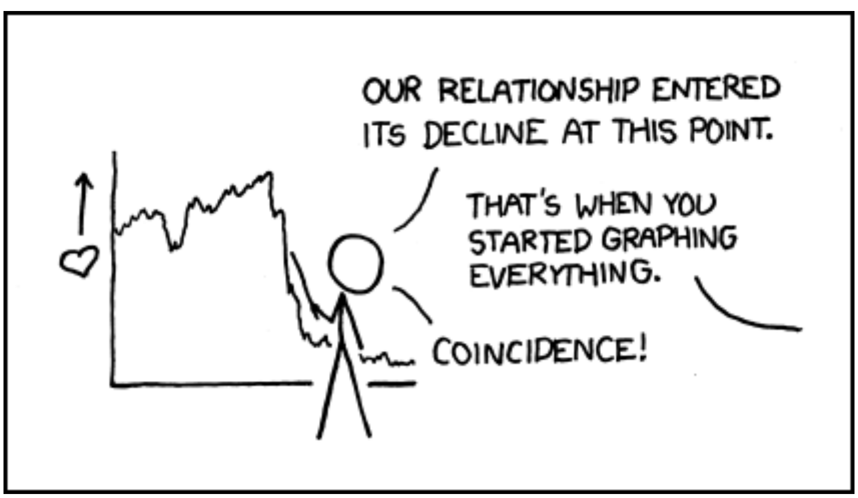
Functions 11

Course Notes

Chapter 1 – Introduction to Functions

We will learn

- *the meaning of the term Function and how to use function notation to calculate and represent functions*
- *the meanings of the terms domain and range, and how a function's structure affects domain and range*
- *how to use transformations to represent and sketch graphs*



1.1 Relations and Functions (*This is a KEY lesson*!)

Learning Goal: We are learning how to identify the difference between a function and a relation. Also, learning to recognize functions in various representations.

This course is called **FUNCTIONS**, so it seems rather important that you know what a function actually is. However, before we define and dive into the world of functions, it is important to be familiar with a few other commonly used terms in the language of functions.

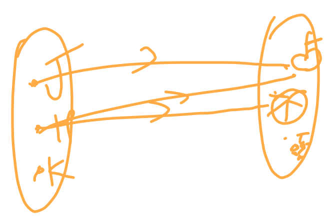
In Grades 9 and 10, you learned about lines ($y = mx + b$) and parabolas ($y = ax^2 + bx + c$). Little did you know, these are called *functions*. Before we get into a formal definition of a function, let's first look at something more familiar, a *relation*.

A ***relation*** is an equation where there is a connection/relationship between two quantities represented by x and y

Ex: $y = 3x + 5$ $y = 5x^2$

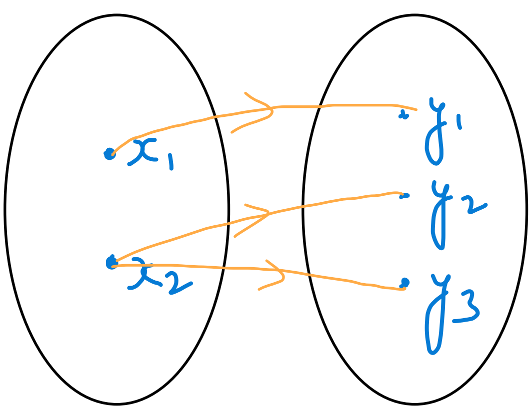
Typically, x is the INDEPENDENT variable (INPUT)

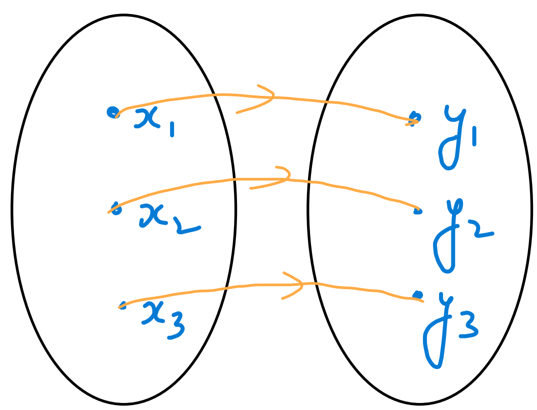
And y is the DEPENDENT variable (OUTPUT)



A relation can be represented in a few ways:

1. **Mapping Diagram**



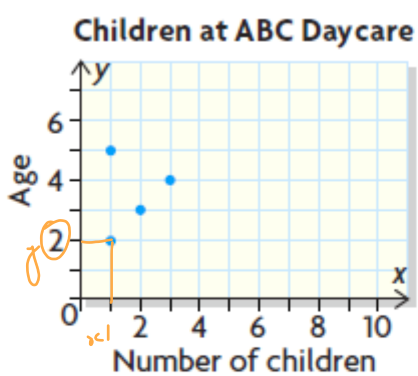


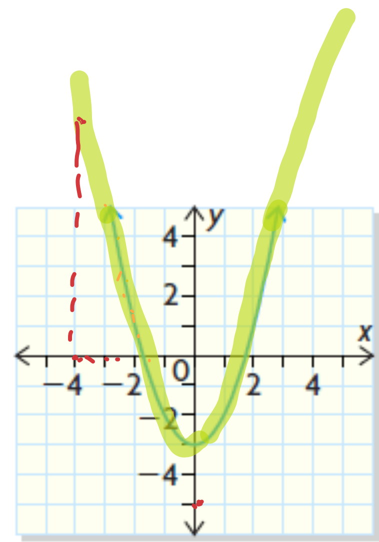
2. **Equation** eg $y = 5x$ $y = 3x^2 + 2x + 5$

3. **Table**

km Driven x	Cost of Rental y
10	50
50	80
70	95
100	110

4. **Graph**





D = {all real values of x}

R = {all real values of y which are greater than −3}

5. **Set of Ordered Pairs**

{(-1,-3), (0,1), (1,1), (2,9)}

D = {−1, 0, 1, 2}

R = {−3, 1, 1, 9}

{(1,4), (3,2), (0,5), (5,6), (3,0)}

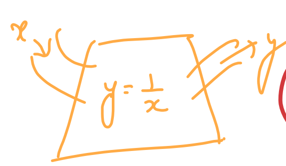
D = {1, 3, 0, 5}

R = {4, 2, 5, 6, 0}

Before we define what a function is, we first need to define a few other things:

Domain:

The set of all acceptable inputs (x-values)



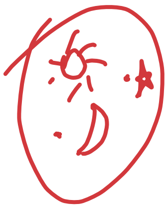
Range:

The set of all acceptable outputs (y-values)

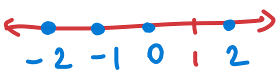
Set Notations: *Some fancy ways to represent sets of Numbers!*

