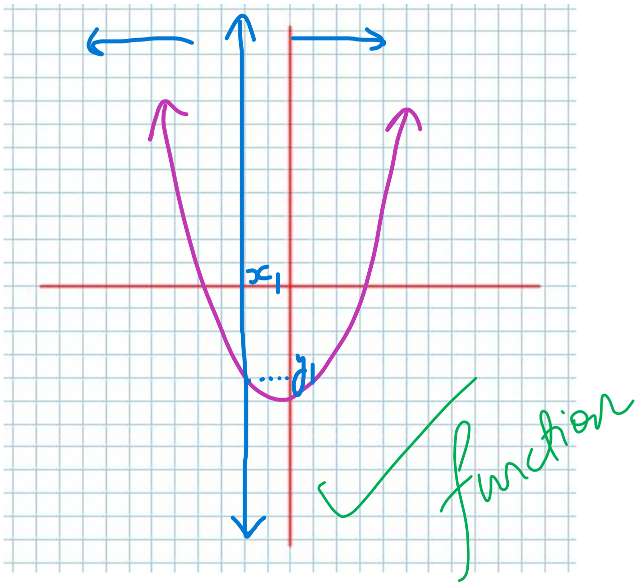
Two other important terms to know are:

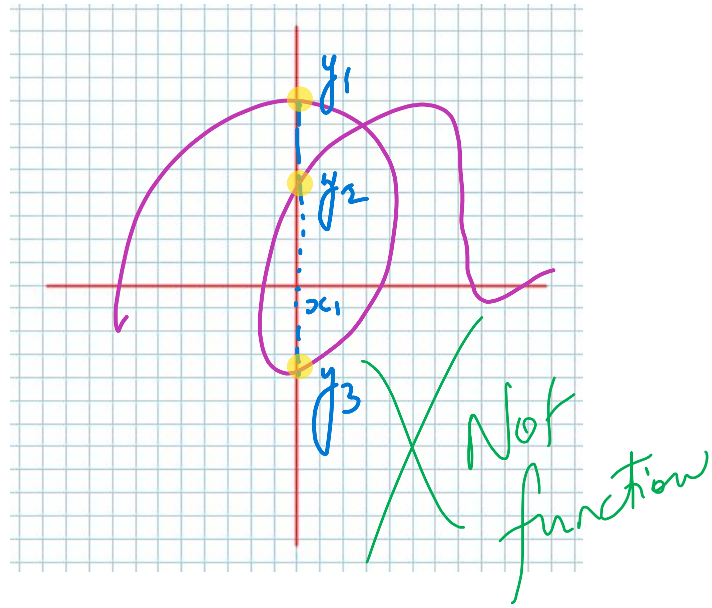
1) The **INDEPENDENT VARIABLE** $\quad x$

2) The **DEPENDENT VARIABLE** $\quad y$

KNOWING WHEN A RELATION IS, AND ISN'T, A FUNCTION

Graphically: *The Vertical Line Test* V.L.T.





Algebraically: (*NOTE: this is a "rough" way of thinking about the problem*)

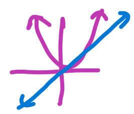
If the Dependent Variable ~~is~~ has an odd exponent, then it is a FUNCTION!

e.g. ① $y^3 = 3x^2 + 2$ odd ✓ function

② $y^6 = 2x^5 + 3x^4 + 1$ even ✗ not function

$y^1 = x^2$

$y^1 = x$



① $y^2 = x$

Not function

x	y
0	0
1	1
1	−1

HW- Section 1.1 (*Suggested problems are from the Nelson Textbook. You are welcome to ask for help from your peers or myself. It is due the next day!*)

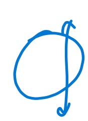
Pg. 10 – 12 #1, 2 (*no ruler needed*), 6, 7 (*no need for the VLT*), 9, 11, 12 (*think carefully about the idea that the domain and range are "limited"*)

Success Criteria:

- I can determine the domain of a relation or function as the set of all values of the independent variable
- I can determine the range of a relation or function as the set of all values of the dependent variable
- I can apply the vertical line test to determine if a graph is a function
- I can recognize whether a relation is a function from its equation

② $x^2 + y^2 = 4$

Not function



V.L.T. Not funct!

1.2 Function Notation

Learning Goal: We are learning to use function notation to represent linear and quadratic functions

Here we learn a **NEW AND IMPROVED WAY** for describing a function, algebraically. You have been using the following form for functions (in this example, for a quadratic):

$$\boxed{y = 3(x-2)^2 + 1}$$

A much more useful way of writing function is to use **FUNCTION NOTATION**. The above quadratic (***which we call a "function of x" because the domain is given as x-values***) can be written as:

$$f(x) = 3(x-2)^2 + 1$$

$$f(2) = 3(2-2)^2 + 1 = 1$$

This new notation is so useful because of the "symbol": $\boxed{f(x) = y}$

This notation shows **BOTH** the **DOMAIN** and the **RANGE** values. Because of that, the function notation shows us **points** on the graph of the function.

$$(x, y) \equiv (x, f(x))$$

Let's do some examples (from your text on pages 23 – 24)

Example 1.2.1

4. Evaluate $f(-1)$, $f(3)$, and $f(1.5)$ for

a) $\boxed{f(x) = (x-2)^2 - 1}$ **b)** $f(x) = 2 + 3x - 4x^2$

$f(-1) = (-1-2)^2 - 1$

$\boxed{f(-1) = 8}$

$f(3) = (3-2)^2 - 1$

$\boxed{f(3) = 0}$

$f(1.5) = (1.5-2)^2 - 1$

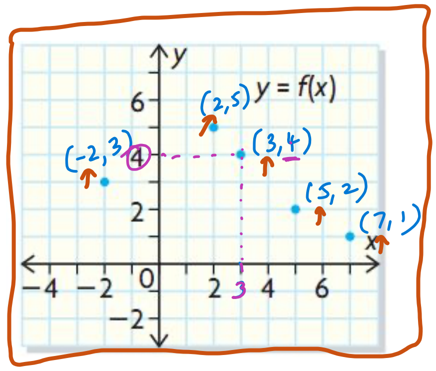
$f(1.5) = (-0.5)^2 - 1$

$= 0.25 - 1.00$

$f(1.5) = -0.75$

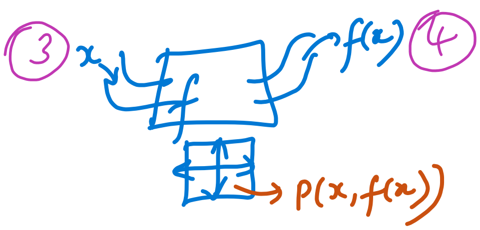
Example 1.2.2

6. The graph of $y = f(x)$ is shown at the right.
   a) State the domain and range of f.
   b) Evaluate.
      i) $f(3)$ iii) $f(5-3)$
      ii) $f(5)$ iv) $f(5) - f(3)$



