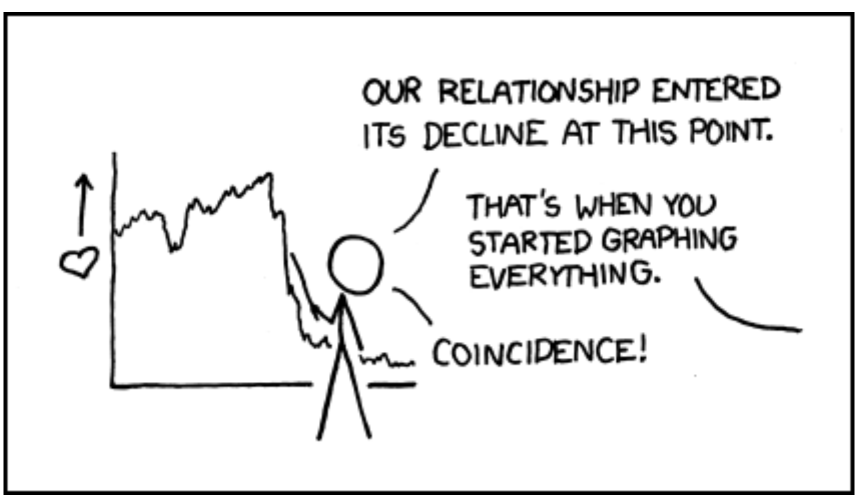
Functions 11

Course Notes

Unit 2 Introduction to Functions

We will learn

- *the meaning of the term Function and how to use function notation to calculate and represent functions*
- *the meanings of the terms domain and range, and how a function's structure affects domain and range*
- *how to use transformations to represent and sketch graphs*



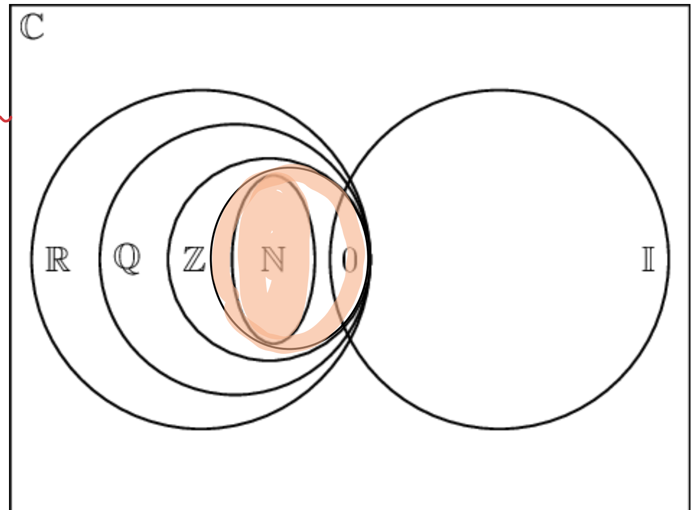
1.1 Relations and Functions (*This is a* *KEY* *lesson*!)

Learning Goal: We are learning to recognize functions in various representations.

This course is called **FUNCTIONS**, so it seems rather important that you know what a function actually is. However, before we define and dive into the world of functions, it is important to be familiar with a few other commonly used terms in the language of functions.

Definition 0.a.

A **Set** is a well-defined collection of things or numbers.



N = { 1, 2, 3, 4, ... }

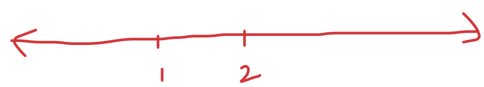
W = { 0, 1, 2, 3, ... }

Z = { . . . . , −2, −1, 0, 1, 2, ... }

Q = { $\frac{N}{D}$; D ≠ 0, N and D are integers }

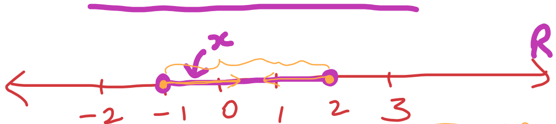
Q' = all Real #s that are not Rational

R → all rationals and irrationals together



Definition 0.b. Set Notations: *Some fancy ways to represent sets of Numbers!*

SET-BUILDER FORM



$\{x \in R \mid -1 \le x \le 2\}$

x < 2
x = 2
x > −1
x = −1

belongs to

such that

*

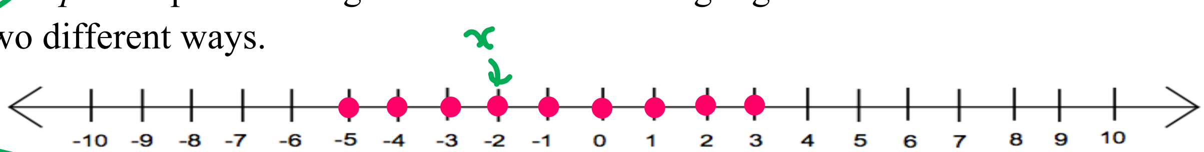
ROSTER FORM

You can list it out

{ ☼, ☽, ★ }

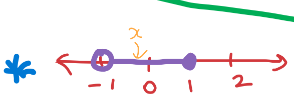
ANOTHER EXAMPLE

- *Example:* Represent the given set of numbers highlighted on the number line in two different ways.



ROSTER = { −5, −4, −3, −2, −1, 0, 1, 2, 3 }

SET BUILDER FORM = $\{x \in Z \mid -5 \le x \le 3\}$

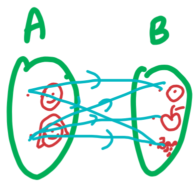


$\{x \in R \mid -1 < x \le 1\}$

Definition 0.c.

A **Relation** is a mathematical relationship between elements of one set with the elements of another set.

Relations can be represented in different ways like an arrow diagram, an equation, a table of values, a set of ordered pairs, or a graph.



ARROW DIAGRAM.

$y = 3x+2$

$x=1 \longrightarrow y=5$

$x=2 \longrightarrow y=8$

EQUATION

x	y
-1	1
0	0
1	1

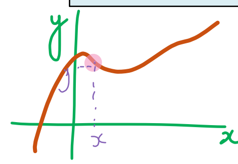
TABLE of VALUES

(x,y)

$(5,2)$

$(-7,9)$

ORDERED PAIR



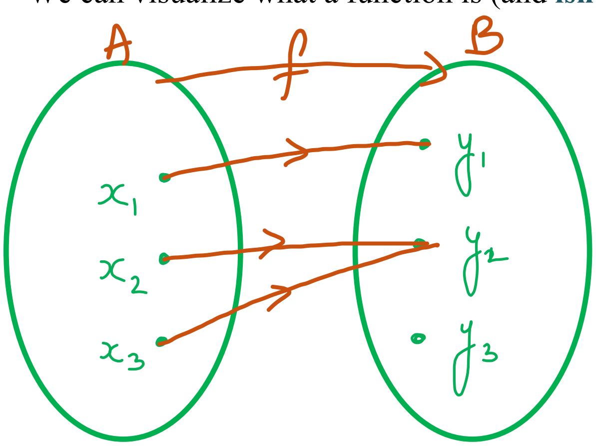
Now comes the moment finally!! We are ready to explore and understand **FUNCTIONS**! 😊

You need to know, very well, the following (algebraic) definition:

Definition 1.1.1

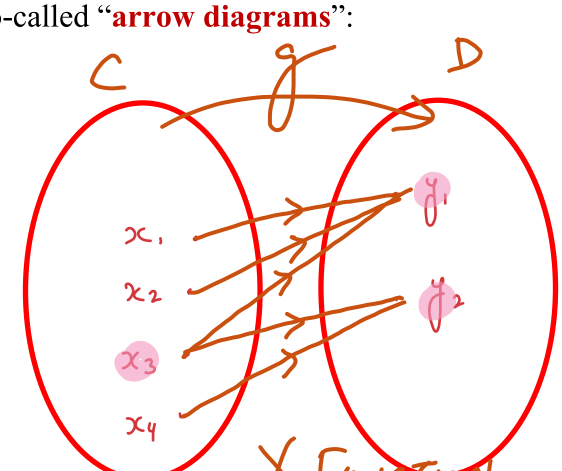
A **FUNCTION** is A VERY SPECIAL RELATION where EACH ELEMENT of ONE SET is CONNECTED (related) to ONE (and ONLY ONE) ELEMENT of ANOTHER SET.