ROSTER FORM

LISTABLE

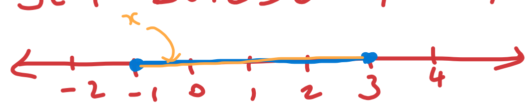


{☼, ☽, ★}



$\{-2, -1, 0, 2\}$

SET BUILDER FORM

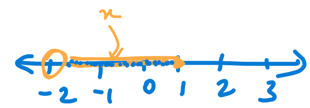


$\{x \in \mathbb{R} \mid -1 \le x \le 3\}$

x belongs to the set of REAL #s

such that

(or) x is a real #



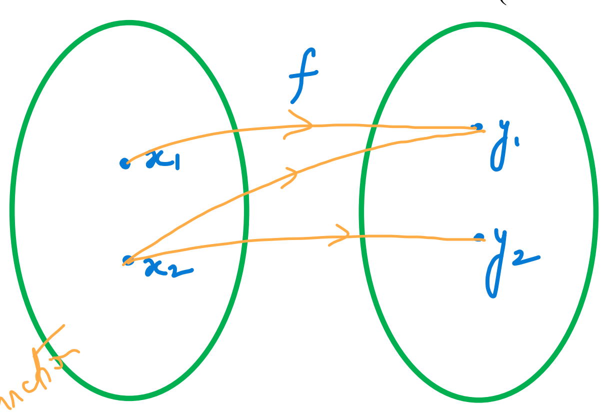
$\{x \in \mathbb{R} \mid -2 < x \le 1\}$

Now comes the moment finally!! We are ready to explore and understand **FUNCTIONS**! 😊

You need to know, very well, the following (algebraic) definition:

> **Definition 1.1.1**
>
> A **FUNCTION** is a very very special relation where each x-value (input value) has a unique (only one) y-value (output value) corresponding it.

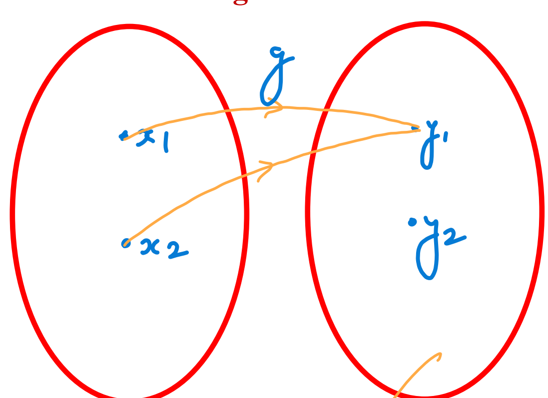
We can visualize what a function is (and **isn't**) by using so-called "**arrow diagrams**":



✗ Not function

$D_f: \{x_1, x_2\}$

$R_f: \{y_1, y_2\}$

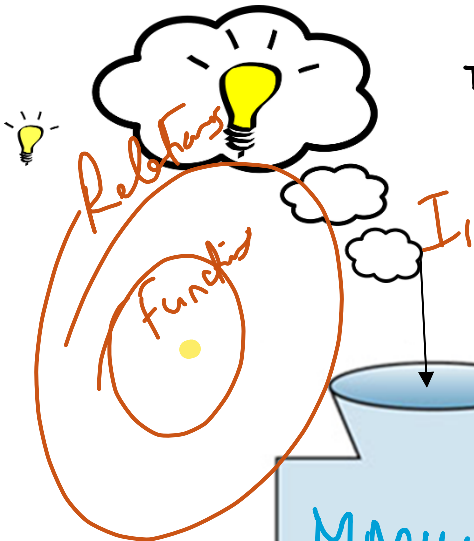


✓ function

$D_g: \{x_1, x_2\}$

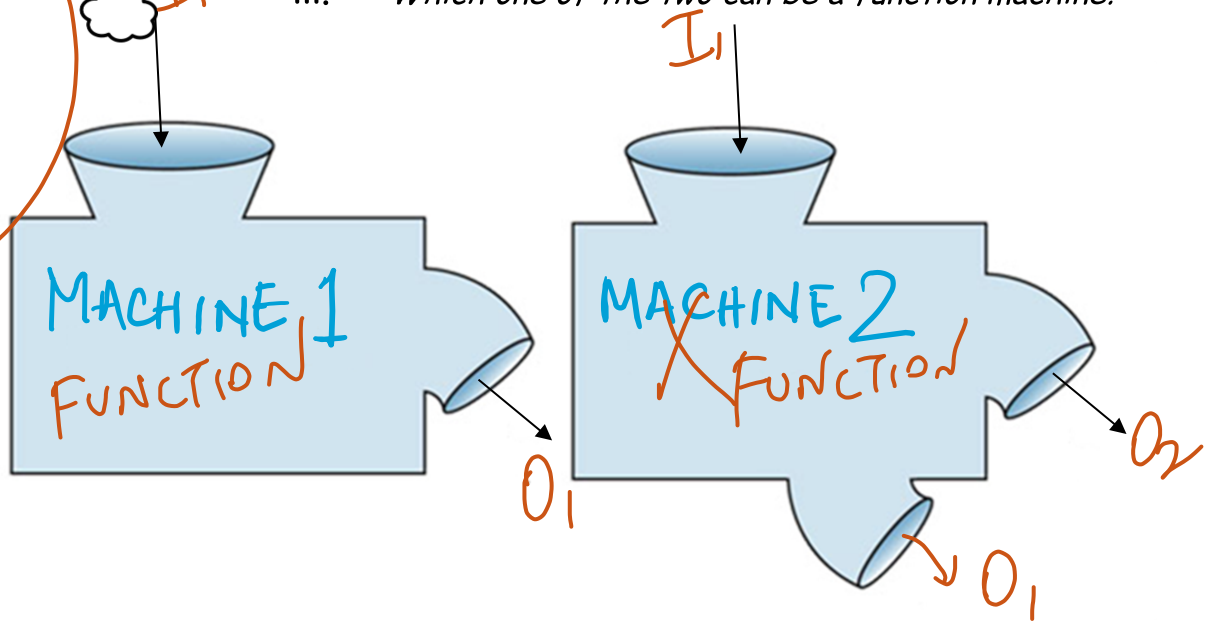
$R_g: \{y_1\}$

We can also view ***Functions*** visually like a ***Vending Machine*** because they are ***PREDICTABLE*** just like our functions***!!***



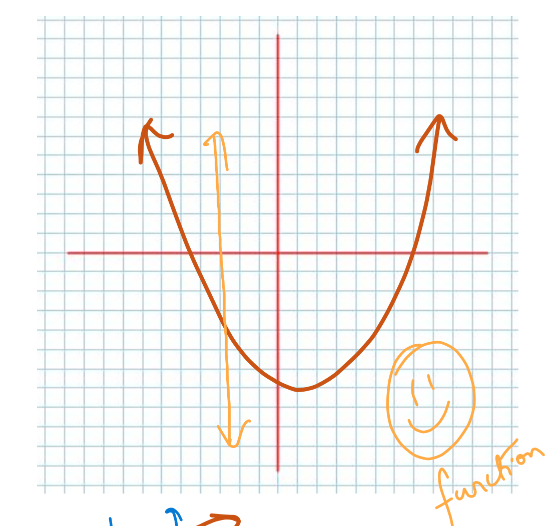
THINKING QUESTION:

