Functions 11

Course Notes

Unit 3 – Quadratic Functions

FUNCTIONS TO THE MAX (OR MIN…AND SOMETIMES ZERO)

We will learn

- *the meaning of a zero, and how to find them algebraically*
- *to determine the max or min value of a quadratic algebraically and graphically*
- *to sketch parabolas (using transformations, zeroes, the vertex and y-intercept)*
- *to solve real-world problems, including linear-quadratic systems*

Chapter 3 – Quadratic Functions   Name______________________.

3.1 Properties of Quadratic Functions- Recall and Review of Grade 10 concepts

Learning Goal: We are learning to represent and interpret quadratic functions in three different forms and recall the properties of quadratics.

This lesson is a review of some quadratics concepts from Grade 10. 😊

Review: **Think and Answer by yourself!**

1. What is the degree of any given polynomial and what is the degree of a quadratic polynomial?

eg x^2+2x+3, x^2-5

degree – Highest Exponent of the Variable.

degree (Quadratic polynomial) = 2

2. Do you remember any algebraic model (algebraic form) of a quadratic relation? Write down everything that you remember.

$y = a(x-h)^2+k \longrightarrow V(h,k)$

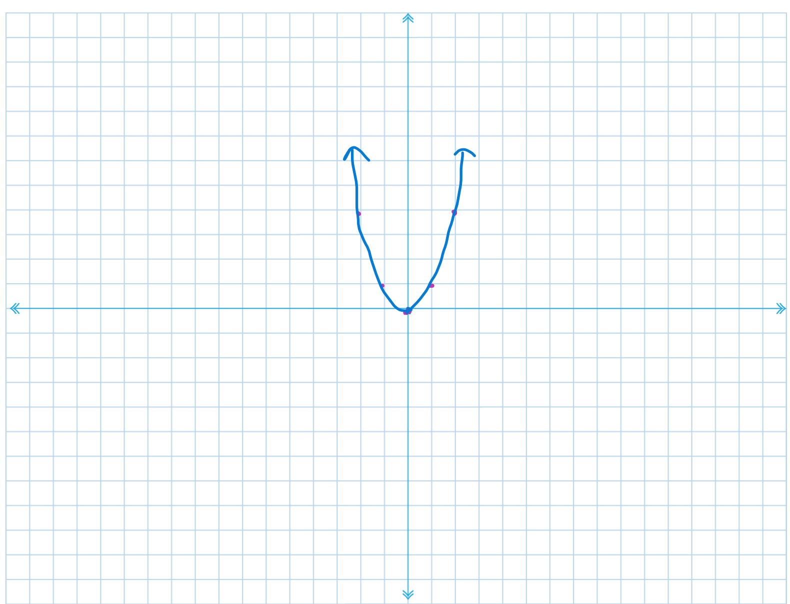
$y = ax^2+bx+c \longrightarrow c \rightarrow$ y intercept

$y = a(x-r)(x-s) \longrightarrow r, s \rightarrow$ x-intercept.

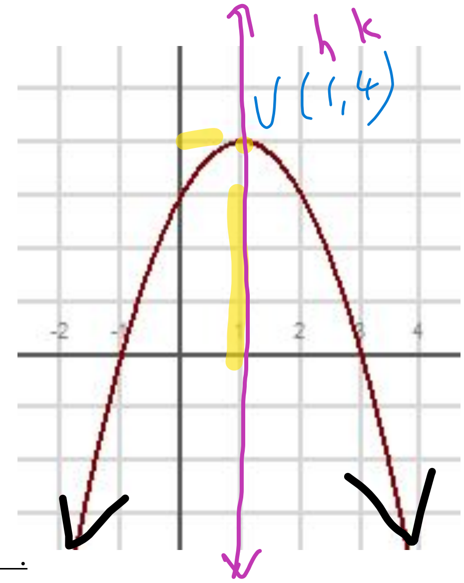
3. In the previous unit, you learned about parent functions. Plot the graph of the parent quadratic function. (We are now calling the quadratic relation, a quadratic function because for every value of x we know that we'll have a unique (predictable) value of y. 😊)

Recall that while dealing with functions, our y = f(x) i.e. our y got a super cool makeover and now looks f(x)

x	$f(x) = x^2$
−2	4
−1	1
0	0
1	1
2	4



4. i. What is the Vertex and the equation of the Axis of Symmetry in the given graph?