We can visualize what a function is (and **isn't**) by using so-called "**arrow diagrams**":



D_f : $\{x_1, x_2, x_3\}$ ✓ FUNCTION

R_f : $\{y_1, y_2\}$



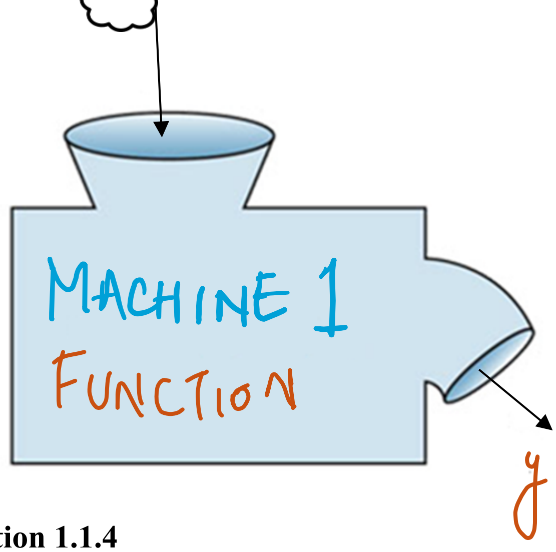
X FUNCTION

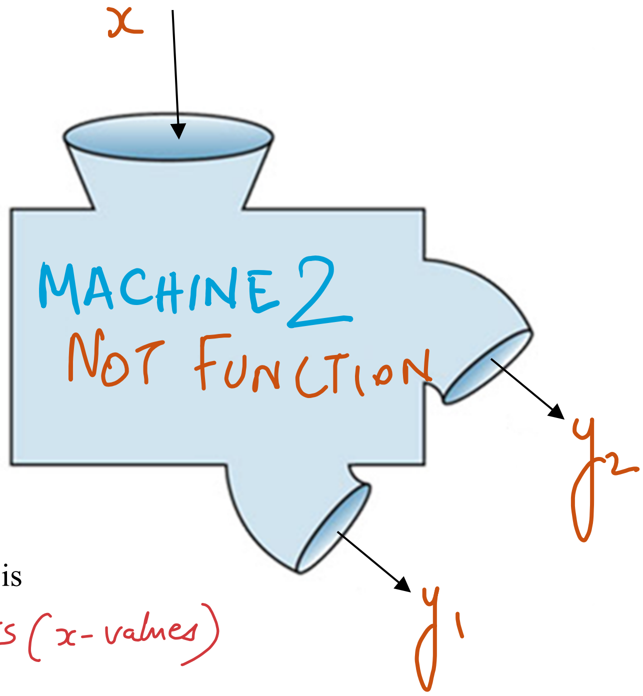
D_g : $\{x_1, x_2, x_3, x_4\}$

R_g : $\{y_1, y_2\}$

We can also view ***Functions*** visually like a ***Vending Machine*** because they are ***PREDICTABLE*** just like our functions*!!*

- **THINKING QUESTION:**
  - i. *Are all Functions also Relations?* ......YES!......
  - ii. *Are all Relations also Functions?* ......No!......
  - iii. *Which one of the two can be a function machine?*

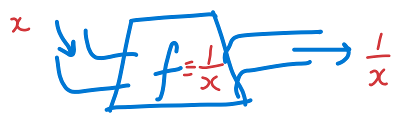




Definition 1.1.4

The **DOMAIN** of a function (or a relation) is

a SET of all ACCEPTABLE INPUTS (x-values)



Definition 1.1.5

The **RANGE** of a function (or a relation) is

a SET of all ACCEPTABLE OUTPUTS (y-values)

$y = 3x + 2$

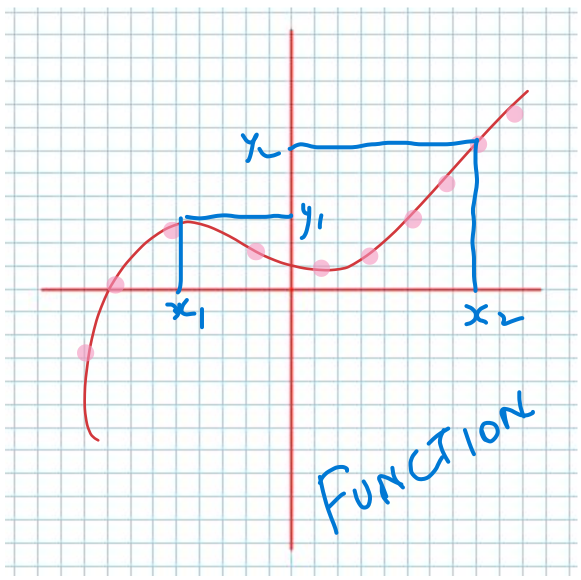
Two other important terms to know are:

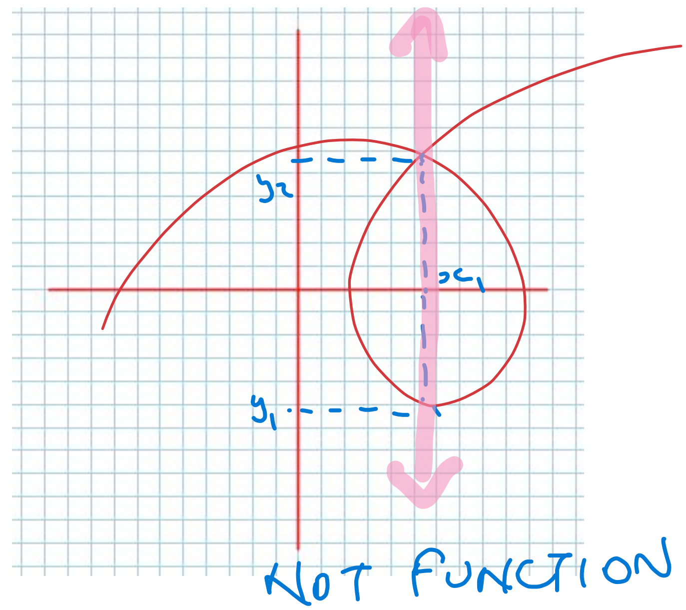
1) The **INDEPENDENT VARIABLE** x (INPUT)

2) The **DEPENDENT VARIABLE** y (OUTPUT)

Knowing when a Relation is, and isn't, a Function

Graphically: *The Vertical Line Test*





Algebraically: (***NOTE: this is a "rough" way of thinking about the problem***)

If the Dependent (y) Variable ~~is~~ has an ODD exponent, then it is a FUNCTION. $y^1 = x^2 + 3$

e.g. has an EVEN exponent, then it is not a function $x^2 + y^2 = 4$

HW- Section 1.1 *(Suggested problems are from the Nelson Textbook. You are welcome to ask for help from your peers or myself. It is due the next day!)*