a) $D_f = \{-2, 2, 3, 5, 7\}$

$R_f = \{1, 2, 3, 4, 5\}$



b) (i) $f(3) = 4$

(ii) $f(5) = 2$

(iii) $f(5-3) = f(2) = 5$

(iv) $f(5) - f(3) = 2 - 4 = -2$

Example 1.2.3

11. For $g(x) = 4 - 5x$, determine the input for x when the output of $g(x)$ is

a) -6 b) 2

a) $g(x) = 4 - 5x$

$\Rightarrow -6 = 4 - 5x$

$\Rightarrow -6 - 4 = -5x$

$\Rightarrow -10 = -5x$

$\Rightarrow \boxed{x = 2}$

b) $2 = 4 - 5x$

$\Rightarrow 2 - 4 = -5x$

$\Rightarrow -2 = -5x$

$\Rightarrow \frac{-2}{-5} = x$

$\Rightarrow \boxed{x = \frac{2}{5}} = 0.4$

Example 1.2.4

7. For $\boxed{h(x) = 2x - 5}$ determine

a) $h(a)$ **c)** $h(3c - 1)$

b) $h(b + 1)$ **d)** $h(2 - 5x)$

a) $h(a) = 2a - 5$

b) $h(b+1) = 2(b+1) - 5$
$= 2b + 2 - 5$
$= 2b - 3$

c) $h(3c-1) = 2(3c-1) - 5$
$= 6c - 2 - 5$
$= 6c - 7$

d) $h(2-5x) = 2(2-5x) - 5$
$= 4 - 10x - 5$
$= -10x - 1$

Example 1.2.5

12. A company rents cars for \$50 per day plus \$0.15/km.

a) Express the daily rental cost as a function of the number of kilometres travelled.

b) Determine the rental cost if you drive 472 km in one day. x

c) Determine how far you can drive in a day for \$80. $f(x)$

a) $f(x) = 0.15x + 50$

where
x = distance in km
$f(x)$ = daily rental cost

b) $f(472) = 0.15(472) + 50$
$= 70.8 + 50$
$= \$120.80$

c) $80 = 0.15x + 50$
$\Rightarrow 80 - 50 = 0.15x$
$\Rightarrow 30 = 0.15x$
$\Rightarrow \dfrac{30}{0.15} = x$
$\Rightarrow x = 200\text{ km}$

HW- Section 1.2

Page 23 #1-2, 5, 8b, 10, 11cd, 15, 16, challenge #17

Success Criteria:

- I can evaluate functions using function notation, by substituting a given value for x in the equation for f(x)
- I can recognize that f(x) = y corresponds to the coordinate (x, y)
- I can, given y = f(x), determine the value of x

1.3 and 1.4 Parent Functions and Domain and Range

Learning Goal: We are learning the graphs and equations of five basic functions; and using their tables, graphs, or equations to find their domains and ranges.

Two **INCREDIBLY IMPORTANT** aspects of functions are their

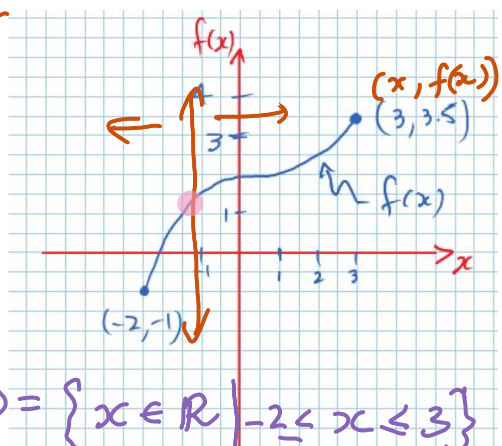
Again, the Domain is the set of all acceptable input values (x-values)

And, the Range is the set of all acceptable output values (y-values)

Example 1.4.1

Given the **SKETCH OF THE GRAPH** of the **RELATION** determine: the domain, the range of the relation, and whether the relation is, or is not, a function. *(Note that domain and range are sets of numbers and can be represented by the fancy* ***set notation****)*