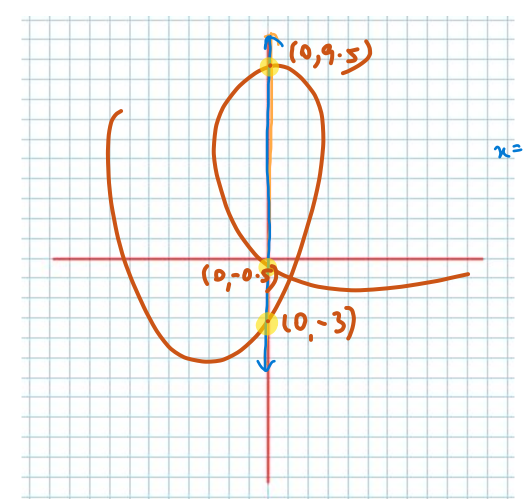
i. *Are all Functions also Relations?* ……YES……
ii. *Are all Relations also Functions?* ……No……
iii. *Which one of the two can be a function machine?*

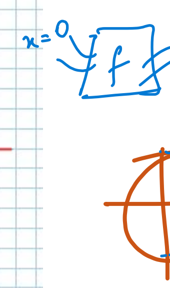


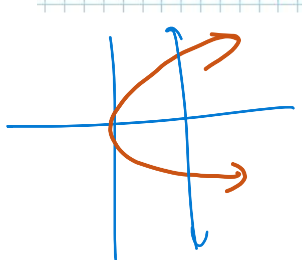
KNOWING WHEN A RELATION IS, AND ISN'T, A FUNCTION

Graphically: *The Vertical Line Test* (VLT)









***Algebraically**: (*NOTE: this is a "rough" way of thinking about the problem*)

If the Dependent Variable ~~is~~ y has an even exponent → Not a function
odd exponent → a function.

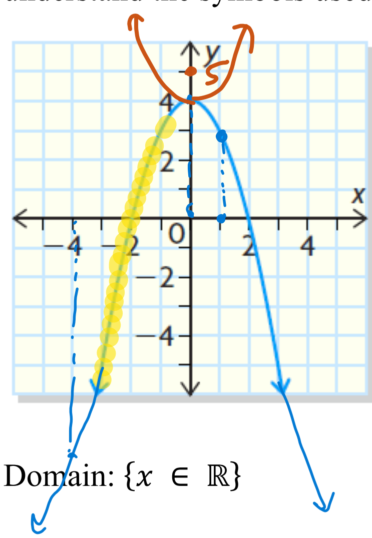
e.g. $y^1 = 5x + 3$

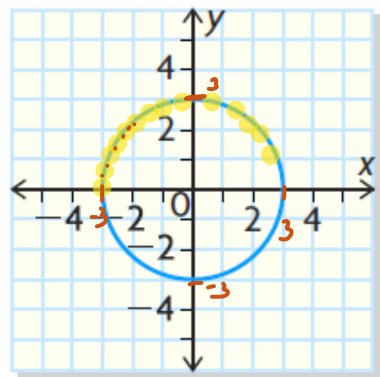
$y^1 = 7(x-2)^2 + 3$

$x^2 + y^2 = 25$

Let's go back and determine the domain and range and whether or not each relation is a function.

Domain and Range can be represented in just words ("x can be any number"), but Math is all about representing things in numbers and symbols. This is what makes math universal, because people in CoCoLoCo island may not understand "x can be any number", but they would understand the symbols used to represent that.





Domain: $\{x \in \mathbb{R}\}$

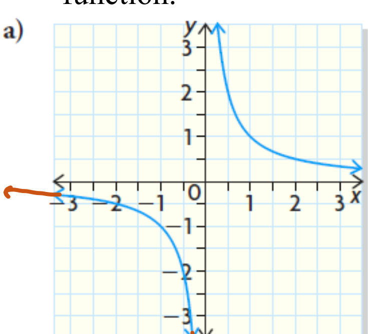
Range: $\{\cancel{x}\, y \in \mathbb{R} \mid y \le 4\}$

Domain: $\{x \in \mathbb{R} \mid -3 \le x \le 3\}$

Range: $\{y \in \mathbb{R} \mid -3 \le y \le 3\}$

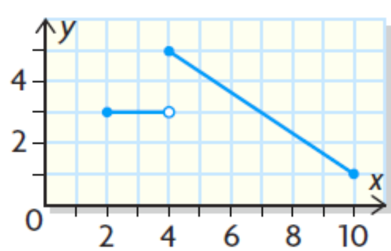
For the following, ~~determine the domain and range using set notation~~, and then state if it is a function.

a)



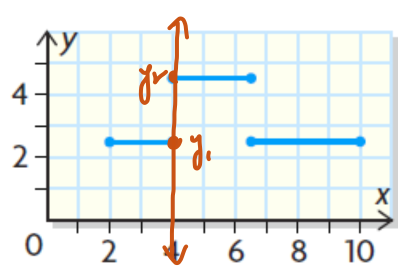
It is a function (passes VLT)

b)



It is a function (passes VLT)

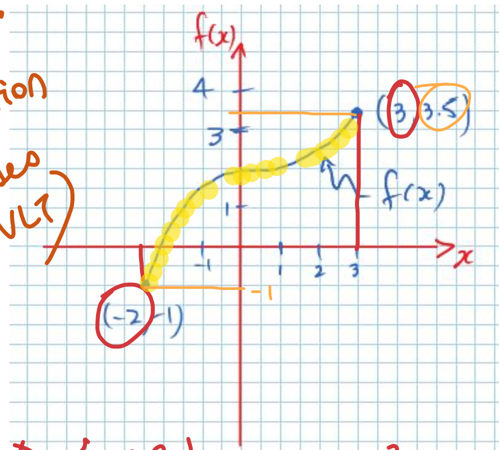
c)



X Not function (fails VLT)

Given the **SKETCH OF THE GRAPH** of the **RELATION** determine: the domain, the range of the relation, and whether the relation is, or is not, a function. *(Note that domain and range are sets of numbers and can be represented by the fancy* ***set notation****)*

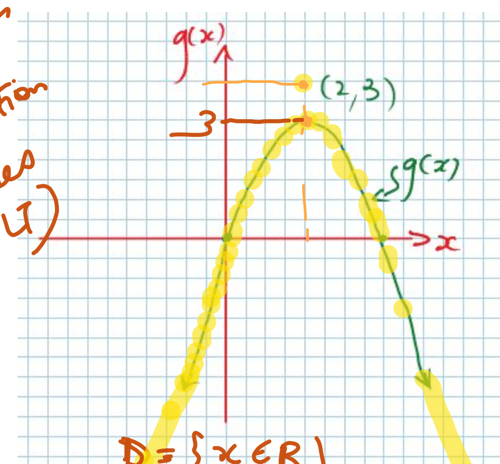
a) ✓ function (passes VLT)