Pg. 10 – 12 #1, 2 (*no ruler needed*), 6, 7 (*no need for the VLT*), 9, 11, 12 (*think carefully about the idea that the domain and range are "limited"*)

Success Criteria:

- I can determine the domain of a relation or function as the set of all values of the independent variable
- I can determine the range of a relation or function as the set of all values of the dependent variable
- I can apply the vertical line test to determine if a graph is a function
- I can recognize whether a relation is a function from its equation

1.2 Function Notation

Learning Goal: We are learning to use function notation to represent linear and quadratic functions

Here we learn a **NEW AND IMPROVED WAY** for describing a function, algebraically. You have been using the following form for functions (in this example, for a quadratic):

$x \to \boxed{f} \to y = f(x)$ $x=1$, $x=2$; $f(1)=4$, $f(2)=1$

$$y = 3(x-2)^2 + 1$$

$f(x) = 3(x-2)^2+1$

INPUT (x) → $\boxed{f(x)}$ → OUTPUT (y) OUTPUT $f(x)$

DOMAIN

A much more useful way of writing function is to use **FUNCTION NOTATION**. The above quadratic (***which we call a "function of x" because the domain is given as x-values***) can be written as:

$x=1 \Rightarrow f(x=1) = 3(1-2)^2+1 = 3(1)+1 = 3+1 = 4$

$x=2 \Rightarrow f(x=2) = 3(2-2)^2+1 = 1$

$(x,y) \equiv (x, f(x))$

This new notation is so useful because of the "symbol": $f(x)$ OUTPUT INPUT

This notation shows **BOTH** the **DOMAIN** and the **RANGE** values. Because of that, the function notation shows us **points** on the graph of the function.

Let's do some examples (from your text on pages 23 – 24)

Example 1.2.1

4. Evaluate $f(-1)$, $f(3)$, and $f(1.5)$ for

a) $f(x) = (x-2)^2 - 1$ **b)** $f(x) = 2 + 3x - 4x^2$

$x \to \boxed{f = (x-2)^2 - 1} \to f(x)$

a) $f(-1) = (-1-2)^2 - 1$
$= (-3)^2 - 1$
$= 9 - 1$
$= 8$

$f(3) = (3-2)^2 - 1$
$\boxed{f(3) = 0}$
$(3, 0)$
$x, f(x)$

$f(1.5) = (1.5-2)^2 - 1$
$= 0.25 - 1$
$f(1.5) = -0.75$

b) $f(-1) = 2 + 3(-1) - 4(-1)^2$
$= 2 - 3 - 4$
$\boxed{f(-1) = -5}$

$f(3) = 2 + 9 - 36$
$= -25$

$f(1.5) = 2 + 4.5 - 4(2.25)$
$= 2 + 4.5 - 9$
$= -2.5$

Example 1.2.2

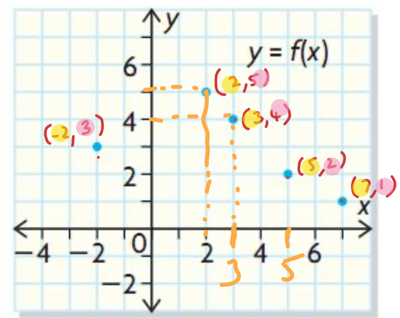
6. The graph of $y = f(x)$ is shown at the right.

a) State the domain and range of f.

b) Evaluate.

i) $f(3) \Rightarrow y$ when $x = 3$ **iii)** $f(5 - 3)$

ii) $f(5)$ **iv)** $f(5) - f(3)$



a) $D_f = \{-2, 2, 3, 5, 7\}$

$R_f = \{1, 2, 3, 4, 5\}$

b) (i) $f(3) = 4$

(iii) $f(5-3) = f(2) = 5$

(ii) $f(5) = 2$

(iv) $f(5) - f(3) = 2 - 4 = -2$

Example 1.2.3

11. For $g(x) = 4 - 5x$, determine the input for x when the output of $g(x)$ is

a) -6 **b)** 2

a) $g(x) = -6$

$g(x) = 4 - 5x$

?

$-6 = +4 - 5x$

$-6 - 4 = -5x$

$-10 = -5x$

$\boxed{2 = x}$

$\boxed{g(2) = -6}$

b) $g(x) = 2$

$g(x) = 4 - 5x$

?

$2 = 4 - 5x$

$2 - 4 = -5x$

$-2 = -5x$

$\frac{-2}{-5} = x$

$x = 0.4$

$\boxed{g(0.4) = 2}$

Example 1.2.4

7. For $h(x) = 2x - 5$, determine

a) $h(a)$

b) $h(b + 1)$

c) $h(3c - 1)$

d) $h(2 - 5x)$

a) $h(a) = 2a - 5$

b) $h(b+1) = 2(b+1) - 5$
$= 2b + 2 - 5$
$= 2b - 3$

c) $h(3c-1) = 2(3c-1) - 5$
$= 6c - 2 - 5$
$= 6c - 7$

d) $h(2-5x) = 2(2-5x) - 5$
$= 4 - 10x - 5$
$= -10x - 1$

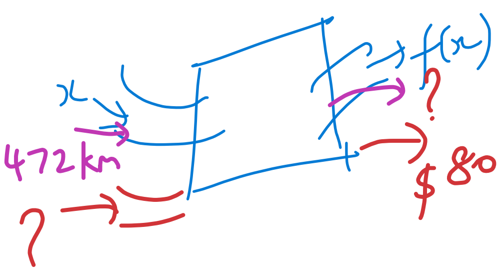
Example 1.2.5

12. A company rents cars for \$50 per day plus \$0.15/km.

a) Express the $\underbrace{\text{daily rental cost}}_{f(x)}$ as a function of the $\underbrace{\text{number of kilometres}}_{x}$ travelled.

b) Determine the rental cost if you drive 472 km in one day.

c) Determine how far you can drive in a day for \$80.



a) $f(x) = 50 + 0.15x$

b) $f(472) = 50 + 0.15(472) = \$120.8$

c) $f(x) = 50 + 0.15x$
$80 = 50 + 0.15x$
$80 - 50 = 0.15x$
$30 = 0.15x$
$\frac{30}{0.15} = x$
$\boxed{x = 200\text{km}}$

HW- Section 1.2

Page 23 #1-2, 5, 8b, 10, 11cd, 15, 16, challenge #17

Success Criteria:

- I can evaluate functions using function notation, by substituting a given value for x in the equation for f(x)
- I can recognize that f(x) = y corresponds to the coordinate (x, y)
- I can, given y = f(x), determine the value of x

1.3 and 1.4 Parent Functions and Domain and Range

Learning Goal: We are learning the graphs and equations of five basic functions; and using their tables, graphs, or equations to find their domains and ranges.