Vertex: (1,4) .

Axis of Symmetry: $x = 1$.

ii. Can you find the maximum point or the minimum point on this graph? If yes, what is that point?

Is it a Maximum / Minimum point? MAX. POINT .

Coordinates of the Point? (1,4) .

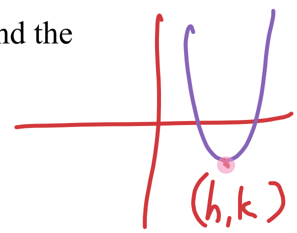
Does this maximum/minimum point that you observed from the graph have a special name? If yes, what is it called? VERTEX .

$a > 0$

$a < 0$

Now time to work together!!!!

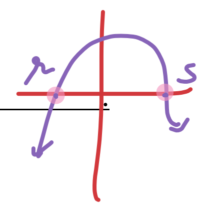
In Grade 10, you studied the **THREE FORMS** of quadratic functions and the **information** they give.



(h,k)
VERTEX

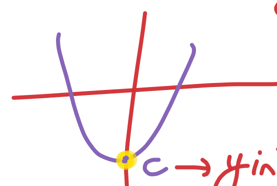
1. Vertex Form: f(x) = $a(x-h)^2+k$.

2. Factored Form: f(x) = $a(x-r)(x-s)$.



(r,0), (s,0) roots, x-intercepts, zeros.

3. Standard Form: f(x) = ax^2+bx+c .



c → y intercept.

Now,

Compare the vertex form of your parabola with the general form of any transformed function (which we learned about in Unit 1) and recall the information that a gives you about the graph.

Vertex Form: f(x) = $a(x-h)^2+k$.

Transformed Function: g(x) = a f [k (x - d)] + c .

So, a is +ve

If a > 0, will the graph open UP or DOWN? UP.

And what if a < 0, will the graph open UP or DOWN? DOWN.

a is -ve

Okay great!! 😊 Now think some more and answer:

If a > 0, will the vertex represent a maximum or a minimum? MINIMUM..

And what if a < 0 ? MAXIMUM..

Fantastic!! 😊 A little more thinking and answer the following:

Should there be a restriction on the leading coefficient **a** for the three different algebraic forms to be a quadratic?

If yes, what should the restriction be? $a \neq 0$.

Also,

If $f(x)=a(x-h)^2+k$ is the vertex form of any quadratic function, then the vertex is represented by $V(h,k)$ and the equation of the axis of symmetry is $x=h$.

Example 3.1.1

Given the quadratic function $f(x)=\frac{1}{2}(x+3)^2-1$, state:

a) The direction the parabola opens
b) The coordinates of the vertex
c) The equation of the axis of symmetry
d) Domain and Range

$f(x)=0.5(x+3)^2-1 \equiv a(x-h)^2+k$

a) $a=0.5>0$ ∴ PARABOLA OPENS UP.

b) VERTEX $(h,k)=(-3,-1)$

c) EoA $x=h \Rightarrow x=-3$

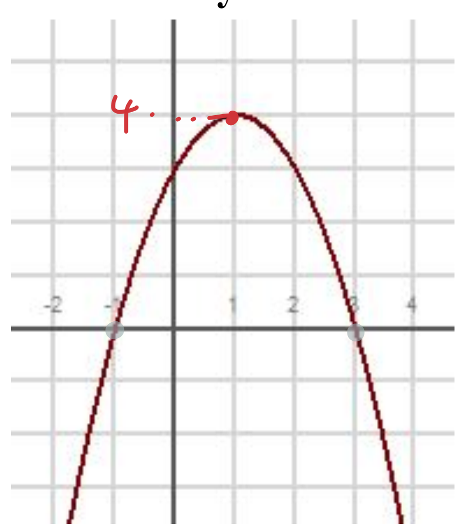
d) $D=\{x\in\mathbb{R}\}$
$R=\{f(x)\in\mathbb{R} \mid f(x)\geq -1\}$