$D = \{x \in \mathbb{R} \mid -2 \le x \le 3\}$

$R = \{y \in \mathbb{R} \mid -1 \le y \le 3.5\}$

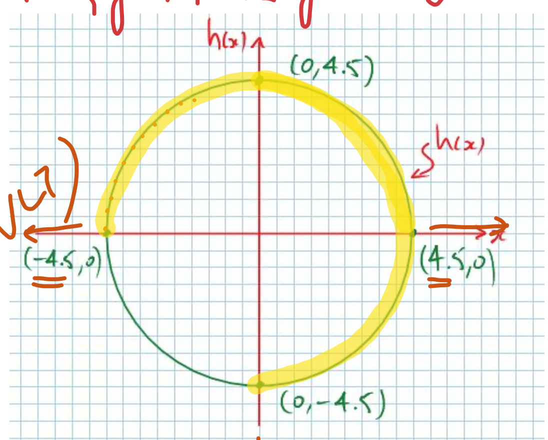
b) ✓ function (passes VLT)



$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R} \mid y \le 3\}$

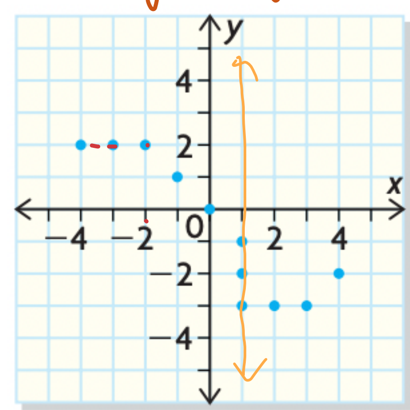
c). ✗ Not function (fails VLT)



$D = \{x \in \mathbb{R} \mid -4.5 \le x \le 4.5\}$

$R = \{y \in \mathbb{R} \mid -4.5 \le y \le 4.5\}$

d) ✗ Not function



$D = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$R = \{-3, -2, -1, 0, 1, 2\}$

HW- Section 1.1 *(Suggested problems are from the Nelson Textbook. You are welcome to ask for help from your peers or myself. It is due the next day!)*

Pg. 10 – 12 #1, 2 (*no ruler needed*), 6, 7 (*no need for the VLT*), 9, 11, 12 (*think carefully about the idea that the domain and range are "limited"*)

Success Criteria:

- I can determine the domain of a relation or function as the set of all values of the independent variable
- I can determine the range of a relation or function as the set of all values of the dependent variable
- I can apply the vertical line test to determine if a graph is a function
- I can recognize whether a relation is a function from its equation

 Name_____________________.

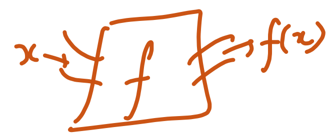
1.2 Function Notation

Learning Goal: We are learning to use function notation to represent linear and quadratic functions

Here we learn a **NEW AND IMPROVED WAY** for describing a function, algebraically.

When we determine that a relation is a function, such as $y = 3x+4$, it is worthwhile to state that it is a function by giving it a name and indicating what the independent variable is.

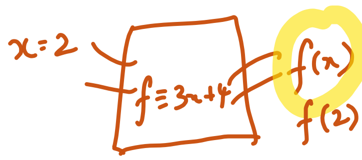
$y = 3x + 4 \longrightarrow f(x) = 3x + 4$



This much more useful way of writing y = f(x) is called the **FUNCTION NOTATION**.

Here, *x* is the independent variable,
which is used to determine the functional value (formerly known as *y*).

Let's look at how this works: Given $\boxed{f(x) = 3x+4}$, evaluate $f(2)$



$f(2) = 3(2)+4$
$= 6+4$
$= 10$
$\boxed{f(2)=10}$

Let's do some examples

1. Given $f(x) = 2x^2 + 3x - 1$, evaluate

a) $f(3)$ b) $f\left(\frac{1}{2}\right)$ c) $f(5-3)$ d) $f(5) - f(4)$

a) $f(3) = 2(3)^2 + 3(3) - 1$
$f(3) = 18 + 9 - 1$
$y = f(3) = 26$

$= f(0.5)$

b) $f(0.5) = 2(0.5)^2 + 3(0.5) - 1$
$f(0.5) = 0.5 + 1.5 - 1$
$f(0.5) = 1$

c) $f(5-3) = f(2) = 2(2)^2 + 3(2) - 1$
$= 8 + 6 - 1$
$f(2) = 13 \rightarrow (2, 13)$ (x, y)

d) $f(5) = 2(5)^2 + 3(5) - 1$
$= 50 + 15 - 1$
$\boxed{f(5) = 64}$

$f(4) = 2(4)^2 + 3(4) - 1$
$= 32 + 12 - 1$
$\boxed{f(4) = 43}$

$\therefore f(5) - f(4) = 64 - 43$
$= 21$

2. Given $g(x) = 5x-8$, determine the x so that $g(x)=18$

$x = \rightarrow \boxed{g} \rightarrow g(x)=18$

$\Rightarrow 18 = 5x - 8$

$\Rightarrow 18+8 = 5x$

$\Rightarrow \frac{26}{5} = \frac{5x}{5}$

$\Rightarrow \boxed{5.2 = x}$

$\boxed{g(5.2) = 18}$

$(x, g(x))$

3. Evaluate $f(3)$ for each of the following.

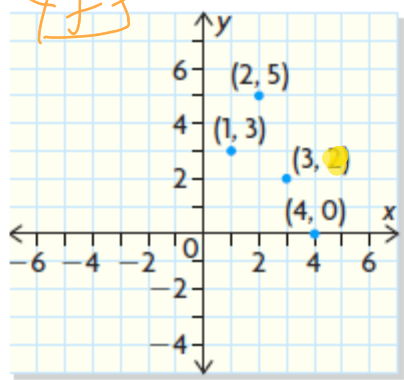
$3 \rightarrow \boxed{f} \rightarrow ?$

a) $\{(1, 2), (2, 0), (3, 1), (4, 2)\}$

b)

x	1	2	3	4
y	2	3	4	5

c)



a) $f(3) = 1$

b) $f(3) = 4$

c) $f(3) = 2$

4. Lastly, something a little strange: given $h(x) = 2x^2 - 3x + 4$ evaluate $h(a)$ and $h(x-2)$

$h(a) = 2a^2 - 3a + 4$

$h(x-2) = 2(x-2)^2 - 3(x-2) + 4$

$= 2(x^2 - 4x + 4) - 3(x-2) + 4$

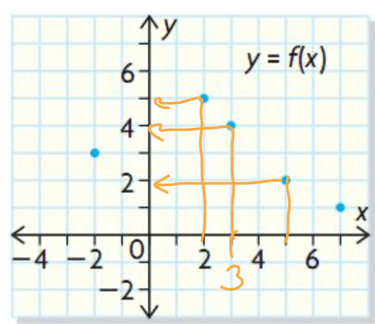
$= 2x^2 - 8x + 8 - 3x + 6 + 4$

$= 2x^2 - 11x + 18$

$\Rightarrow h(x-2) = 2x^2 - 11x + 18$

One more Example-

6. The graph of $y = f(x)$ is shown at the right.
   a) State the domain and range of f.
   b) Evaluate.
      i) $f(3)$
      ii) $f(5)$
      iii) $f(5-3)$
      iv) $f(5) - f(3)$