Two **INCREDIBLY IMPORTANT** aspects of functions are their

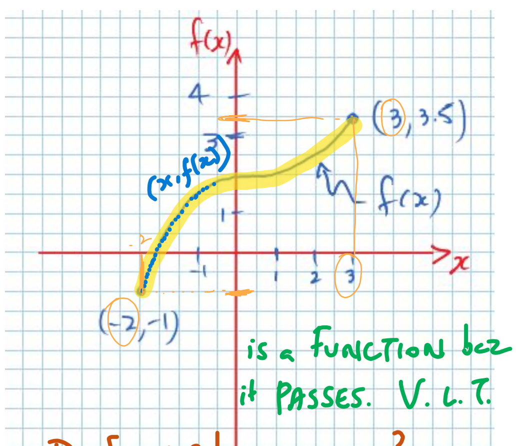
Again, the Domain is a set of all acceptable input values (x-values)

And, the Range is a set of all acceptable output values (y-values) f(x)

Example 1.4.1

Given the **SKETCH OF THE GRAPH** of the **RELATION** determine: the domain, the range of the relation, and whether the relation is, or is not, a function. *(Note that domain and range are sets of numbers and can be represented by the fancy **set notation**)*

a)

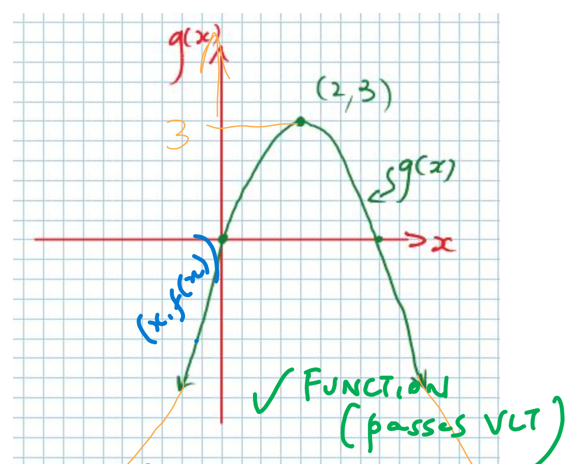


is a FUNCTION bcz it PASSES. V. L. T.

$D = \{x \in \mathbb{R} \mid -2 \le x \le 3\}$

$R = \{y \in \mathbb{R} \mid -1 \le y \le 3.5\}$

b)

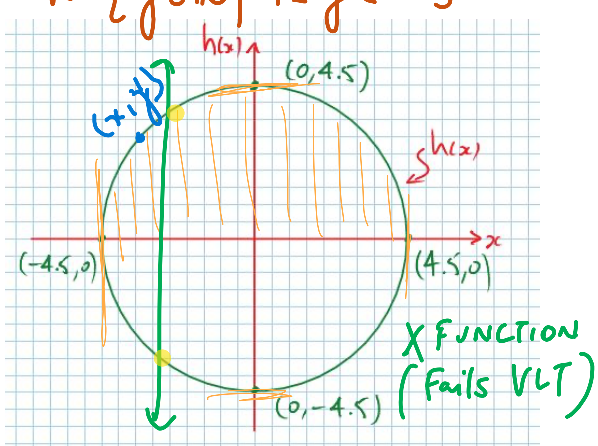


✓ FUNCTION (passes VLT)

$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R} \mid y \le 3\}$

c)

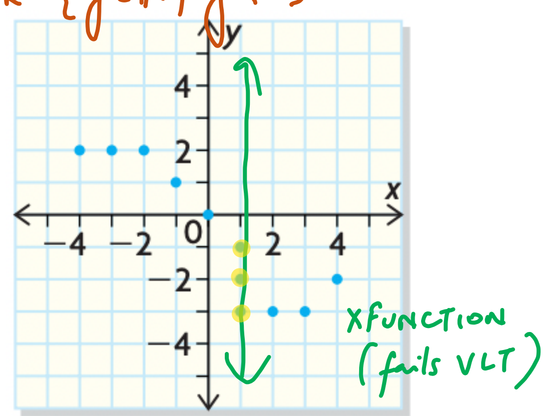


X FUNCTION (Fails VLT)

$D = \{x \in \mathbb{R} \mid -4.5 \le x \le 4.5\}$

$R = \{y \in \mathbb{R} \mid -4.5 \le y \le 4.5\}$

d)

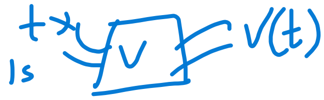


X FUNCTION (fails VLT)

$D = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$R = \{-3, -2, -1, 0, 1, 2\}$

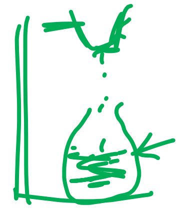
1mL/s.



1s	1ml
2s	2ml
3s	3ml

Example 1.4.2 (*From Pg. 36 in your text*)

8. Write a function to describe coffee dripping into a 10-cup carafe at a rate of 1 mL/s. State the domain and range of the function (1 cup = 250 mL).



10cups $= 10 \times 250$ $= 2500$ml ~ 2500s.

$\boxed{V(t) = t \quad \text{where } V \equiv \text{Volume},\ t = \text{time}}$

$D = \{t \in \mathbb{R} \mid 0 \leq t \leq 2500\text{sec.}\}$

$R = \{V(t) \in \mathbb{R} \mid 0 \leq V(t) \leq 2500\text{ml}\}$

THE PARENT FUNCTIONS (*for Grade 11*)

Together we will explore (graphically) basic properties of the five *parent* functions:

BASE function

$y = mx + b$

a) Linear $y = x$

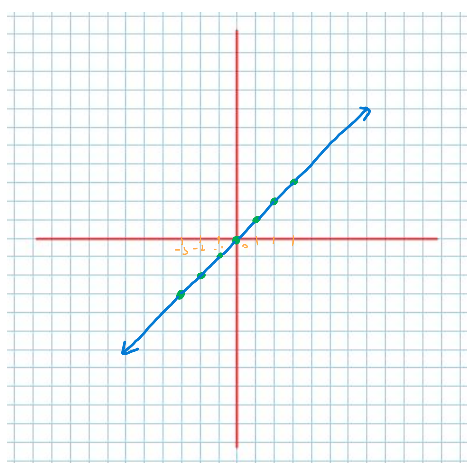


TABLE OF VALUES

x	$f(x)$	$(x, f(x))$
−3	−3	(−3, −3)
−2	−2	(−2, −2)
−1	−1	(−1, −1)
0	0	(0, 0)
1	1	(1, 1)
2	2	(2, 2)
3	3	(3, 3)

$D_f = \{x \in \mathbb{R}\}$

$R_f = \{f(x) \in \mathbb{R}\}$

The graph is a LINE

b) Quadratic $y = x^2$

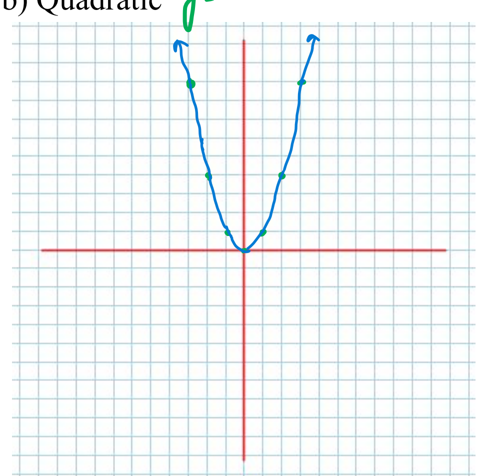


TABLE OF VALUES

x	$g(x)$	$(x, g(x))$
−3	9	(−3, 9)
−2	4	(−2, 4)
−1	1	(−1, 1)
0	0	(0, 0)
1	1	(1, 1)
2	4	(2, 4)
3	9	(3, 9)

$D_g = \{x \in \mathbb{R}\}$

$R_g = \{g(x) \in \mathbb{R} \mid g(x) \geq 0\}$

The graph is a PARABOLA

c) Absolute Value $y = |x|$

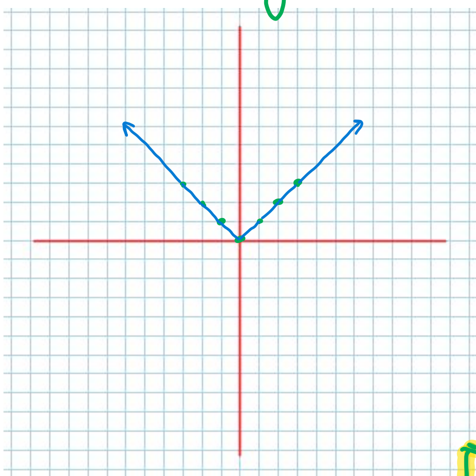


TABLE OF VALUES

x	$h(x)$ ($\lvert x \rvert$)	$(x, h(x))$
−3	3	(−3, 3)
−2	2	(−2, 2)
−1	1	(−1, 1)
0	0	(0, 0)
1	1	(1, 1)
2	2	(2, 2)
3	3	(3, 3)

