Applications of Quadratics

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Today we will review and explore again 3 real-world applications of quadratics, namely, projectile problems, profit problems and area maximization problems.

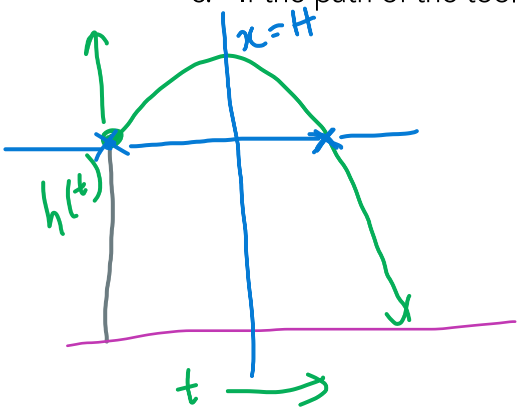
Projectile Problems

Projectile problems are problems where something is being launched in the air. Because of gravity the shape of its flight makes a parabola. These are the easiest because the equation is always given so you don't have to figure it out.

Example 1: A construction worker repairing a window tosses a tool to his partner across the street (this is dangerous, never do this!). The height of the tool above the ground is modelled by the quadratic function $h(t) = -5t^2 + 20t + 25$ where $h(t)$ is height in metres and t is the time in seconds after the toss.

a. How high above the ground is the window?
b. If his partner misses the tool (as he probably should), when will it hit the ground?
c. If the path of the tool's height were graphed, where would the axis of symmetry be?

$x = H \qquad V(H, K)$



(a) $C = 25$

$\therefore$ The window is 25m above the ground.

b) $h(t) = 0$ when it hits the ground.

To Find: $t = ?$

$0 = h(t) = -5t^2 + 20t + 25$

$0 = -5t^2 + 20t + 25$

$0 = -5(t^2 - 4t - 5)$

$0 = -5(t-5)(t+1)$

$t_1 = 5 \qquad \cancel{t_2 = -1}$

$\therefore$ The tool hits the ground at 5sec.

c) $x = \dfrac{5 + (-1)}{2}$

$= \dfrac{4}{2} = 2$

AoS: $x = 2$

c) $h(t) = -5t^2 + 20t + 25$

PARTIAL FACTORING

$\cancel{25} \; 0 = -5t(t-4) + \cancel{25}$

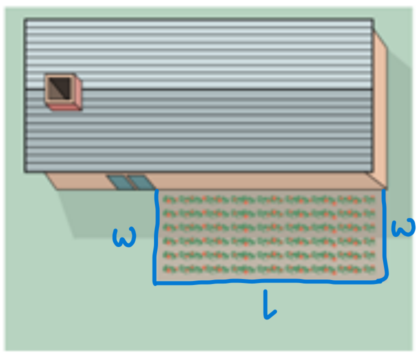
$t_1 = 0 \qquad t_2 = 4$

$\therefore H = \dfrac{0+4}{2} = 2$

$\boxed{\therefore \text{AoS: } x = 2}$

Area Maximization Problems

In these problems, there is some sort of area to be maximized within a Constraint on the perimeter. You need to remember the formulas for the areas of simple shapes like rectangles.



$l + 2w = 24 \Rightarrow \boxed{l = (24 - 2w)}$

Example 2: Mr. Greenthumb has 24 m of fencing to enclose a rectangular garden at the back of their house.

a. Write an expression to model the area as a function of the width of the garden

b. Determine the width that will maximize the area of the garden.

a) $A(w) = lw = (24-2w)w = 24w - 2w^2$

$$\boxed{\begin{aligned} A(w) &= 24w - 2w^2 \\ &= -2w^2 + 24w \end{aligned}}$$

b) $0 = -2w^2 + 24w$

$0 = -2w(w-12)$

$w_1 = 0 \qquad w_2 = 12$

$\therefore$ AoS $\rightarrow$ H $= \frac{w_1 + w_2}{2} = \frac{0+12}{2} = 6$

$\therefore$ The width that will max. the area is $w = 6$ m.

Profit Problems

Example 3: Mrs. Businesswoman owns a small company that produces and sells reusable water bottles. The revenue and cost equations are:

$$R(x)=(-50x^2+2500x)$$

$$C(x)=(150x+9500)$$

Where x is the selling price in dollars.

a. Write a simplified expression for the profit $P(x)$.
b. What is the price that will maximize profit? $H=?$
c. What is the maximum profit?

a) $P(x) = R(x) - C(x)$

$= (-50x^2+2500x) - (150x+9500)$

$= -50x^2+2500x-150x-9500$

$$\boxed{P(x) = -50x^2+2350x-9500}$$

b) PARTIAL FACTORING

$0 = -50x^2+2350x$

$0 = -50x(x-47)$

$x_1 = 0$ $\qquad$ $x_2 = 47$

AoS: $H = \frac{x_1+x_2}{2} = \frac{47+0}{2} = 23.5$

$\therefore$ The price that will max the profit is \$23.50

c) $P(23.5) = -50(23.5)^2+2350(23.5)-9500 =$

$= 18112.50$

$\therefore$ The max profit = \$18112.50.