(i) $f(3) = 4$

(ii) $f(5) = 2$

(iii) $f(2) = 5$

(iv) $2 - 4 = -2$

Some more practice:

Textbook Example 1.2.4

7. For $\boxed{h(x) = 2x - 5,}$ determine

a) $h(a)$ **b)** $h(b + 1)$ **c)** $h(3c - 1)$ **d)** $h(2 - 5x)$

a) $h(a) = 2a - 5$

b) $h(b+1) = 2(b+1) - 5$
$= 2b + 2 - 5$
$= 2b - 3$

c) $h(3c-1) = 2(3c-1) - 5$
$= 6c - 2 - 5$
$= 6c - 7$

d) $h(2-5x)$
$= 2(2-5x) - 5$
$= 4 - 10x - 5$
$= -10x - 1$

Example 1.2.5

12. A company rents cars for \$50 per day plus \$0.15/km.

a) Express the daily rental cost as a function of the number of kilometres travelled. $y = f(x)$ x

b) Determine the rental cost if you drive 472 km in one day.

c) Determine how far you can drive in a day for \$80.

a) $f(x) = 0.15x + 50$

b) $f(x = 472) = 0.15(472) + 50 = 70.8 + 50 = \$120.80$

c) $f(x) = 80 \Rightarrow x = ?$

$\therefore \; 80 = 0.15x + 50$

$x = \dfrac{80 - 50}{0.15} = \dfrac{30}{0.15} = 200$

HW- Section 1.2

Page 23 #1-2, 5, 8b, 10, 11cd, 15, 16, challenge #17

Success Criteria:

- I can evaluate functions using function notation, by substituting a given value for x in the equation for f(x)
- I can recognize that f(x) = y corresponds to the coordinate (x, y)
- I can, given y = f(x), determine the value of x

1.3 and 1.4 Parent Functions and Domain and Range

Learning Goal: We are learning the graphs and equations of five basic functions; and using their tables, graphs, or equations to find their domains and ranges.

THE PARENT FUNCTIONS *(for Grade 11U)*

Together we will explore (graphically) basic properties of the five *parent* functions:

a) Linear $f(x) = x$

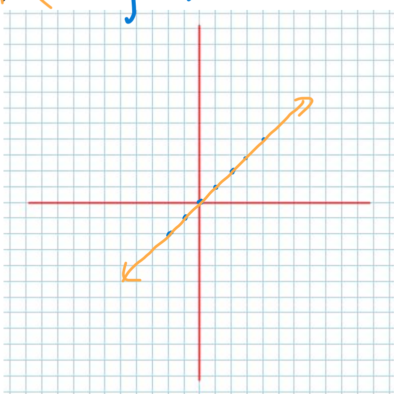


TABLE OF VALUES

x	$f(x)$	$(x, f(x))$
-2	-2	$(-2, -2)$
-1	-1	$(-1, -1)$
0	0	$(0, 0)$
1	1	$(1, 1)$
2	2	$(2, 2)$

$D = \{x \in \mathbb{R}\}$

$R = \{f(x) \in \mathbb{R}\}$

The graph is a LINE

b) Quadratic $g(x) = x^2$

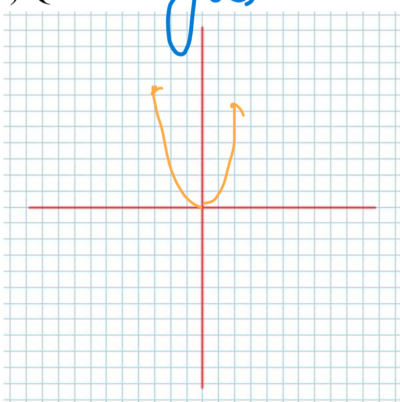


TABLE OF VALUES

x	$g(x)$	$(x, g(x))$
-2	4	$(-2, 4)$
-1	1	$(-1, 1)$
0	0	$(0, 0)$
1	1	$(1, 1)$
2	4	$(2, 4)$

The graph is a Parabola

$D = \{x \in \mathbb{R}\}$

$R = \{g(x) \in \mathbb{R} \mid g(x) \geq 0\}$

c) Absolute Value $h(x) = |x|$

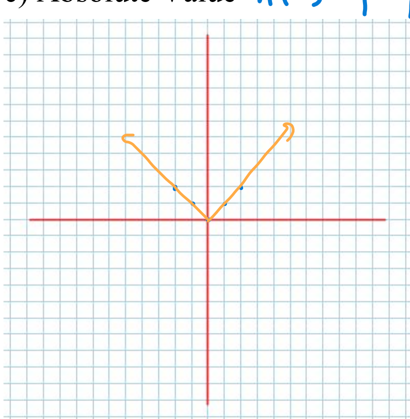


TABLE OF VALUES

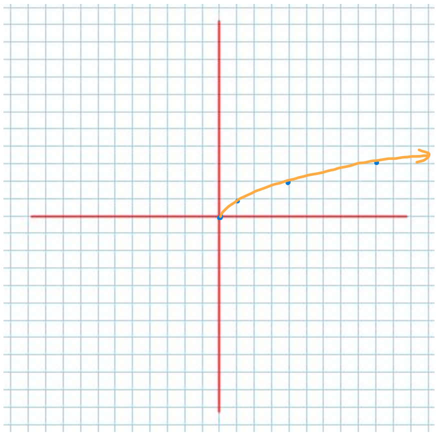
x	$h(x)$	$(x, h(x))$
-2	2	$(-2, 2)$
-1	1	$(-1, 1)$
0	0	$(0, 0)$
1	1	$(1, 1)$
2	2	$(2, 2)$

The graph is a V-shape

$D = \{x \in \mathbb{R}\}$

$R = \{h(x) \in \mathbb{R} \mid h(x) \geq 0\}$

d) Square Root $i(x) = \sqrt{x}$



$D = \{x \in R \mid x \geq 0\}$

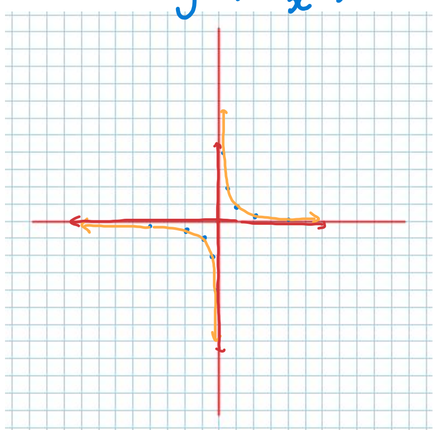
$R = \{i(x) \in R \mid i(x) \geq 0\}$

TABLE OF VALUES

x	$i(x)$	$(x, i(x))$
0	0	(0,0)
1	1	(1,1)
4	2	(4,2)
9	3	(9,3)
16	4	(16,4)
25	5	(25,5)
36	6	