$D_h = \{x \in \mathbb{R}\}$

$R_h = \{h(x) \in \mathbb{R} \mid h(x) \geq 0\}$

The graph is a V-shape

RADICAL

RADICAND

d) Square Root $y = \sqrt{x}$

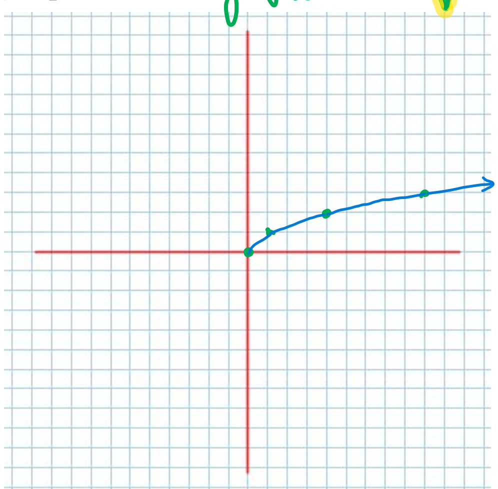


TABLE OF VALUES

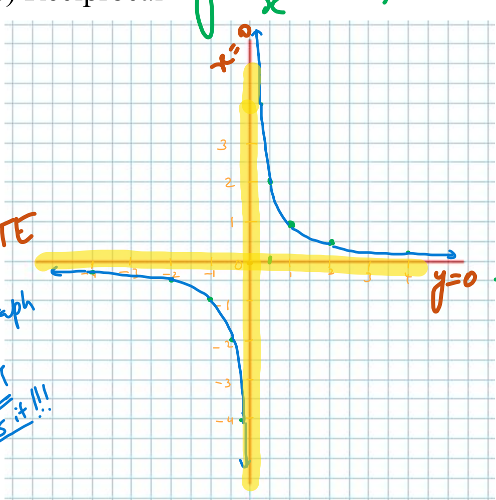
x	$i(x)$ ($\sqrt{x}$)	$(x, i(x))$
0	0	(0, 0)
1	1	(1, 1)
4	2	(4, 2)
9	3	(9, 3)
16	4	(16, 4)
25	5	(25, 5)

$D_i = \{x \in \mathbb{R} \mid x \geq 0\}$

$R_i = \{i(x) \in \mathbb{R} \mid i(x) \geq 0\}$

The graph is a ½ parabola opening to the right

e) Reciprocal $y = \frac{1}{x}$; $x \neq 0$



ASYMPTOTE

It is a line to which a given graph seems to be approaching BUT it never meets it!!!

TABLE OF VALUES

~~x~~ 0.5	$j(x)$ ($\frac{1}{x}$) 2	$(x, j(x))$
−4	−0.25	(−4, −0.25)
−2	−0.5	(−2, −0.5)
−1	−1	(−1, −1)
~~0~~		
1	1	(1, 1)
2	0.5	(2, 0.5)
4	0.25	(4, 0.25)
0.5	2	

$D_j = \{x \in \mathbb{R} \mid x \neq 0\}$

$R_j = \{j(x) \in \mathbb{R} \mid j(x) \neq 0\}$

The graph is a hyperbola

The parent reciprocal graph has 2 ASYMPTOTES

(i) $x = 0$

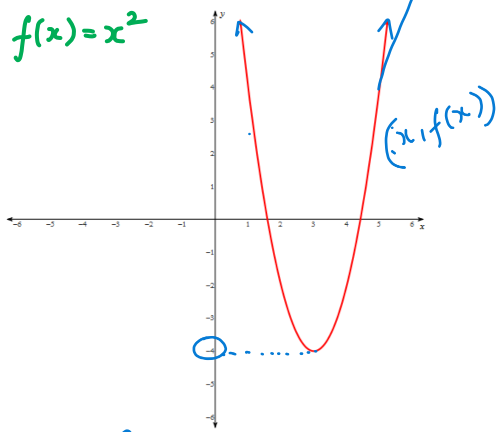
(ii) $y = 0$

$\frac{1}{0.5}$

9. Determine the domain and range of the following graphed functions:

$f(x) = 2(x-3)^2 - 4$

$f(x) = x^2$

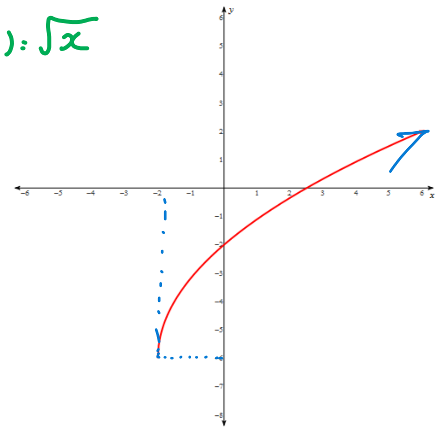


$D_f = \{x \in \mathbb{R}\}$

$R_f = \{f(x) \in \mathbb{R} \mid f(x) \geq -4\}$

$f(x) = 2\sqrt{2x+4} - 6$

$f(x) = \sqrt{x}$

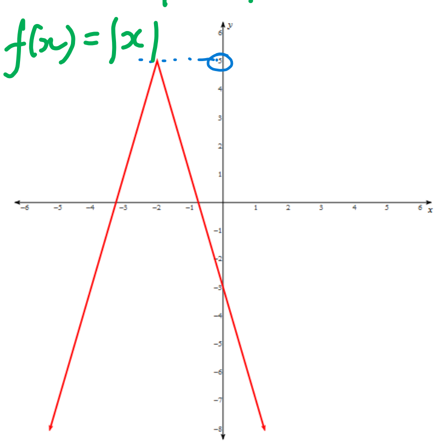


$D_f = \{x \in \mathbb{R} \mid x \geq -2\}$

$R_f = \{f(x) \in \mathbb{R} \mid f(x) \geq -6\}$

$f(x) = -4|x+2| + 5$

$f(x) = |x|$

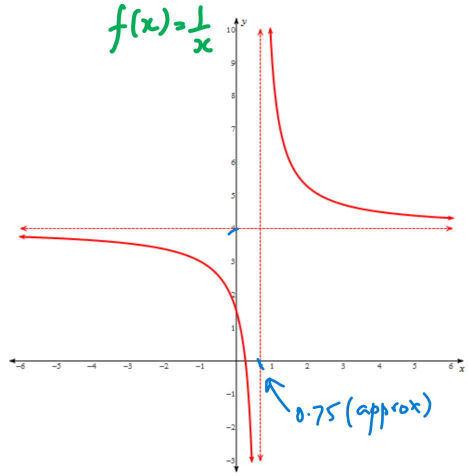


$D_f : \{x \in \mathbb{R}\}$

$R_f = \{f(x) \in \mathbb{R} \mid f(x) \leq 5\}$

$f(x) = \dfrac{5}{3x-2} + 4$

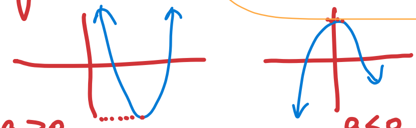
$f(x) = \frac{1}{x}$



$D_f = \{x \in \mathbb{R} \mid x \neq 0.75\}$

$R_f = \{f(x) \in \mathbb{R} \mid f(x) \neq 4\}$

$y = a(x-h)^2 + k \longrightarrow$ x is unrestricted (D)