5. (i) The horizontal intercepts or x- intercepts (when y = 0) of a parabola are called its Zeros or Roots.
(ii) The vertical intercepts or y- intercepts can be found by taking x = 0
i.e. y-intercept = f(0) = C in standard form.

KEY thing to REMEMBER:

Zeros of a quadratic function are the value of x when y = 0

State the zeros of the graphed quadratic function to the right.
Also state its domain and range



Zeros $=-1$ and 3
(α) (β)

$D=\{x\in\mathbb{R}\}$

$R=\{f(x)\in\mathbb{R} \mid f(x)\leq 4\}$

6. The difference between consecutive values of y for constant increments of x are known as **finite differences**.

- If the first difference is the same, then a given table of values represents a linear relation.
- If the second difference is the same, then a given table of values represents a quadratic relation
- If the third difference is the same, then a given table of values represents a cubic relation
- If the fourth difference is the same, then a given table of values represents a quartic relation
- If the fifth difference is the same, then a given table of values represents a quintic relation
- And so on…

Determine if the function represented by the table of values is quadratic or not.

i)

$\Delta y = y_n - y_{n-1}$

x	f(x)	1st Diff.	2nd Diff.
0	5		
1	2	$2-5=-3$	
2	-1	$-1-2=-3$	
3	-4	$-4--1=-3$	
4	-7	$-7--4=-3$	

NOT QUADRATIC

· It represents a LINEAR function.

(ii)

Δ(1st diff)

x	f(x)	1st Diff.	2nd Diff.
-1	2		
0	3	$3-2=1$	
1	6	$6-3=3$	$3-1=2$
2	11	$11-6=5$	$5-3=2$

It represents a QUADRATIC function :)

Not Linear :(

Key idea to remember-

- Quadratic Functions have a constant non-zero second difference.
- If the 2nd Diff. > 0, then the leading coefficient a > 0. So, parabola opens UP.
- If the 2nd Diff. < 0, then the leading coefficient a < 0. So, parabola opens DOWN.

Example 3.1.2

Given the quadratic function $g(x) = -2(x+3)(x-1)$, state

a) The direction the parabola opens
b) The zeros of the quadratic
c) The equation of the axis of symmetry
d) The coordinates of the vertex
e) The function in vertex form

Sketch the graph of the function.

e) $a = -2$
$(h,k) = (-1,8)$
$y = a(x-h)^2 + k$
$f(x) = -2(x+1)^2 + 8$

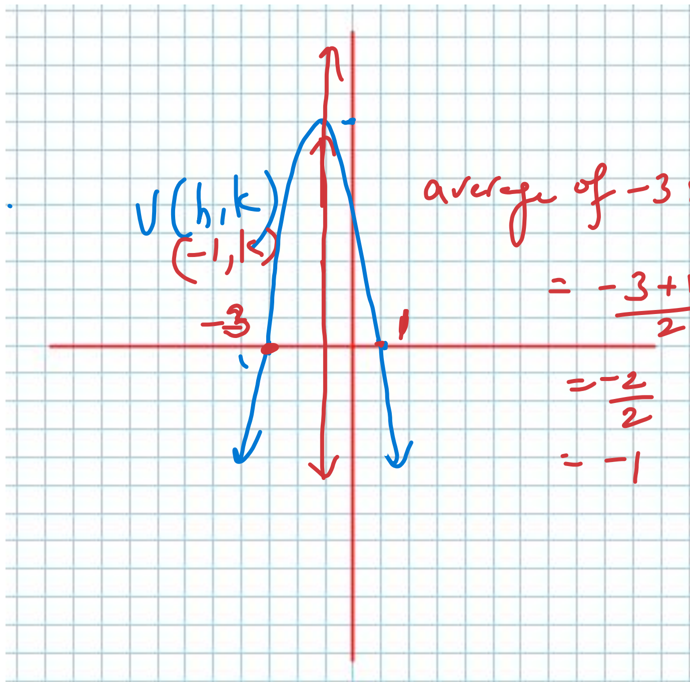
$g(x) = -2(x+3)(x-1) \equiv a(x-r)(x-s)$