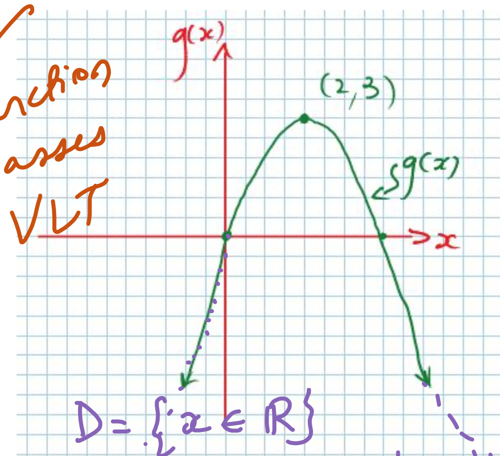
a) ✓ function passes VLT



$D = \{x \in \mathbb{R} \mid -2 \le x \le 3\}$

$R = \{f(x) \in \mathbb{R} \mid -1 \le f(x) \le 3.5\}$

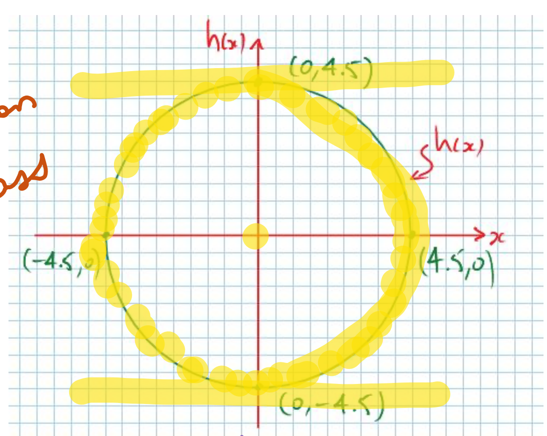
b) ✓ function passes VLT



$D = \{x \in \mathbb{R}\}$

$R = \{f(x) \in \mathbb{R} \mid f(x) \le 3\}$

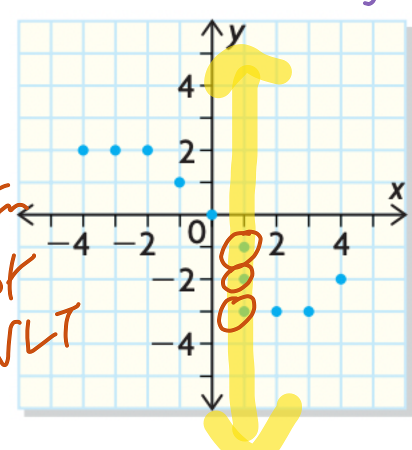
c) ✗ not function does not pass VLT



$D = \{x \in \mathbb{R} \mid -4.5 \le x \le 4.5\}$

$R = \{f(x) \in \mathbb{R} \mid -4.5 \le x \le 4.5\}$

✗ Not function does not pass VLT



$D = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$R = \{-3, -2, -1, 0, 1, 2\}$

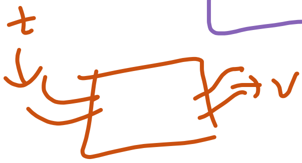
1 cup = 250 mL
10 cups = 2500 mL

Example 1.4.2 (*From Pg. 36 in your text*)

8. Write a function to describe coffee dripping into a 10-cup carafe at a rate of 1 mL/s. State the domain and range of the function (1 cup = 250 mL).

$$\boxed{V(t) = t}$$

t = time (in sec)
V = Volume (in mL)



$D = \{t \in \mathbb{R} \mid 0 \leq t \leq 2500\}$

$R = \{V(t) \in \mathbb{R} \mid 0 \leq V \leq 2500\}$

FUN WITH DOMAIN AND RANGE

Work with a partner to create a simple logo for a future start-up company (using at least 16-line segments) on Desmos.

THE PARENT FUNCTIONS (*for Grade 11*)

Together we will explore (graphically) basic properties of the five *parent* functions:

a) Linear $f(x) = x$

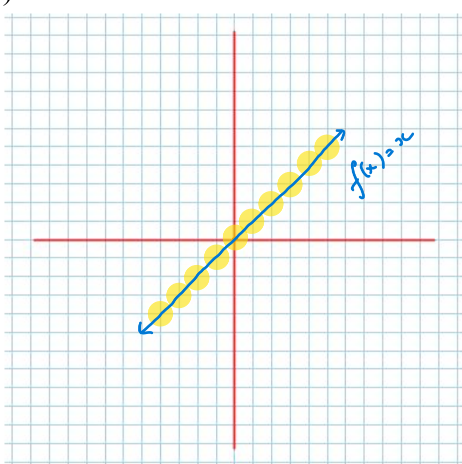


TABLE OF VALUES

$f(x) = x$

x	$f(x)$	$(x, f(x))$
−3	−3	(−3, −3)
−2	−2	(−2, −2)
−1	−1	(−1, −1)
0	0	(0, 0)
1	1	(1, 1)
2	2	(2, 2)
3	3	(3, 3)

$D = \{x \in \mathbb{R}\}$

$R = \{f(x) \in \mathbb{R}\}$

The graph is a straight line

b) Quadratic $g(x) = x^2$

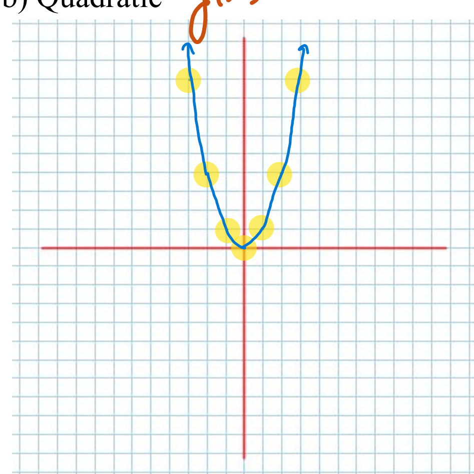


TABLE OF VALUES

x^2

x	$g(x)$	$(x, g(x))$
−3	9	(−3, 9)
−2	4	(−2, 4)
−1	1	(−1, 1)
0	0	(0, 0)
1	1	(1, 1)
2	4	(2, 4)
3	9	(3, 9)

$D = \{x \in \mathbb{R}\}$

$R = \{g(x) \in \mathbb{R} \mid g(x) \geq 0\}$

The graph is a parabola

c) Absolute Value $h(x) = |x|$

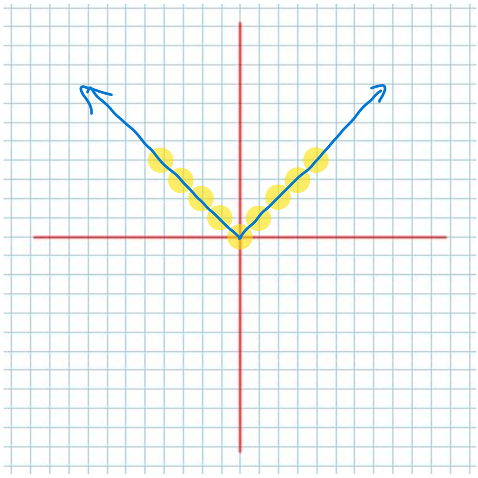


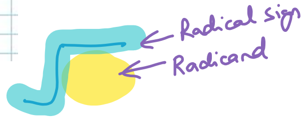
TABLE OF VALUES

x	$h(x)$ ($|x|$)	$(x, h(x))$
-3	3	$(-3, 3)$
-2	2	$(-2, 2)$
-1	1	$(-1, 1)$
0	0	$(0, 0)$
1	1	$(1, 1)$
2	2	$(2, 2)$
3	3	$(3, 3)$