RANGE: QUADRATIC, SQUARE ROOT, ABSOLUTE VALUE

$a > 0 \Rightarrow R = \{f(x) \in \mathbb{R} \mid f(x) \geq c\}$

$a < 0 \Rightarrow R = \{f(x) \in \mathbb{R} \mid f(x) \leq c\}$

Example 1.4.3 (*From Pg. 37 in your text…use Desmos*)

9. Determine the domain and range of each function.

a) $f(x) = -3x + 8$

LINEAR $\rightarrow f(x) = x$

$D_f = \{x \in \mathbb{R}\}$

$R_f = \{f(x) \in \mathbb{R}\}$

All lines are usually UNRESTRICTED

EXCEPTIONS

HORIZONTAL LINES $y = a$
Domain is unrestricted but Range is restricted

VERTICAL LINES $x = b$
Domain is restricted but Range is Unrestricted.

d) $p(x) = \frac{2}{3}(x-2)^2 - 5$

$V(2, -5)$

QUADRATIC $f(x) = x^2$

$D_p = \{x \in \mathbb{R}\}$

$R_p = \{p(x) \in \mathbb{R} \mid p(x) \geq -5\}$

f) $r(x) = \sqrt{5 - x}$

should be positive $+0$

SQUARE ROOT $f(x) = \sqrt{x}$

$5 - x \geq 0$

$\boxed{5 \geq x}$

$D_r: \{x \in \mathbb{R} \mid x \leq 5\}$

$R_r = \{r(x) \in \mathbb{R} \mid r \geq 0\}$

g) $f(x) = \frac{5}{3x - 2} + 4$

c

RECIPROCAL $f(x) = \frac{1}{x}$

$3x - 2 \neq 0 \Rightarrow x \neq \frac{2}{3}$

$D_f = \{x \in \mathbb{R} \mid x \neq \frac{2}{3}\}$

$R_f = \{f(x) \in \mathbb{R} \mid f(x) \neq 4\}$

Range for RECIPROCAL: $\{f(x) \in \mathbb{R} \mid f(x) \neq c\}$

Example 1.4.4

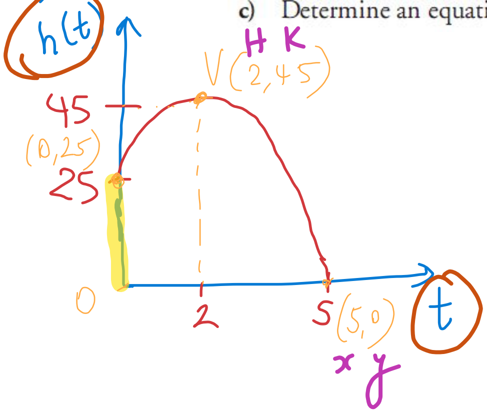
10. A ball is thrown upward from the roof of a 25 m building. The ball reaches a max height of 45 m above the ground after 2 s and hits the ground 5 s after being thrown.

a) Sketch a graph that shows the height of the ball as a function of time.

b) State the domain and range of the function.

c) Determine an equation for the function.

a)



b) $D_h = \{t \in \mathbb{R} \mid t \geq 0\}$

$R_h = \{h(t) \in \mathbb{R} \mid 0 \leq h(t) \leq 45\}$

c) $y = a(x - H)^2 + K$

$0 = a(5-2)^2 + 45$

$0 = 9a + 45$

$-45 = 9a$

$\boxed{-5 = a}$

$\therefore$ EQUATION:-

$y = -5(x-2)^2 + 45$

$\boxed{h(t) = -5(t-2)^2 + 45}$

HW- Section 1.3/1.4
Domain and Range Handout

Success Criteria:

- I can identify the unique characteristics of five basic types of functions
- I can identify the domain and ranges of five basic types of functions
- I can identify when there are restrictions given real-world situations

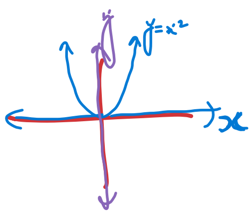
1.6 – 1.8: Transformations of Functions *(Part 1)*

Learning Goal: We are learning to apply combinations of transformations in a systematic order to sketch graphs of functions.

To **TRANSFORM** something is to CHANGE THE ORIGINAL FORM (PARENT)

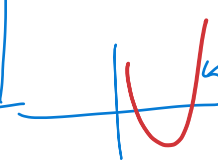
TRANSFORMATIONS OF FUNCTIONS can be seen in two ways: algebraically, and graphically. We'll begin by examining transformations graphically.

But before we do, we need to remember that the **GRAPH OF A FUNCTION**, $f(x)$, is given by:



SET of all points

$$\boxed{f(x)=\left\{\left(x,f(x)\right)\mid x\in D_f\right\}}$$



SKETCH of the graph.

So, for functions we have two things (NUMBERS!) to "transform". We can apply transformations to

1) **Domain** values (which we call **HORIZONTAL TRANSFORMATIONS**)
2) **Range** values (which we call **VERTICAL TRANSFORMATIONS**)

There are **THREE BASIC FUNCTIONAL TRANSFORMATIONS**

1) Flips (*Reflections "across" an axis*)
2) Stretches (*Dilations*)
3) Shifts (*Translations*)

So, we can have **Horizontal** flips, stretches and/or shifts, and **Vertical** flips, stretches and/or shifts. Now let's take a look at how transformations can be applied to functions.

***Note*:** We'll (mostly) be applying transformations to our so-called "parent functions" (although applying transformations to linear functions can seem pretty silly!)