a) $a = -2 < 0$ ∴ PARABOLA opens DOWN.

b) $r = -3$; $s = 1$ ← zeros/roots/x-intercepts.
$(-3,0)$ $(1,0)$

c) $x = h$ $\boxed{x = \frac{r+s}{2}}$

$x = -1$



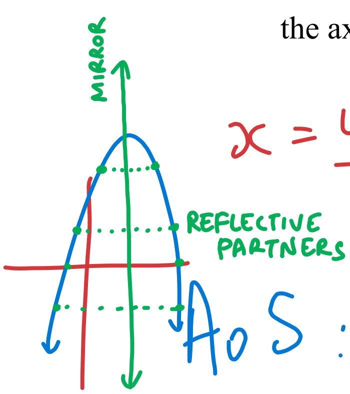
average of -3 & 1
$= \frac{-3+1}{2}$
$= \frac{-2}{2}$
$= -1$

d) $y = -2(x+3)(x-1)$
$(-1,k)$ is a point on it.

$k = -2(-1+3)(-1-1)$
$k = -2(2)(-2)$ ∴ $V(h,k) = (-1,8)$
$\boxed{k = 8}$

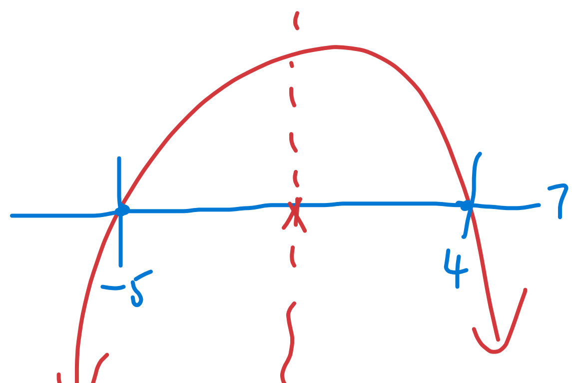
Example 3.1.3

Given the two points $(4,7)$, $(-5,7)$ which are on a parabola, determine the equation of the axis of symmetry.



$x = \frac{4 + -5}{2} = \frac{-1}{2} = -0.5$

AoS: $x = -0.5$



Average of -5 & 4
$\boxed{x = h}$

Example 3.1.4 (From Pg. 147 in your text)

11. The height of a rocket above the ground is modelled by the quadratic function $\boxed{h(t) = -4t^2 + 32t}$, where $h(t)$ is the height in metres t seconds after the rocket was launched.

a) Graph the quadratic function.

b) How long will the rocket be in the air? How do you know?

c) How high will the rocket be after 3 s?

d) What is the maximum height that the rocket will reach?