Example 4: You are running a smoothie shop. The shop incurs both fixed costs and variable costs for each smoothie sold. Here's the breakdown:

- Fixed cost: $500 per day (for rent, utilities, and staff).
- Variable cost: $2 per smoothie (for ingredients and packaging).

You currently charge $8 per smoothie and sell 100 smoothies per day. Market research suggests that for every $0.50 decrease in the price, you will sell 10 more smoothies per day.

a. Write an expression for the price per smoothie if there have been x number of $0.50 decreases in price.

$$=(8-0.50x)$$

b. Write an expression for the number of smoothies sold if there have been x number of $0.50 decreases in price.

$$=(100+10x)$$

c. Write an equation to model your shop's total revenue, $R(x)$, where x represents the number of $0.50 decreases in price. Simplify.

$$R(x)=\left(\text{Cost per Smoothie}\right)\left(\text{\# smoothies sold}\right)$$

$$R(x)=(8-0.5x)(100+10x)$$

$$R(x)=800+80x-50x-5x^2$$

$$R(x)=-5x^2+30x+800$$

d. Write an equation to model your shop's total cost, $C(x)$. Simplify the function.

$$C(x)=\text{Fixed}+\text{Variable}$$

$$C(x)=500+2(100+10x)$$

$$=500+200+20x$$

$$C(x)=700+20x$$

e. Write an equation to model your shop's total profit, $P(x)$. Simplify the function.

$$P(x)=R(x)-C(x)$$

$$=(-5x^2+30x+800)-(700+20x)$$

$$\boxed{P(x)=-5x^2+10x+100}$$

f. Determine the smoothie price that will maximize your profit. What is the maximum profit?

PARTIAL FACTORING

$$P(x)=-5x^2+10x+100$$

$$0=-5x^2+10x$$

$$=-5x(x-2)$$

$$x_1=0 \qquad x_2=2$$

$$\therefore \text{AoS: } x=\frac{0+2}{2}=1$$

∴ The smoothie price that will max profit is by dropping $0.50 once.

i.e. price $=8-1(0.5)$

$=\$7.50$

$\therefore$ Max Profit $=P(1)=-5(1)^2+10(1)+100$

$=-5+10+100=\$105$

Classwork/Homework:

1: Basketball Shot

A basketball player takes a shot, and the height $h(t)$, n meters of the ball after t seconds is given by the equation:

$$h(t)=-5t^2+10t+2$$

a. How long does it take for the ball to reach its maximum height?
b. What is the maximum height of the ball?

2: Rocket Launch

A model rocket is launched from the ground, and its height $h(t)$ in meters after t seconds is given by:

$$h(t)=-4.9t^2+29t$$

a. How long will it take for the rocket to reach its maximum height?
b. What is the maximum height the rocket reaches?
c. After how many seconds will the rocket hit the ground?

3: Fencing a Garden

A gardener wants to fence a **rectangular garden** along a river, where only **three sides** need fencing (the side along the river does not require fencing). The gardener has **60 meters of fencing**.

a. Write a function for the area of the garden in terms of the width w
b. What dimensions will maximize the area of the garden?
c. What is the maximum area?

4: Building a Dog Pen

A farmer wants to build a **rectangular dog pen** that must be divided into **two equal sections** by a fence running parallel to one of the shorter sides. The total amount of fencing available is **80 meters**.

a. Write a function for the area of the pen in terms of the width w
b. What dimensions will maximize the area of the pen?
c. What is the maximum area?

5: Coffee Shop Optimization

A coffee shop sells lattes for **$8** each and usually sells **120 lattes per day**. Market research shows that **for every $1 decrease in price**, the shop will sell **27 more lattes**. The variable cost per latte is **$2**, and the fixed daily cost is **$200**.

a. Write an expression for the price per latte if there have been x number of $1 decreases in price.
b. Write an expression for the number of lattes sold if there have been x number of $1 decreases in price.
c. Write an equation for the revenue function and simplify.
d. Write an equation for the cost function and simplify.
e. Write an equation for the profit function and simplify.
f. What price should the coffee shop charge to maximize profit?
g. What is the maximum profit?

Answers:
1a) 1s b) 7m
2a) 3s b) 44.1m c) 6s
3a) $A(w)=-2w^2+60w$
b) $w=15m, l=30m$
c) $A=450m^2$

4a) $A(w)=-1.5w^2+40w$
b) $w=13.3m, l=20m$
c) $A=265.3m$

5a) $price\ per\ latte=8-x$
b) $\#lattes\ sold=120+27x$
c)$R(x)=-27x^2+96x+960$
$d)$ $C(x)=54x+440$
e) $P(x)=-27x^2+42x+520$
f) Optimal price is $7.22
g) Max profit is $536.30