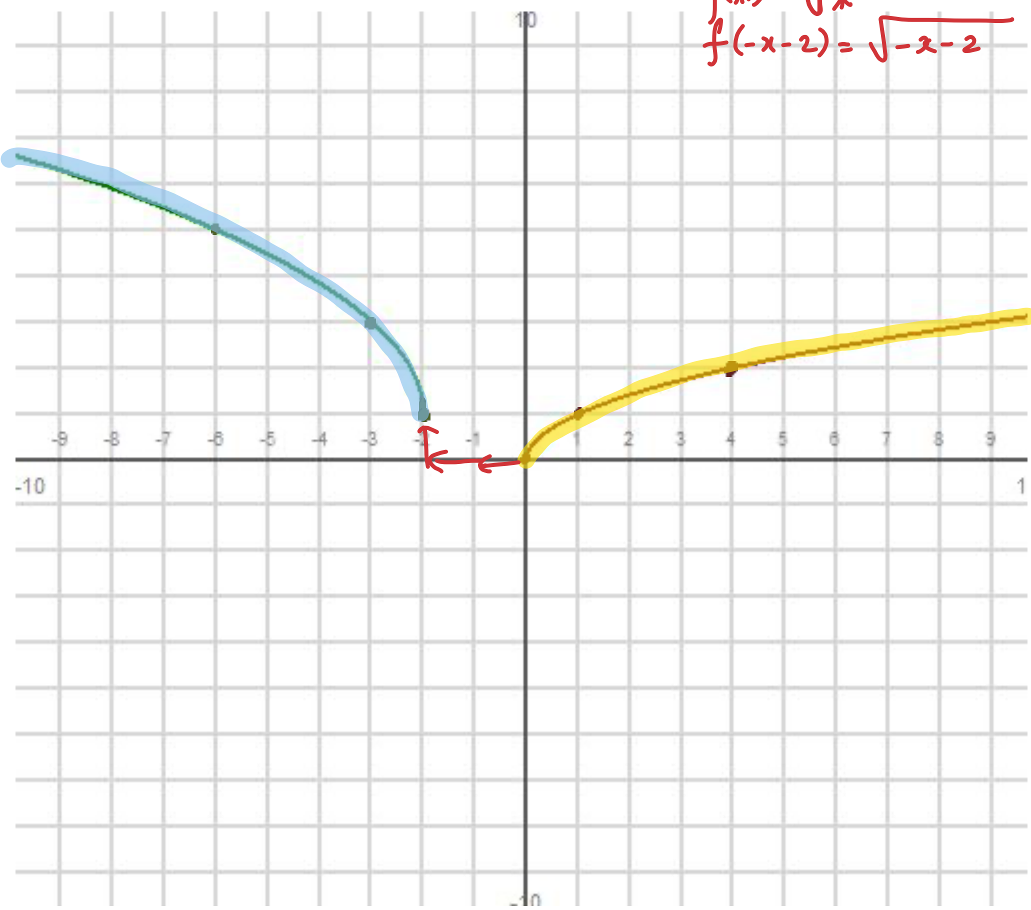
Example 1.8.1

Consider, and make observations concerning the sketch of the graph of the parent function $f(x)=\sqrt{x}$ and the transformed function $g(x)=2\sqrt{-x-2}+1$. $= 2f(-x-2)+1$

$2f(-(x+2))+1$

$f(*)=\sqrt{*}$

$f(-x-2)=\sqrt{-x-2}$



Horizontal Transformations

H. SHIFT TO LEFT (2 UNITS)

H. FLIP.

Vertical Transformations

V. SHIFT UP (1 UNIT)

V. STRETCH

Note: In the above example we can **algebraically** describe $g(x)$ as a transformed $f(x)$ with the functional equation $g(x)=2f\left(-x-2\right)+1$

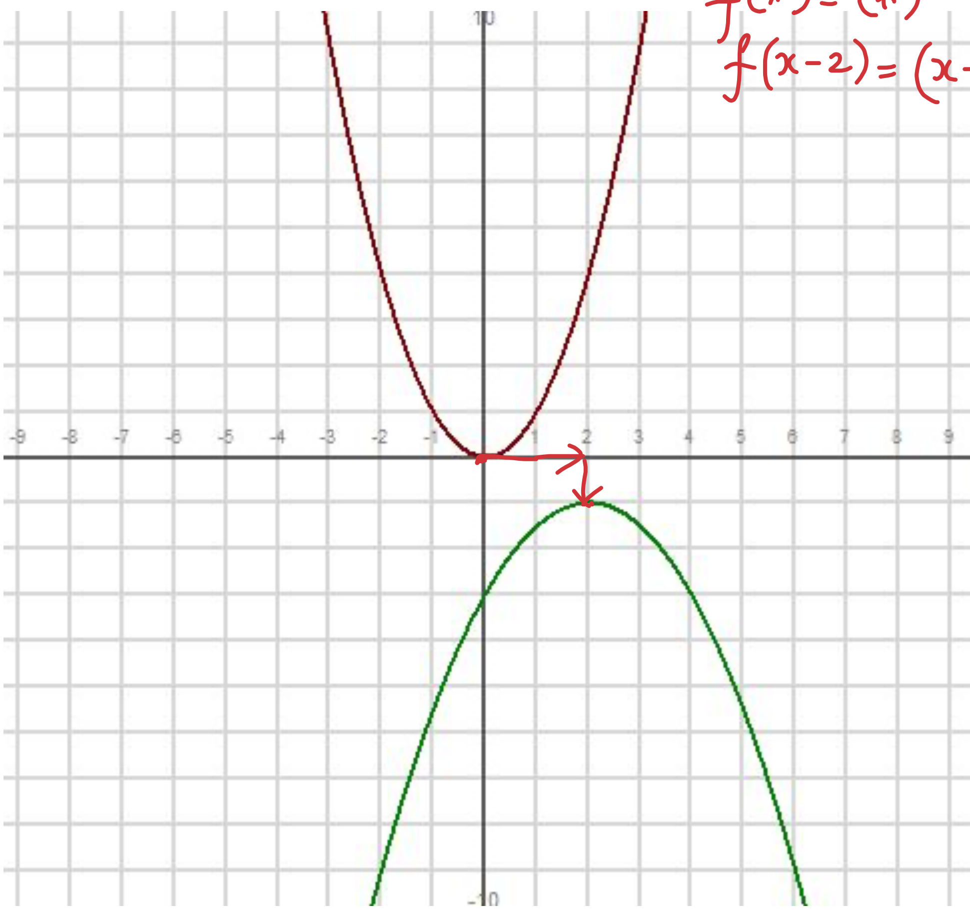
Example 1.8.2

Consider, and make observations concerning the sketch of the graph of the parent function $f(x)=x^2$ and the transformed function $g(x)=-\frac{1}{2}(x-2)^2-1$ $= -0.5 f(x-2)^2 -1$

H. SHIFT RIGHT

$f(*)=(*)^2$

$f(x-2)=(x-2)^2$



Horizontal Transformations

H. SHIFT (2 UNITS RIGHT)

H. STRETCH

Vertical Transformations

V. SHIFT (1 UNIT DOWN)

V. FLIP

V. STRETCH

Note: In the above example we can **algebraically** describe $g(x)$ as a transformed $f(x)$ with the functional equation

1.6 – 1.8: Transformations of Functions *(Part 2)*

We now turn to examining Transformations of Functions from an algebraic point of view (although a geometric perspective will still shine though!)

Definition 1.8.1

Given a function $f(x)$ we can obtain a related function through functional transformations as

$$g(x)=af\left(k(x-d)\right)+c$$, where

VERTICAL TRANSFORMATIONS:

$a \rightarrow$ LEADING COEFFICIENT

V. FLIP if $a<0$

V. STRETCH by factor a

$c \rightarrow$ CONSTANT at the end.

V. SHIFT (up or down)

HORIZONTAL TRANSFORMATIONS

$k \rightarrow$ MUST ALWAYS BE FACTORED OUT !!! **

H. FLIP if $k<0$

H. STRETCH by a factor of $\frac{1}{k}$

$d \rightarrow$ CONSTANT on the INSIDE

H. SHIFT (left or right) (INSIDE SIGN is OPPOSITE)

Example 1.8.3

Consider the given function. State its parent function, and all transformations.

$$f(x)=3\sqrt{-x+2}-1 = 3\sqrt{-(x-2)}-1$$

$f(x)=\sqrt{x}$

Horizontal Transformations

H. FLIP

(NO H. STRETCH)

H. SHIFT (2 UNITS RIGHT)

Vertical Transformations

(NO V. FLIP)

V. STRETCH ($a=3$)

V. SHIFT (1 UNIT DOWN)

Example 1.8.4

The basic absolute value function $\boxed{f(x)=|x|}$ has the following transformations applied to it: **Vertical Stretch** -3**, Vertical Shift** 1 **up, Horizontal Shift** 5 **right.**

Determine the equation of the transformed function.

$$g(x) = af(k(x-d))+c$$

$$g(x) = -3|x-5|+1$$

Back to a geometric point of view

Sketching the graph of a transformed function can be relatively easy if we know:

1) The shape of the parent function AND a few (3 or 4) points on the parent.
2) How transformations affect the points on the parent
   i) Horizontal transformations affect the domain values (***OPPOSITE!!!!!!***)
   ii) Vertical transformations affect the range values

Note: Given a point on some parent function which has transformations applied to it is called an ***IMAGE POINT*** on the transformed function.