$y = -4t^2 + 32t + 0$

$h(t) = -4t^2 + 32t$

$= -4t(t-8)$

$= -4(t-0)(t-8)$

$r = 0$; $s = 8$

$\therefore h = \frac{0+8}{2} = 4$

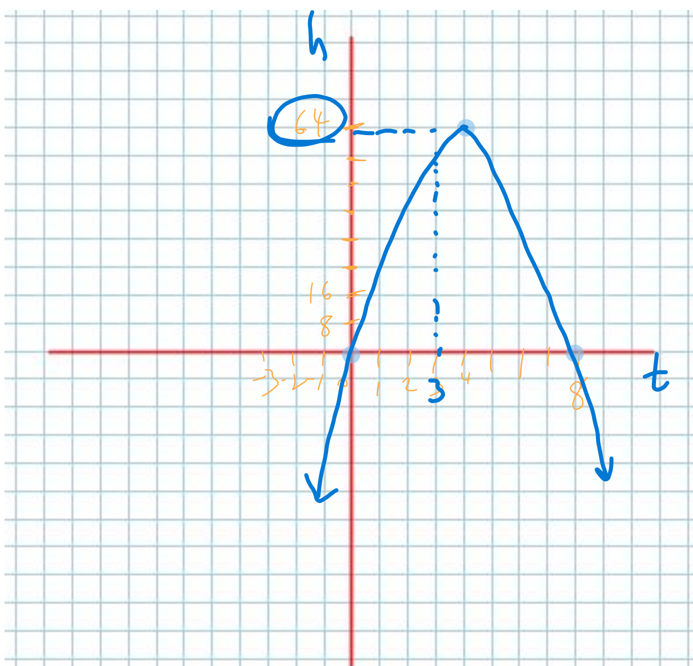
$k = y_{t=4} = -4(4)^2 + 32(4)$

$= -64 + 128$

$\boxed{k = 64}$

$\boxed{V(h,k) = (4,64)}$

$\boxed{y_{int} = C = 0}$



b) for 8 sec.

c) $t = 3 \therefore h(t=3) = -4t^2 + 32t$

$= -4(3)^2 + 32(3)$

$= -36 + 96$

$= 60$ m.

At 3 sec, rocket reaches 60m.

d) max height of 64m.

HW- Section 3.1

Pg. 145 – 147 #3, 4, 5bc, 6 (*expand*!), 7, 8, 9de, 12 (*tricky*!)

Success Criteria:

- I can recognize a quadratic function in standard, factored, and vertex form
- I can determine the zeros, direction of opening, axis of symmetry, vertex, domain and range from the graph or the algebraic form of a parabola
- I can determine the equation of quadratic function from its parabola (i.e. graph)

3.2 The Maximum or Minimum of Quadratic Functions

Learning Goal: We are learning to determine the maximum/minimum value of a quadratic function.

One very important aspect of quadratic functions is that they all have either a maximum (if the associated parabola opens down) or a minimum (if the parabola opens up). **Max/Min's** have so many **applications** in the real world that it's **ridiculous**.

The **BIG QUESTION** we are faced with is this:

k. value of the vertex (h,k)

How do we find the Maximum or Minimum Value for some given Quadratic?

The maximum or minimum value of a quadratic function is the y-coordinate of the vertex.

So, clearly we do need to find the vertex.

In order to find the vertex using algebra, we will consider three techniques:

1) USING THE ZEROS, TO FIND THE AXIS OF SYMMETRY, and then the vertex (**this is the easiest technique**, ***assuming we can factor the quadratic***).
2) COMPLETING THE SQUARE to find the vertex (this is the toughest technique, but it's nice because you ***end up with the quadratic in vertex form***).
3) ☆ USE PARTIAL FACTORING TO FIND THE AXIS OF SYMMETRY, and then the vertex.

Note: We can also use graphing calculators to find the max/min of a quadratic!

Example 3.2.2

Determine the max or min value for the function $f(x) = -3x^2 - 12x + 15$ by finding **THE ZEROS** of the quadratic.

$y = -3x^2 - 12x + 15$

$f(x) = -3x^2 - 12x + 15$

$0 = -3(x^2 + 4x - 5)$

$0 = -3(x+5)(x-1)$

$x_1 + 5 = 0$ $\qquad$ $x_2 - 1 = 0$

$x_1 = -5$ $\qquad$ $x_2 = 1$

r $\qquad$ s

$M = -5 = 5 \times -1$

$A = 4 = 5 + (-1)$

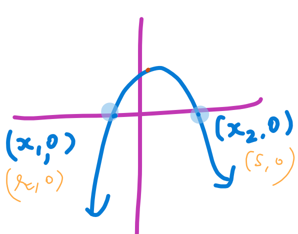
$h = \frac{r+s}{2} = \frac{-5+1}{2} = \frac{-4}{2} = -2$