$D = \{x \in \mathbb{R}\}$

$R = \{h(x) \in \mathbb{R} \mid h(x) \geq 0\}$

The graph is a V-shape



d) Square Root $i(x) = \sqrt{x}$

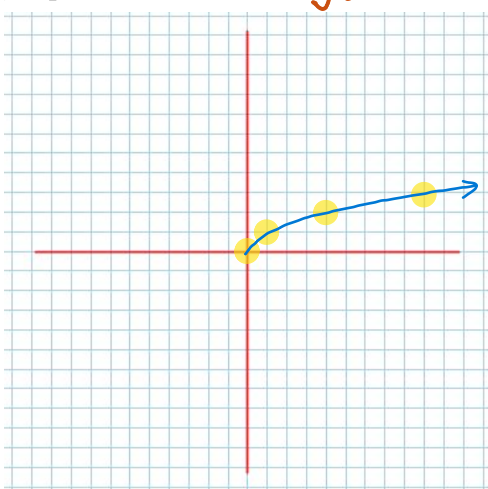


TABLE OF VALUES

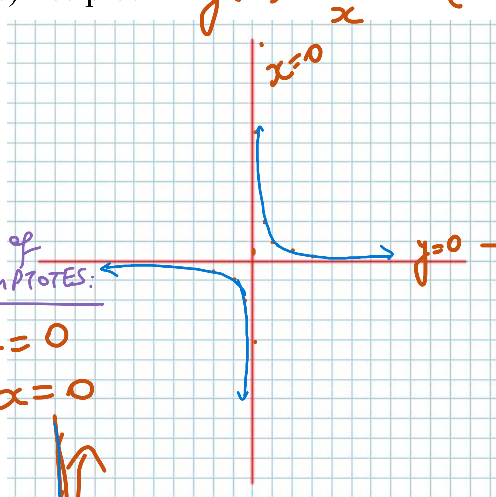
x	$i(x)$ ($\sqrt{x}$)	$(x, i(x))$
0	0	$(0, 0)$
1	1	$(1, 1)$
4	2	$(4, 2)$
9	3	$(9, 3)$
16	4	$(16, 4)$
25	5	$(25, 5)$
36	6	$(36, 6)$

$D = \{x \in \mathbb{R} \mid x \geq 0\}$

$R = \{i(x) \in \mathbb{R} \mid i(x) \geq 0\}$

The graph is a $\frac{1}{2}$ parabola opening to the Right.

e) Reciprocal $j(x) = \frac{1}{x}$ (RESTRICTION : $x \neq 0$)



Here, EQUATIONS of the ASYMPTOTES:

① x-axis $y = 0$

② y-axis $x = 0$

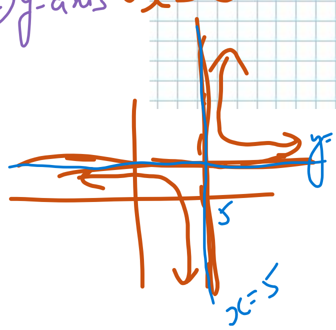
TABLE OF VALUES

x	$j(x)$	$(x, j(x))$
-4 / -2	-0.25 / -0.5	$(-4, -0.25)$ / $(-2, -0.5)$
-1	-1	$(-1, -1)$
~~0~~	~~X X~~	~~X X~~
1	1	$(1, 1)$
2	0.5	$(2, 0.5)$
4	0.25	$(4, 0.25)$
-0.5	-2	$(-0.5, -2)$
0.5	2	$(0.5, 2)$

$D = \{x \in \mathbb{R} \mid x \neq 0\}$

$R = \{j(x) \in \mathbb{R} \mid j(x) \neq 0\}$

The graph is a hyperbola



ASYMPTOTE

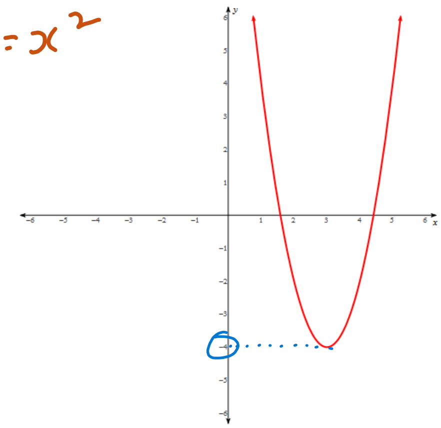
These are lines to which the graph seems to touch but never actually touches.