The graph is a ½ Parabola to the Right

e) Reciprocal $j(x) = \frac{1}{x}, x \neq 0$



$D = \{x \in R \mid x \neq 0\}$

$R = \{j(x) \in R \mid j(x) \neq 0\}$

ASYMPTOTE ($x = 0$, $y = 0$)

TABLE OF $\frac{1}{x}$ VALUES

x	$j(x)$	$(x, j(x))$
−2	$\frac{1}{-2} = -0.5$	(−2, −0.5)
−1	$\frac{1}{-1} = -1$	(−1, −1)
−0.5	$\frac{1}{-0.5} = -2$	(−0.5, −2)
0	$\frac{1}{0}$ X Not Defined.	
0.5	$\frac{1}{0.5} = 2$	(0.5, 2)
1	$\frac{1}{1} = 1$	(1, 1)
2	$\frac{1}{2} = 0.5$	(2, 0.5)

The graph is a Hyperbola

ASYMPTOTE: It is a line to which the graph seems to be approaching but never touches.

So, we see that the **GRAPH OF A FUNCTION**, $f(x)$, is given by: $f(x) = \{(x, f(x)) \mid x \in D_f\}$

In Grade 10, we learned about transformation of quadratic functions. To **TRANSFORM** something is to

Transformations are values which change the shape, direction, and position of the function. In a quadratic function,

$$f(x) = x^2 \rightarrow f(x) = a(x-h)^2 + k$$

where **a** → Direction of Opening ($a > 0 \rightarrow$ UP, $a < 0 \rightarrow$ DOWN)
h → Horizontal Shift (Left ⟷ Right)
k → Vertical Shift (Up ⟷ Down)

Recall that **y = a(x-h)² + k** is the **Vertex Form** and is considered the ***strongest form*** for its ability to tell us about all the ***transformations***.

So, for quadratic functions(and functions in general) we have two things (NUMBERS!) to "transform". We can apply transformations to

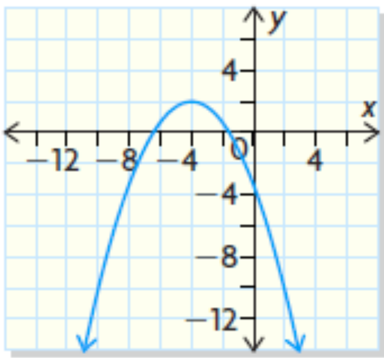
1) **Domain** values (which we call **HORIZONTAL TRANSFORMATIONS**)
2) **Range** values (which we call **VERTICAL TRANSFORMATIONS**)

$$(x, y) \longrightarrow (x + h, ay + k)$$

Also, there are **THREE BASIC FUNCTIONAL TRANSFORMATIONS**

1) Flips (*Reflections "across" an axis*)
2) Stretches (*Dilations*)
3) Shifts (*Translations*)

Example: Given *f(x)* and its graph, state the vertex, domain and range



$f(x) = a(x-h)^2 + k$

$f(x) = -\frac{1}{3}(x+4)^2 + 2$

VERTEX : $V(h,k) = (-4,2)$

$D = \{x \in R\}$

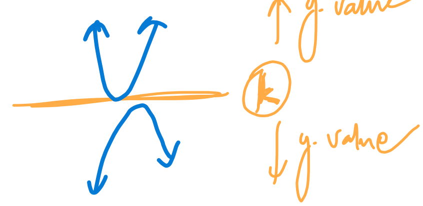
$R = \{f(x) \in R \mid f(x) \leq 2\}$

In general, given $f(x) = a(x-h)^2 + k$, the domain is ***ALWAYS*** $\{x \in R\}$

The range, however, depends on the vertical stretch, or "*a*":

If $a > 0$ $R = \{f(x) \in R \mid f(x) \geq k\}$

If $a < 0$ $R = \{f(x) \in R \mid f(x) \leq k\}$



Determine the domain and range of each quadratic function:

$f(x) = 3(x-4)^2 - 8$

$D = \{x \in R\}$

$R = \{f(x) \in R \mid f(x) \geq -8\}$

$g(x) = -23(x+365)^2 + 4303$ 4303

$D = \{x \in R\}$

$R = \{g(x) \in R \mid g(x) \leq 4303\}$

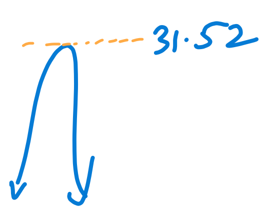
Note that sometimes the domain needs to be *restricted*. This means that instead of $\in R$ there will be some limitations to both the domain and the range.

Example: A baseball thrown from the top of a building falls to the ground below. The path of the ball is modelled by the function $h(t) = -5t^2 + 5t + 30$, where $h(t)$ is the height of the ball above ground, in metres, and t is the elapsed time in seconds. What are the domain and range of this function?

(For this unit, let's use Desmos/GeoGebra to find the vertex form of the quadratic function)

$D = \{t \in R \mid t \geq 0\}$

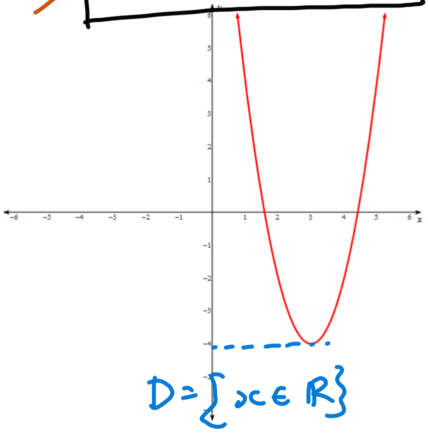
$R = \{h(t) \in R \mid h(t) \leq 31.25\}$



***Now, let's explore the transformations of other parent graphs as well.**

Determine the domain and range of the following graphed functions:

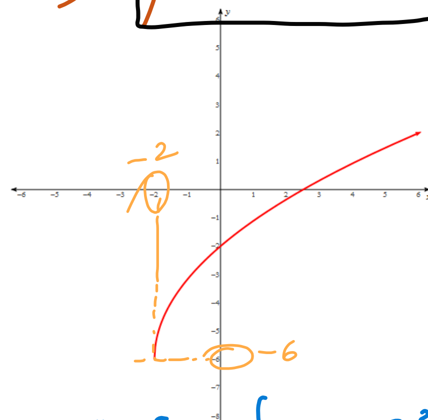
a) $\boxed{f(x) = 2(x-3)^2 - 4}$



$D = \{x \in \mathbb{R}\}$

$R = \{f(x) \in \mathbb{R} \mid f(x) \geq -4\}$

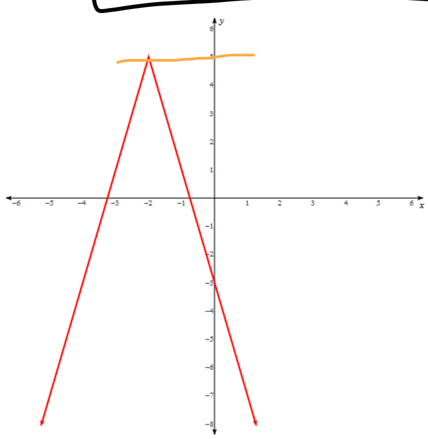
b) $\boxed{f(x) = 2\sqrt{2x+4} - 6}$



$D = \{x \in \mathbb{R} \mid x \geq -2\}$

$R = \{f(x) \in \mathbb{R} \mid f(x) \geq -6\}$

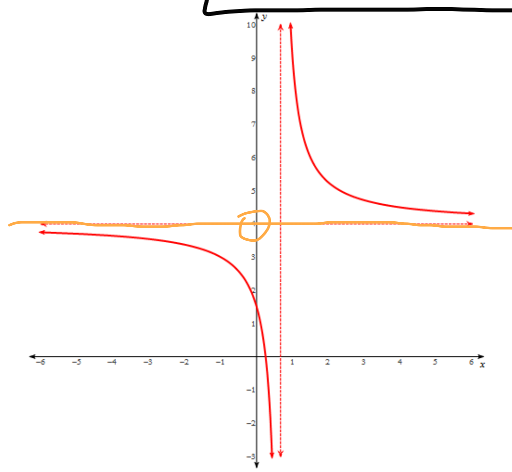
c) $\boxed{f(x) = -4|x+2| + 5}$



$D = \{x \in \mathbb{R}\}$

$R = \{f(x) \in \mathbb{R} \mid f(x) \leq 5\}$

d) $\boxed{f(x) = \frac{5}{3x-2} + 4}$



$D = \{x \in \mathbb{R} \mid x \neq 0.75\}$

$R = \{f(x) \in \mathbb{R} \mid f(x) \neq 4\}$

Rule for RANGE (Parabola, V-shape, square root)

① $R = \{f(x) \in R \mid f(x) \geq c\}$ if $a > 0$

② $R = \{f(x) \in R \mid f(x) \leq c\}$ if $a < 0$

Example 1.4.3

9. Determine the domain and range of each function.

$y = mx + b$

a) $f(x) = -3x + 8$

$D = \{x \in R\}$

$R = \{f(x) \in R\}$

$y = a(x-h)^2 + k$

d) $p(x) = \frac{2}{3}(x-2)^2 - 5$

c

(2, -5)

$D = \{x \in R\}$

$R = \{p(x) \in R \mid p(x) \geq -5\}$

f) $r(x) = \sqrt{5 - x} + 0$

c

Always move the negative in an inequality

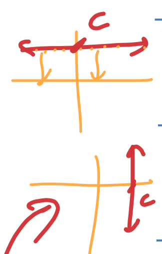
$5 - x \geq 0$

$5 \geq x$

$D = \{x \in R \mid x \leq 5\}$

$R = \{r(x) \in R \mid r(x) \geq 0\}$

EXCEPTIONS



$D = \{x \in R\}$

$R = \{f(x) \in R \mid f(x) = c\}$

$D = \{x \in R \mid x = c\}$

$R = \{f(x) \in R\}$

Not function (vertical line)

g) $f(x) = \frac{5}{3x-2} + 4$

c

$3x - 2 \neq 0$

$3x \neq 2$

$x \neq \frac{2}{3}$

$D = \{x \in R \mid x \neq \frac{2}{3}\}$

$R = \{f(x) \in R \mid f(x) \neq 4\}$

In general, R for reciprocal $f^n = \{f(x) \neq c\}$

Example 1.4.4

10. A ball is thrown upward from the roof of a 25 m building. The ball reaches a height of 45 m above the ground after 2 s and hits the ground 5 s after being thrown.

a) Sketch a graph that shows the height of the ball as a function of time.

b) State the domain and range of the function.

c) Determine an equation for the function.

HW- Section 1.3/1.4

Domain and Range Handout

Success Criteria:

- I can identify the unique characteristics of five basic types of functions
- I can identify the domain and ranges of five basic types of functions
- I can identify when there are restrictions given real-world situations

1.6 – 1.8: Transformations of Functions and Graphing them

Learning Goal: We are learning to use transformations to sketch the graphs of functions

Graphing a Quadratic function (and other functions) requires an understanding of *transformations*. Transformations are values which change the shape, direction, and position of the function. Recall from Grade 10 that in a quadratic function,

$\boxed{f(x) = x^2} \longrightarrow f(x) = a(x-h)^2 + k$

In general: $f(x) = a(x-h)^2 + k$

VERTICAL TRANSFORMATIONS:

$a \rightarrow$ V. STRETCH
V. FLIP

$a > 0$ (POSITIVE) $\Rightarrow$ PARABOLA OPENS UP

$a < 0$ (NEGATIVE) $\Rightarrow$ PARABOLA OPENS DOWN

$k \rightarrow$ V. SHIFT (UP or DOWN)

HORIZONTAL TRANSFORMATION

$h \rightarrow$ H. SHIFT (LEFT or RIGHT)

eg $y = 2(x+3)^2 + 5$
$h = -3$

$y = -0.5(x-2)^2 + 1$
$h = 2$

$(x, y) \longrightarrow (x+h, ay+k)$ IMAGE POINT

$y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

$x+h$	$ay+k$

The process to graphing is straight-forward.
1. Identify the transformations
2. Create starting points from the base "parent" function
3. Transform the starting points
4. Graph the transformed points

Now, let's generalize the transformations for all functions..

Definition 1.8.1

Given a function $f(x)$ we can obtain a related function through functional transformations as

$$g(x) = a f(k(x-d)) + c$$

where

VERTICAL TRANSFORMATION

$a \rightarrow$ V. STRETCH & V. FLIP.

$a > 0 \rightarrow$ POSITIVE $\rightarrow$ GRAPH OPENS UP

$a < 0 \rightarrow$ NEGATIVE $\rightarrow$ GRAPH OPENS DOWN

$c \rightarrow$ V. SHIFT (UP or DOWN)

HORIZONTAL TRANSFORMATION.

$\frac{1}{k} \rightarrow$ H. stretch & H. flip if (−)

$d \rightarrow$ H. SHIFT (LEFT or RIGHT)

$$(x, y) \longrightarrow \left(\frac{1}{k}x + d,\ ay + c\right)$$

Example 1.8.3

Consider the given function. State its parent function, and all transformations.

$$g(x) = 3\sqrt{\underset{-1}{-x}+\underset{-1}{2}} - 1$$

PARENT: $f(x) = \sqrt{x}$

$g(x) = a\sqrt{k(x-d)} + c$

k must always be FACTORED out

$$\boxed{g(x) = 3\sqrt{-1(x-2)} - 1}$$

PROPER FORM

Horizontal Transformations

$\frac{1}{k} = \frac{1}{-1} = -1$ H. flip

$d = 2$ H. shift RIGHT

Vertical Transformations

$a = 3$ V. stretch

$c = -1$ V. shift DOWN

Example 1.8.4

The basic absolute value function $f(x) = |x|$ has the following transformations applied to it: **Vertical Stretch −3**, **Vertical Shift 1 up**, **Horizontal Shift 5 right.**

Determine the equation of the transformed function.