$k = y_{x=h} = y_{(x=-2)}$

$= -3(-2)^2 - 12(-2) + 15$

$= -12 + 24 + 15$

$= 27$



$a = -3 < 0$

$\therefore$ Max value $= 27$

k

h

Example 3.2.3

Complete the square to find the vertex of the quadratic and state **where** the max (min) is and **what** the max (min) is.

$$g(x) = \boxed{2x^2 + 8x} - 5$$

$a = 2 > 0 \therefore$ Min Value

STEP 1: $g(x) = 2(x^2 + 4x) - 5$

STEP 2: $\pm\left(\dfrac{\text{COEFFICIENT of } x}{2}\right)^2 = \pm\left(\frac{4}{2}\right)^2 = \pm 2^2$

$g(x) = 2(x^2 + 4x + 2^2 - 2^2) - 5$

STEP 3: $g(x) = 2((x+2)^2 - 4) - 5$

STEP 4: $g(x) = 2(x+2)^2 - 8 - 5$

$g(x) = 2(x+2)^2 - 13$

$a(x-h)^2 + k$

$\therefore$ VERTEX $(h, k) = (-2, -13)$

$\therefore$ min value $= -13$ at $x = -2$

Example 3.2.4

Using **partial factoring** determine the axis of symmetry. Then find the vertex and state the min or max value.

$$h(x) = 5x^2 + 15x - 3$$

STEP 1: Take $y = h(x) = -3$

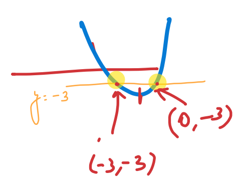
$-3 = 5x^2 + 15x - 3$

$\Rightarrow -3 + 3 = 5x^2 + 15x$

$\Rightarrow 0 = 5x^2 + 15x$

$\Rightarrow 0 = 5x(x+3)$

$x_1 = 0$; $x_2 = -3$



STEP 2: $h = \dfrac{x_1 + x_2}{2}$; x_1 and x_2 are SYMMETRIC PARTNERS.

$h = \dfrac{0 + -3}{2} = -1.5$

STEP 3: $k = y_{x=h} = y_{x=-1.5} = 5(-1.5)^2 + 15(-1.5) - 3$

$= 11.25 - 22.5 - 3$

$= -14.25$

$V(h, k) = (-1.5, -14.25)$

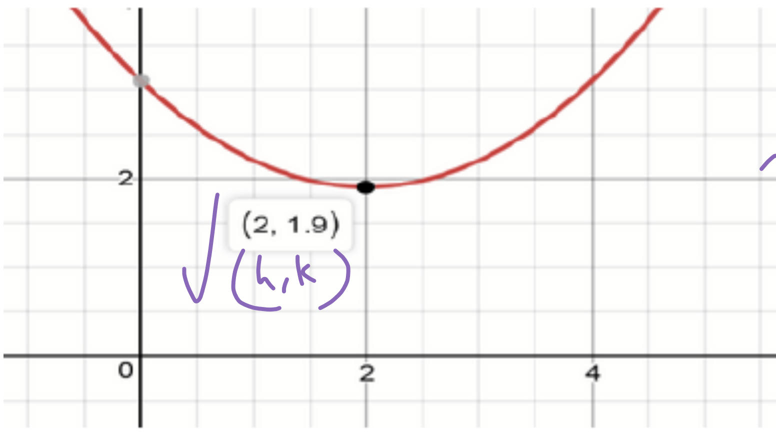
$\therefore a = 5 > 0$

$\therefore$ Min. Value $= -14.25$

Example 3.2.5

Using graphing technology, determine the max/min value of the quadratic

$$f(x) = 0.3x^2 - 1.2x + 3.1$$



min. value = 1.9.