$-\frac{1}{2}$

9. Determine the domain and range of the following graphed functions:

$f(x) = 2(x-3)^2 - 4$

$f(x) = x^2$

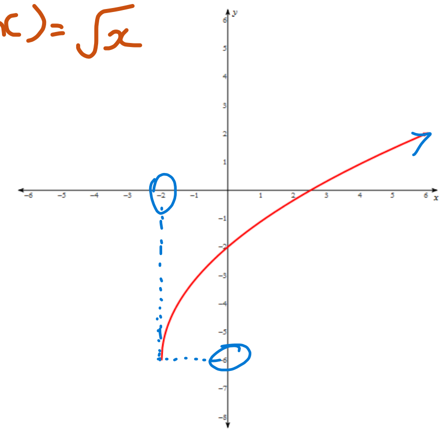


$D = \{x \in \mathbb{R}\}$

$R = \{f(x) \in \mathbb{R} \mid f(x) \geq -4\}$

$f(x) = 2\sqrt{2x+4} - 6$

$f(x) = \sqrt{x}$

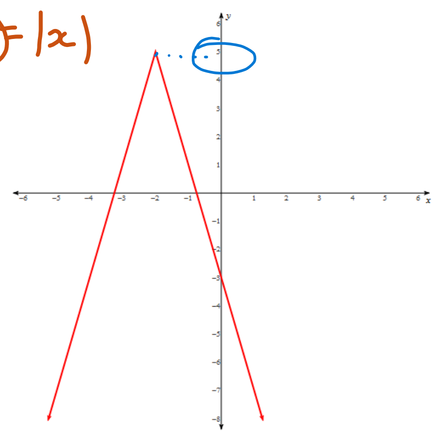


$D = \{x \in \mathbb{R} \mid x \geq -2\}$

$R = \{f(x) \in \mathbb{R} \mid f(x) \geq -6\}$

$f(x) = -4|x+2| + 5$

$f(x) = |x|$

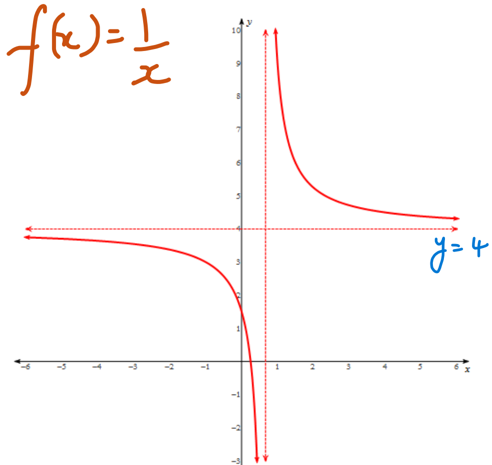


$D = \{x \in \mathbb{R}\}$

$R = \{f(x) \in \mathbb{R} \mid f(x) \leq 5\}$

$f(x) = \dfrac{5}{3x-2} + 4$

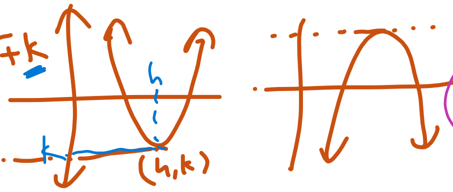
$f(x) = \dfrac{1}{x}$



$D = \{x \in \mathbb{R} \mid x \neq 0.75\}$

$R = \{f(x) \in \mathbb{R} \mid f(x) \neq 4\}$

$f(x) = a(x-h)^2 + k$



Range (TRICK)

① $a > 0$ (+ve) Then, $R = \{f(x) \in \mathbb{R} \mid f(x) \geq c\}$

② $a < 0$ (−ve) Then, $R = \{f(x) \in \mathbb{R} \mid f(x) \leq c\}$

Example 1.4.3 (*From Pg. 37 in your text…use Desmos*)

9. Determine the domain and range of each function.

a) $f(x) = -3x + 8$

$D = \{x \in \mathbb{R}\}$

$R = \{f(x) \in \mathbb{R}\}$

lines are usually unrestricted

d) $p(x) = \frac{2}{3}(x - 2)^2 - 5$

c

$D = \{x \in \mathbb{R}\}$

$R = \{p(x) \in \mathbb{R} \mid p(x) \geq -5\}$

f) $r(x) = \sqrt{5 - x}$

Restriction on x

$5 - x \geq 0$

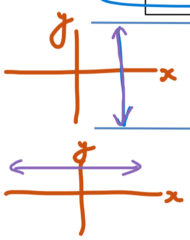
$\Rightarrow 5 \geq x$

$x \leq 5$

$D = \{x \in \mathbb{R} \mid x \leq 5\}$

$R = \{r(x) \in \mathbb{R} \mid r(x) \geq 0\}$

EXCEPTIONS



VERTICAL Line
restricted domain
but unrestricted range

HORIZONTAL Line
restricted range
but unrestricted domain

g) $f(x) = \frac{5}{3x - 2} + 4$

Restriction on x

$3x - 2 \neq 0$

$\Rightarrow 3x \neq 2$

$\Rightarrow x \neq \frac{2}{3}$

$D = \{x \in \mathbb{R} \mid x \neq \frac{2}{3}\}$

$R = \{f(x) \in \mathbb{R} \mid f(x) \neq 4\}$

Example 1.4.4

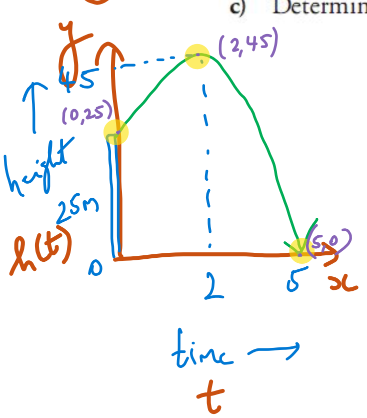
10. A ball is thrown upward from the roof of a 25 m building. The ball reaches a height of 45 m above the ground after 2 s and hits the ground 5 s after being thrown.

a) Sketch a graph that shows the height of the ball as a function of time.

b) State the domain and range of the function.

c) Determine an equation for the function.

ⓐ



ⓑ $D = \{t \in \mathbb{R} \mid t \geq 0\}$

$R = \{h(t) \in \mathbb{R} \mid 0 \leq h(t) \leq 45\}$

ⓒ $y = a(x-h)^2 + k$

$y = a(x-2)^2 + 45$

$\Rightarrow 0 = a(5-2)^2 + 45$

$\Rightarrow 0 = a(3)^2 + 45$

$\Rightarrow 0 = 9a + 45 \Rightarrow -45 = 9a$

$\Rightarrow \frac{-45}{9} = a$

$\Rightarrow a = -5$

$\therefore$ Equation for the function:-

$y = -5(x-2)^2 + 45$

or

$\boxed{h(t) = -5(t-2)^2 + 45}$

HW- Section 1.3/1.4

Domain and Range Handout

Success Criteria:

- I can identify the unique characteristics of five basic types of functions
- I can identify the domain and ranges of five basic types of functions
- I can identify when there are restrictions given real-world situations

1.6 – 1.8: Transformations of Functions *(Part 1)*

Learning Goal: We are learning to apply combinations of transformations in a systematic order to sketch graphs of functions.

To **TRANSFORM** something is to change the form.