$g(x) = a\,|k(x-d)| + c$

$a = -3$; $c = 1$; $d = 5$; $k = 1$

$g(x) = -3\,|1(x-5)| + 1$

$g(x) = -3\,|x-5| + 1$

Sketching the graph of a transformed function can be relatively easy if we know:

1) The shape of the parent function AND a few (3 or 4) points on the parent.
2) How transformations affect the points on the parent
   i) Horizontal transformations affect the domain values (***OPPOSITE!!!!!!***)
   ii) Vertical transformations affect the range values

Note: Given a point on some parent function which has transformations applied to it is called an ***IMAGE POINT*** on the transformed function.

Example 1.8.5

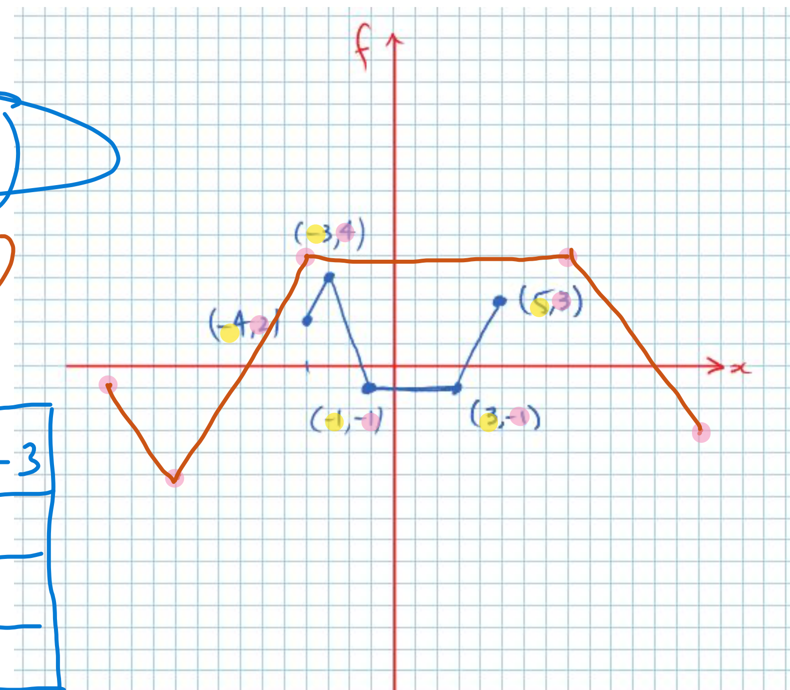
Given the sketch of the function $f(x)$ determine the image points of the transformed function $\boxed{-2f\left(\frac{1}{3}(x+1)\right)+3}$ and sketch the graph of the transformed function.

$a=-2$ $\quad \frac{1}{k}=3$

$c=3$ $\quad d=-1$

$$(x,y)\to\left(\frac{1}{k}x+d,\ ay+c\right)$$

$$(x,y)\to(3x-1,-2y+3)$$



x	y
-4	2
-3	4
-1	-1
3	-1
5	3

$3x-1$	$-2y+3$
-13	-1
-10	-5
-4	5
8	5
14	-3

$-3x+12 \geq 0$
$\frac{12}{3} \geq \frac{3x}{3}$
$4 \geq x$

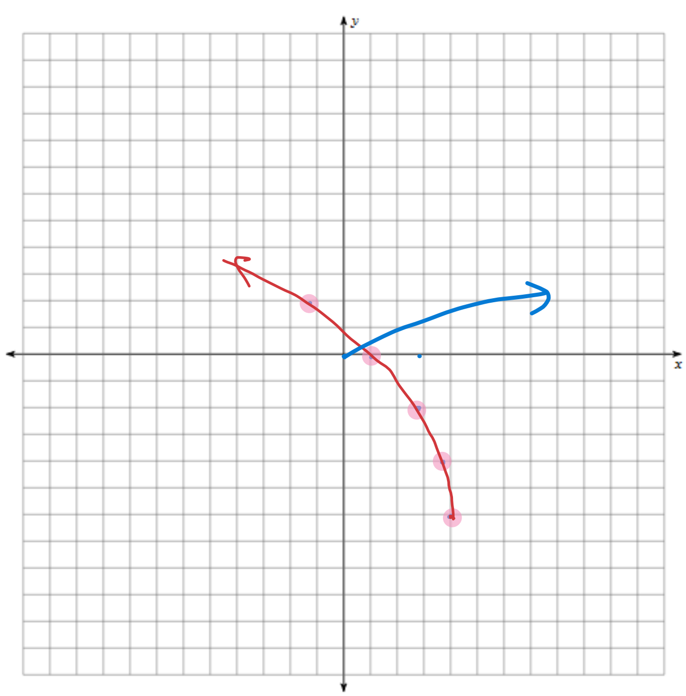
$e(x=0) = 2\sqrt{-3(0)+12} - 6$
$= 2\sqrt{12} - 6$

Function	Proper Function $g(x) = af(k(x-d))+c$	Vertical Stretch *a*	Horizontal Stretch *1/k*	Horizontal Shift *d*	Vertical Shift *c*
$e(x) = 2\sqrt{-3x+12} - 6$	$e(x) = 2\sqrt{-3(x-4)} - 6$	2	$\frac{1}{-3} = -\frac{1}{3}$	4	-6

Domain	Range	y-int (x=0)
$\{x \in R \mid x \leq 4\}$	$\{e(x) \in R \mid e(x) \geq -6\}$	0.9

Table Of Values

Parent Function: $f(x) = \sqrt{x}$		Transformed Function	
		$\frac{1}{k}x+d$	$ay+c$
x	$f(x)$	$-\frac{1}{3}x+4$	$2y-6$
0	0	$-\frac{1}{3}(0)+4$ 4	-6 START POINT
1	1	$-\frac{1}{3}(1)+4$ 3.7	-4
4	2	$-\frac{1}{3}(4)+4$ 2.7	-2
9	3	$-\frac{1}{3}(9)+4$ 1	0
16	4	-1.3	2



Extra work space.

HW- Section 1.6-1.8

Big Handout

Success Criteria:

- I can use the value of a to determine if there is a vertical stretch/reflection in the x-axis
- I can use the value of k to determine if there is a horizontal stretch/reflection in the y-axis
- I can use the value of d to determine if there is a horizontal translation
- I can use the value of c to determine if there is a vertical translation
- I can transform x coordinates by using the expression $\frac{1}{k}x + d$
- I can transform y coordinates by using the expression $ay + c$