HW- Section 3.2

Pg. 153 #1, 3, 4abc (*one method is fine*), 6 (*Desmos*), 7bc, 8, 9 (*try Partial Factoring*), 11 (*ask for help on c if you feel the need!*)

Success Criteria:

- I can recognize when a function has a maximum or minimum value (based on "*a*")
- I can find the max/min (vertex) value using various methods (*partial factoring* ☺)

RADICAL SIGN

Chapter 3 – Quadratic Functions $\sqrt{\bigcirc}$ Name______________________.

3.4 Operations with Radicals

RADICAND.

Learning Goal: We are learning to simplify and perform operations on radicals.

First we need to understand that **RADICALS** (*square roots, cube roots, etc*) **ARE NUMBERS**, and working with them should not induce any kind of fear in your spirit. So, **FEAR NOT!**

A COUPLE OF THINGS TO REMEMBER:

1) The square root of a square number/perfect square is a nice integer.

e.g. $\sqrt{25} =$ 5 $\qquad 5^2 = 25 \longleftrightarrow 5 = \sqrt{25}$

$\sqrt{49} =$ 7 $\qquad 7^2 = 49 \longleftrightarrow 7 = \sqrt{49}$

2) The cube root of a cubed number/perfect cube is a nice integer

e.g. $\sqrt[3]{27} = \sqrt[3]{3^3} = 3$ $\qquad 3^3 = (3)(3)(3) = 27 \longleftrightarrow 3 = \sqrt[3]{27}$

$\sqrt[3]{125} = \sqrt[3]{5^3} = 5$

Now, if we don't have a radical with a perfect square (or cube as the case may be) we could use a calculator to find the root.

e.g. $\sqrt{24} = 4.898979485566356196394568114941 18...$

BUT the "**DECIMAL EXPANSION**" is **unending** and **doesn't repeat** and so we can only **APPROXIMATE THE VALUE** of $\sqrt{24}$ because of the need to **ROUND-OFF**.

However,

"**EXACT NUMBERS**" like $\sqrt{24}$ are sometimes preferred in mathematical solutions and so **we do need to know how to work with these radical NUMBERS.** Working with radical numbers means we'll be:

- adding/subtracting
- multiplying/dividing them.

$\sqrt{-25}$ ← THIS IS NOT REAL!!!

Before beginning, there is one thing to keep in mind: We will not be dealing with negative square roots as this will require the use of Complex numbers which you may not have learnt about yet.

COEFFICIENTS WITH COEFFICIENTS, RADICALS WITH RADICALS

e.g. The number $2\sqrt{5}$ has a coefficient part of 2 and a radical part of $\sqrt{5}$

$2x$

Such a number (with both a coefficient and a radical part) is called a Mixed Radical.

***Rules:**

1.) When multiplying radicals $\boxed{\sqrt{a} \times \sqrt{b} = \sqrt{ab}}$; $a \geq 0, b \geq 0$

2.) When dividing radicals $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$; $b \neq 0$

3.) We can add or subtract only like radicals. For example: $3\sqrt{5}$ and $-7\sqrt{5}$ are like radicals but $7\sqrt{5}$ and $7\sqrt{3}$ are unlike radicals.

Example 3.4.1

Multiply the following:

a) $\sqrt{5} \times \sqrt{3}$ $= \sqrt{(5)(3)} = \sqrt{15}$

b) $-2\sqrt{7} \times 3\sqrt{6}$ $= -6\sqrt{42}$

c) $5\sqrt{10} \times \sqrt{5}$ $= 5\sqrt{50}$

d) $\sqrt{2} \times \sqrt{2}$ $= \sqrt{4} = 2$

Example 3.4.2

Simplify the following:

a) $\sqrt{50}$ $= \sqrt{(25)(2)} = \sqrt{25}.\sqrt{2} = 5\sqrt{2}$

b) $-3\sqrt{27}$ $= -3\sqrt{(9)(3)} = -3\sqrt{9}\sqrt{3} = -3(3)\sqrt{3}$
$= -9\sqrt{3}$

c) $2\sqrt{50}\times\left(-3\sqrt{24}\right)$
$= 2\left(\sqrt{25}\sqrt{2}\right).-3\left(\sqrt{4}\sqrt{6}\right) = 2(5)\sqrt{2}.-3(2)\sqrt{6}$
$= -60\sqrt{12} = -60\left(\sqrt{4}\sqrt{3}\right)$
$= (-60)(2)\sqrt{3} = -120\sqrt{3}$