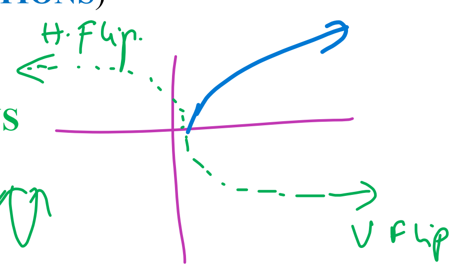
TRANSFORMATIONS OF FUNCTIONS can be seen in two ways: algebraically, and graphically. We'll begin by examining transformations graphically.

But before we do, we need to remember that the **GRAPH OF A FUNCTION**, $f(x)$, is given by:

$$f(x)=\left\{\left(x,f(x)\right)\middle|\,x\in D_f\right\}$$

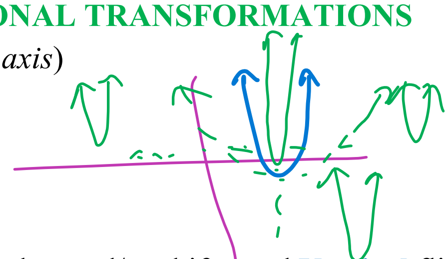
So, for functions we have two things (NUMBERS!) to "transform". We can apply transformations to

1) **Domain** values (which we call **HORIZONTAL TRANSFORMATIONS**)
2) **Range** values (which we call **VERTICAL TRANSFORMATIONS**)



There are **THREE BASIC FUNCTIONAL TRANSFORMATIONS**

1) Flips (*Reflections "across" an axis*)
2) Stretches (*Dilations*)
3) Shifts (*Translations*)



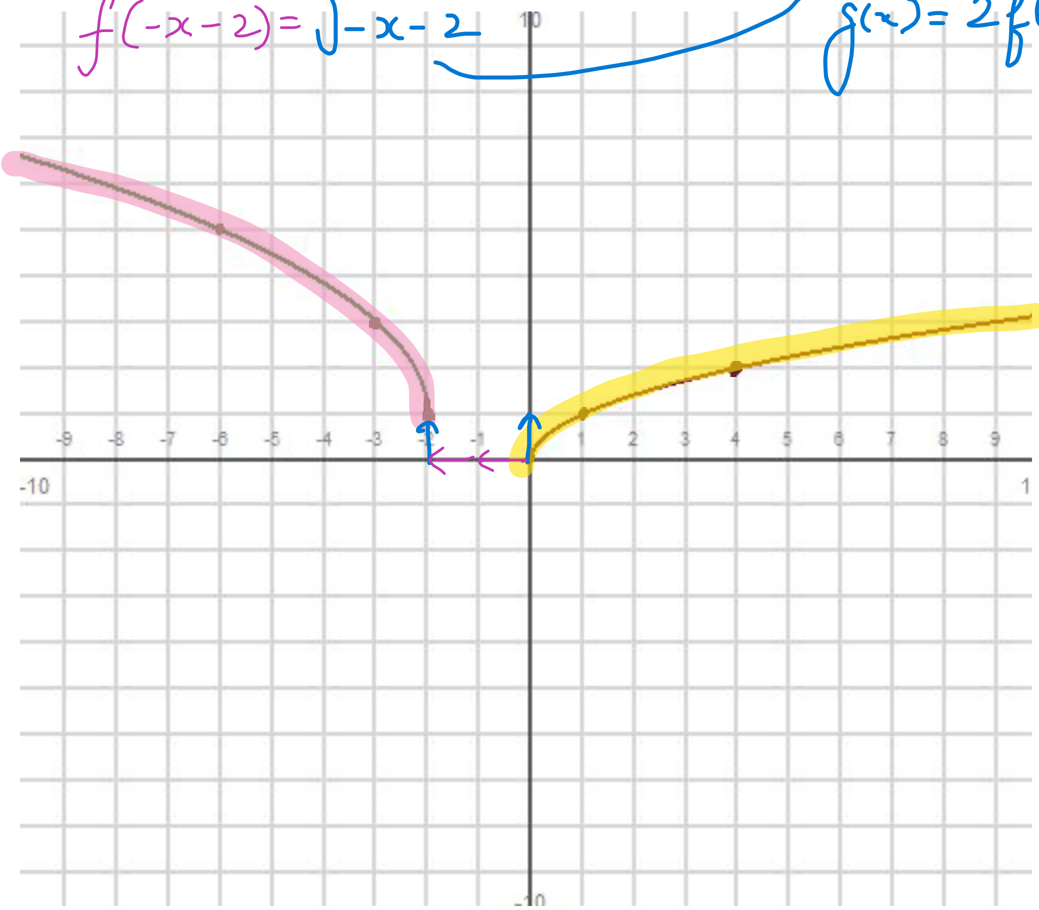
So, we can have **Horizontal** flips, stretches and/or shifts, and **Vertical** flips, stretches and/or shifts. Now let's take a look at how transformations can be applied to functions.

***Note*:** We'll (mostly) be applying transformations to our so-called "parent functions" (although applying transformations to linear functions can seem pretty silly!)

Example 1.8.1

Consider, and make observations concerning the sketch of the graph of the parent function $f(x)=\sqrt{x}$ and the transformed function $g(x)=2\sqrt{-x-2}+1$.