Example 1.8.5

Given the sketch of the function $f(x)$ determine the image points of the transformed function $-2f\left(\frac{1}{3}(x+1)\right)+3$ and sketch the graph of the transformed function.

Points of transformed functions

$$\left(\frac{1}{k}x+d,\ ay+c\right)$$

$k=\frac{1}{3}\ \therefore\ \frac{1}{k}=3$

$d=-1$

$a=-2\ ;\ c=3$

$$-2f\left(\frac{1}{3}(x+1)\right)+3$$

PARENT TABLE

x	y
-4	2
-3	4
-1	-1
3	-1
5	3

TRANSFORMED TABLE

$3x-1$ $\frac{1}{k}x+d$	$-2y+3$ $ay+c$
$3(-4)-1$ -13	$-2(2)+3$ -1
$3(-3)-1$ -10	$-2(4)+3$ -5
$3(-1)-1$ -4	$-2(-1)+3$ 5
$3(3)-1$ 8	$-2(-1)+3$ 5
$3(5)-1$ 14	$-2(3)+3$ -3

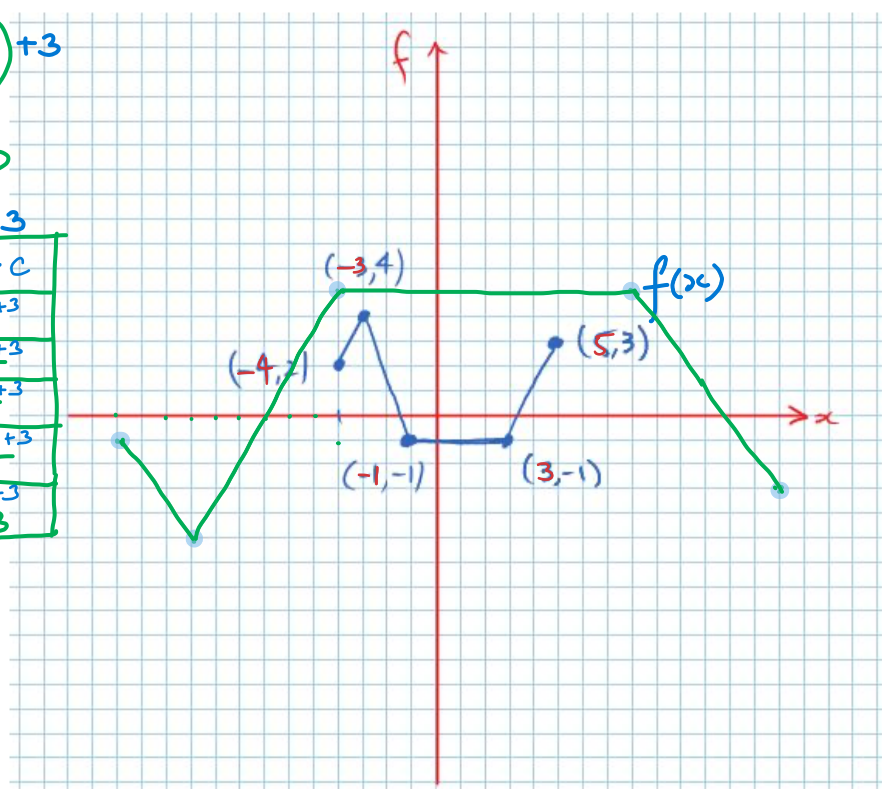


IMAGE POINTS:

$(-13,-1)$
$(-10,-5)$
$(-4,5)$
$(8,5)$
$(14,-3)$

$-3x+12 \geq 0$
$12 \geq 3x$
$4 \geq x$

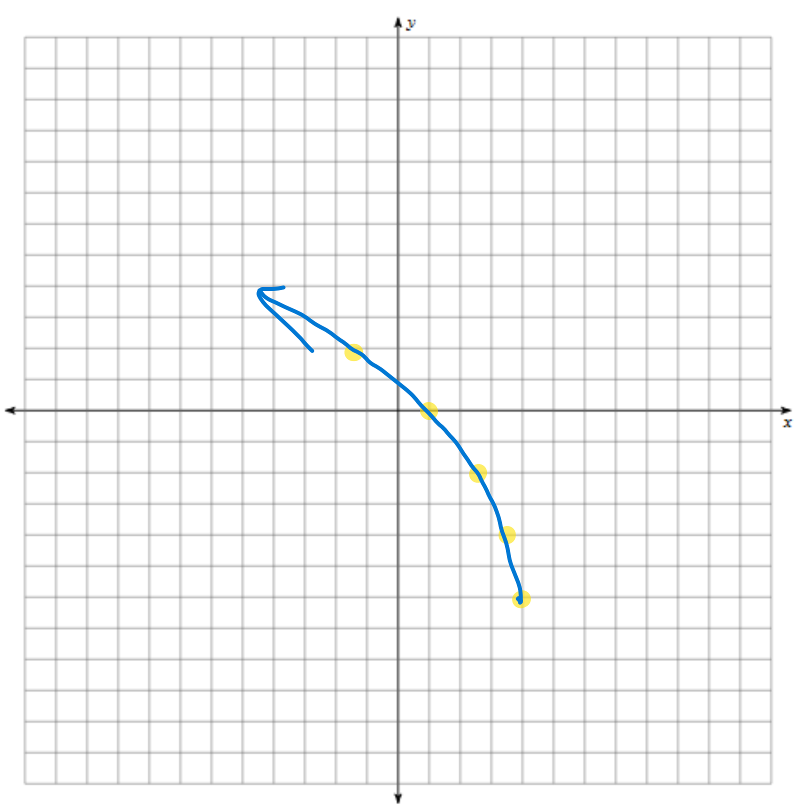
$f(x)=\sqrt{x} \Rightarrow f(-3x+12)=\sqrt{-3x+12}$

Function	Proper Function $f(x) = a\,f\left(k(x-d)\right)+c$	Vertical Stretch *a*	Horizontal Stretch *1/k*	Horizontal Shift *d*	Vertical Shift *c*
$e(x)=2\sqrt{-3x+12}-6$ $\frac{-3x}{-3} \quad \frac{12}{-3}$	$=2f(-3(x-4))-6$	2	$-1/3$	4	-6

Domain		Range		y-int (x=0)	
Domain	$D=\{x\in\mathbb{R} \mid x\leq 4\}$	Range	$R=\{f(x)\in\mathbb{R} \mid f(x)\geq -6\}$	y-int (x=0)	$y_{int}=2\sqrt{-3(0)+12}-6$ $=2\sqrt{12}-6$ ≈ 0.93

Table Of Values	Parent Function: x	y	Transformed Function $\frac{1}{k}x+d$	$ay+c$
	x	$e(x)=\sqrt{x}$	$-\frac{1}{3}x+4$	$2y-6$
	0	0	4	−6
	1	1	3.7	−4
	4	2	2.7	−2
	9	3	1	0
	16	4	−1.3	2

STARTING POINT



Extra work space.

HW- Section 1.6-1.8

Big Handout

Success Criteria:

- I can use the value of a to determine if there is a vertical stretch/reflection in the x-axis
- I can use the value of k to determine if there is a horizontal stretch/reflection in the y-axis
- I can use the value of d to determine if there is a horizontal translation
- I can use the value of c to determine if there is a vertical translation
- I can transform x coordinates by using the expression $\frac{1}{k}x + d$
- I can transform y coordinates by using the expression $ay + c$