$f(-x-2)=\sqrt{-x-2}$ $\qquad$ $g(x)=2f(-x-2)+1$



Horizontal Transformations

H. Flip

H. shift 2 units left

Vertical Transformations

V- stretch

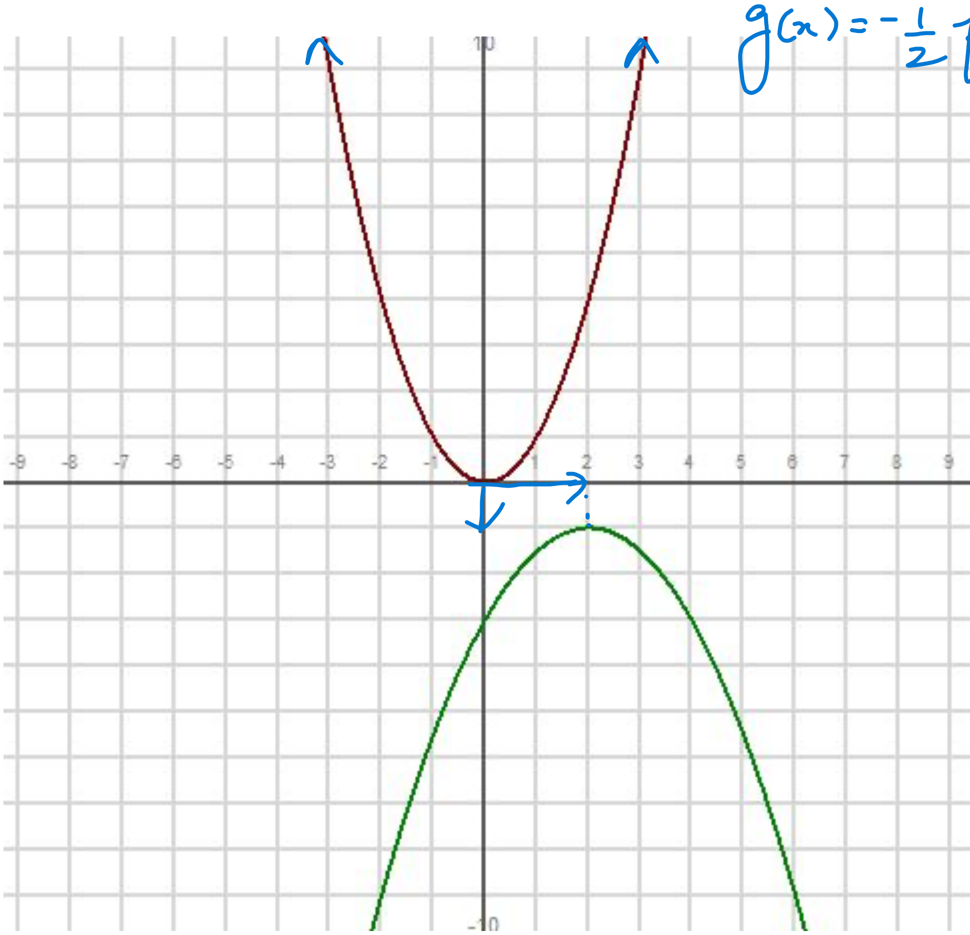
V. shift 1 unit UP.

Note: In the above example we can **algebraically** describe $g(x)$ as a transformed $f(x)$ with the functional equation $g(x)=2f(-x-2)+1$

Example 1.8.2

Consider, and make observations concerning the sketch of the graph of the parent function $f(x)=x^2$ and the transformed function $g(x)=-\frac{1}{2}(x-2)^2-1$

$g(x)=-\frac{1}{2}f(x-2)-1$



Horizontal Transformations

H. stretch

H. shift 2 units right

Vertical Transformations

V. flip

V. shift 1 unit down

Note: In the above example we can **algebraically** describe $g(x)$ as a transformed $f(x)$ with the functional equation

1.6 – 1.8: Transformations of Functions *(Part 2)*

We now turn to examining Transformations of Functions from an algebraic point of view (although a geometric perspective will still shine though!)

Definition 1.8.1

Given a function $f(x)$ we can obtain a related function through functional transformations as

$$\boxed{g(x) = af\left(k(x-d)\right) + c}$$, where

VERTICAL TRANSFORMATIONS

① (i) a → vertical stretch

(ii) a > 0 → new graph resembles parent
a < 0 → vertical flip.

② c → vertical shift
(up or down)
+ −

HORIZONTAL TRANSFORMATIONS

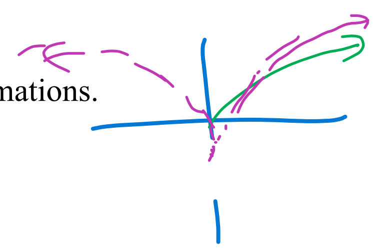
① (i) k → horizontal stretch by 1/k
(ii) k < 0 → Horizontal Flip.

② d → horizontal Shift.

* k must always* be factored out

Example 1.8.3

Consider the given function. State its parent function, and all transformations.



$$g(x) = 3\sqrt{-x+2} - 1$$

$$g(x) = 3\sqrt{-1(x-2)} - 1$$

$$g(x) = af(k(x-d)) + c$$

$$-1\,(x-(2))$$

Horizontal Transformations

k = −1 H. flip.

d = 2 H. shift 2 units right

Vertical Transformations

a = 3 V. stretch by factor of 3

c = −1 V. shift 1 unit down

Example 1.8.4

The basic absolute value function $f(x)=|x|$ has the following transformations applied to it: **Vertical Stretch** -3, **Vertical Shift** 1 **up, Horizontal Shift** 5 **right.**

Determine the equation of the transformed function.

$a=-3$
$c=1$
$d=5$
$k=1$

$g(x)=af(k(x-d))+c$

$g(x)=-3|x-5|+1$

Back to a geometric point of view

Sketching the graph of a transformed function can be relatively easy if we know:

1) The shape of the parent function AND a few (3 or 4) points on the parent.
2) How transformations affect the points on the parent
   i) Horizontal transformations affect the domain values (***OPPOSITE!!!!!!***)
   ii) Vertical transformations affect the range values

Note: Given a point on some parent function which has transformations applied to it is called an ***IMAGE POINT*** on the transformed function.

$(x,y) \longrightarrow \left(\frac{1}{k}x+d,\ ay+c\right)$

PARENT (image point) TRANSFORMED

Example 1.8.5

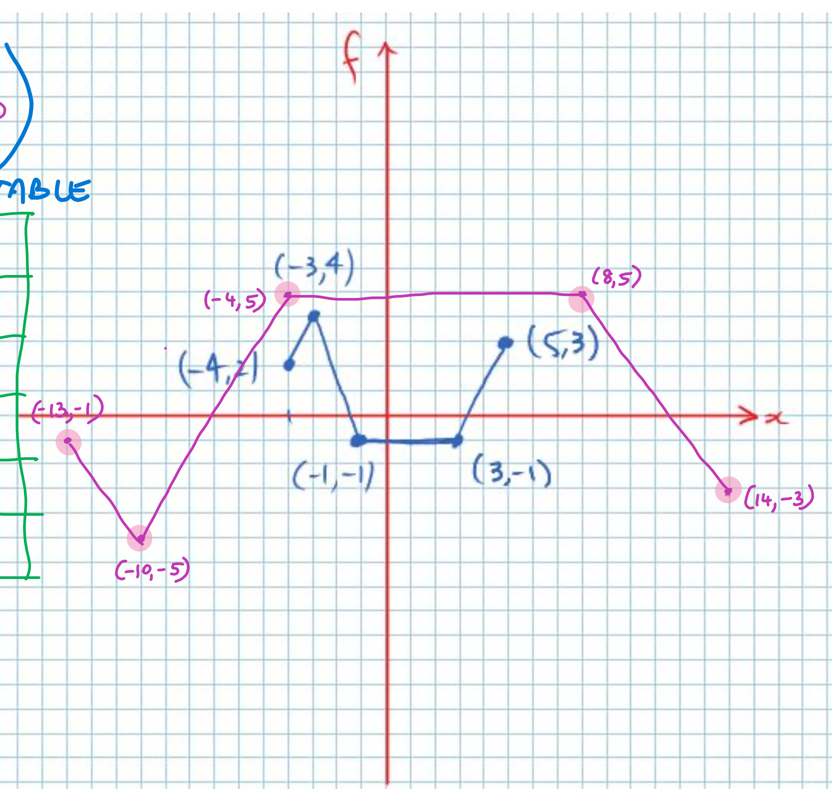
Given the sketch of the function $f(x)$ determine the image points of the transformed function $\boxed{\underset{a}{-2}f\left(\underset{k}{\frac{1}{3}}\left(x+\underset{d=-1}{1}\right)\right)+\underset{c}{3}}$ and sketch the graph of the transformed function.

$(x,y) \longrightarrow (3x-1,\ -2y+3)$

$\longrightarrow$ TRANSFORMED TABLE

x	y
-4	2
-3	4
-1	-1
3	-1
5	3

$3x-1$	$-2y+3$
-13	-1
-10	-5
-4	5
8	5
14	-3



RESTRICTION on x

$-3x+12 \geq 0$

$12 \geq 3x$

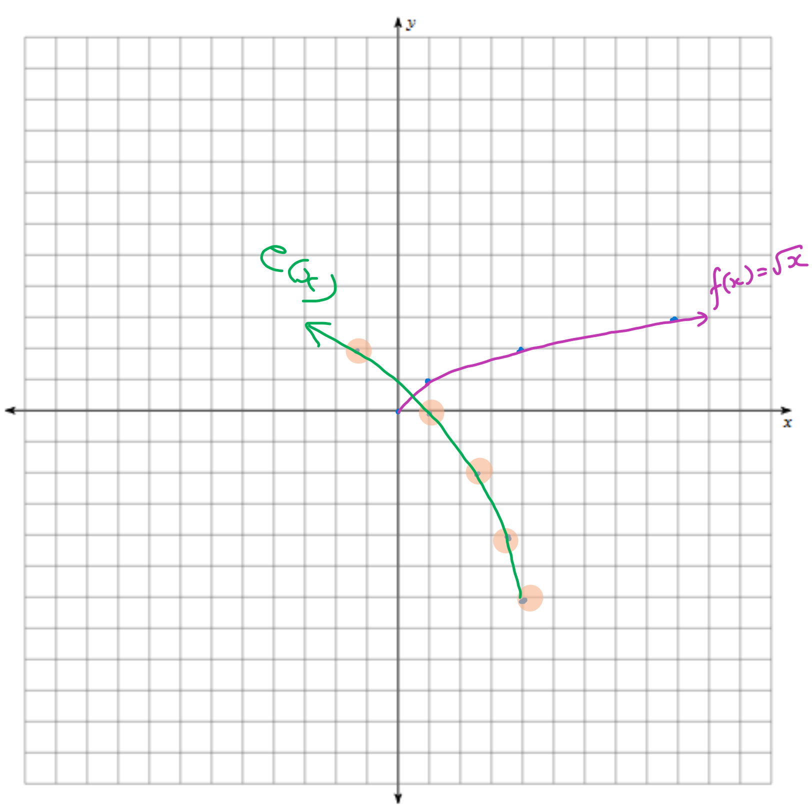
$4 \geq x$

Function	Proper Function $g(x) = a\,f\left(k(x-d)\right)+c$ $f(x)=\sqrt{x}$	Vertical Stretch *a*	Horizontal Stretch *1/k*	Horizontal Shift *d*	Vertical Shift *c*
$e(x) = 2\sqrt{-3x+12}-6$	$e(x) = 2\sqrt{-3(x-4)}-6$	2	$-\frac{1}{3}$	4	-6

Domain	$\{x \in \mathbb{R} \mid x \leq 4\}$	Range	$\{e(x) \in \mathbb{R} \mid e(x) \geq -6\}$	y-int (x=0)	$e(0) = y_{int}$ 0.93 (approx)

Table Of Values	Parent Function:		Transformed Function	
	x	$f(x)=\sqrt{x}$	$-\frac{1}{3}x+4$	$2y-6$
	0	0	$-\frac{1}{3}(0)+4 = 4$	$2(0)-6 = -6$
	1	1	$-\frac{1}{3}(1)+4 = 3.7$	$2(1)-6 = -4$
	4	2	$-\frac{1}{3}(4)+4 = 2.7$	$2(2)-6 = -2$
	9	3	$-\frac{1}{3}(9)+4 = 1$	$2(3)-6 = 0$
	16	4	$-\frac{1}{3}(16)+4 = -1.3$	$2(4)-6 = 2$

Starting point



Extra work space.

HW- Section 1.6-1.8

Big Handout

Success Criteria:

- I can use the value of a to determine if there is a vertical stretch/reflection in the x-axis
- I can use the value of k to determine if there is a horizontal stretch/reflection in the y-axis
- I can use the value of d to determine if there is a horizontal translation
- I can use the value of c to determine if there is a vertical translation
- I can transform x coordinates by using the expression $\frac{1}{k}x + d$
- I can transform y coordinates by using the expression $ay + c$