Example 3.4.3

Add the following:

a) $3\sqrt{2} + 7\sqrt{2}$

$= 10\sqrt{2}$

b) $5\sqrt{7} - 3\sqrt{5} - 7\sqrt{7}$

$= -2\sqrt{7} - 3\sqrt{5}$

c) $2\sqrt{5} - 3\sqrt{20}$

$= 2\sqrt{5} - 3\sqrt{4}\sqrt{5}$
$= 2\sqrt{5} - 6\sqrt{5}$
$= -4\sqrt{5}$

d) $-3\sqrt{300} + \sqrt{243}$

$= -3\sqrt{100}\sqrt{3} + \sqrt{81}\sqrt{3}$
$= -30\sqrt{3} + 9\sqrt{3}$
$= -21\sqrt{3}$

Note: **We can only** ADD OR SUBTRACT "LIKE" RADICALS. **e.g.** $2\sqrt{3}$ and $-5\sqrt{3}$ ARE LIKE, **but** $2\sqrt{5}$ and $3\sqrt{20}$ ARE NOT ***(or aren't they?.....)***

Example 3.4.4

Simplify:

a) $2\sqrt{3}\,(3\sqrt{2}-5\sqrt{6})$

$=(2\sqrt{3})(3\sqrt{2}) - (2\sqrt{3})(5\sqrt{6})$

$= 6\sqrt{6} - 10\sqrt{18}$

$= 6\sqrt{6} - 10\sqrt{9}\sqrt{2}$

$= 6\sqrt{6} - 30\sqrt{2}$

b) $(3\sqrt{12}-5\sqrt{2})(2\sqrt{3}+3\sqrt{2})$

$(3\sqrt{4}\sqrt{3}-5\sqrt{2})(2\sqrt{3}+3\sqrt{2})$

$(6\sqrt{3}-5\sqrt{2})(2\sqrt{3}+3\sqrt{2})$ foil

$=12\sqrt{9}+18\sqrt{6}-10\sqrt{6}-15\sqrt{4}$

$=36+18\sqrt{6}-10\sqrt{6}-30$

$=6+8\sqrt{6}$

c) $(5-2\sqrt{2})^2$

$(5-2\sqrt{2})(5-2\sqrt{2})$

$=25-10\sqrt{2}-10\sqrt{2}+4\sqrt{4}$

$=25-10\sqrt{2}-10\sqrt{2}+8$

$=33-20\sqrt{2}$

HW- **Section 3.4**

Pg. 167 – 168 #3 – 5abc, 6 – 7acef, 8 - 13

Success Criteria:

- I can recognize "like" radicals. Totally awesome dude!
- I can write a radical in simplest form
- I can simplify radicals by adding, subtracting, multiplying, and dividing
- I can appreciate that a radical is an EXACT answer and therefore SUPERIOR to decimals

3.6 Zeros of Quadratic Functions

Learning Goal: We are learning to determine the number of zeros of a quadratic function.

Before we begin, let's think about a couple of things…

Remember – **FUNCTIONS CAN BE DESCRIBED AS A SET OF ORDERED PAIRS**, where the "ordered pair" is a pair of numbers: a **domain value** and a **range value** which can look like $(x, f(x))$. We have talked about the vertex of a parabola. Consider a parabola opening down (which means it will have a maximum value.

The vertex of that parabola is NOT the maximum. Instead, the vertex is a POINT which is made up of two special numbers. The domain value is WHERE the max occurs and the functional value (the "y" value) is the maximum.

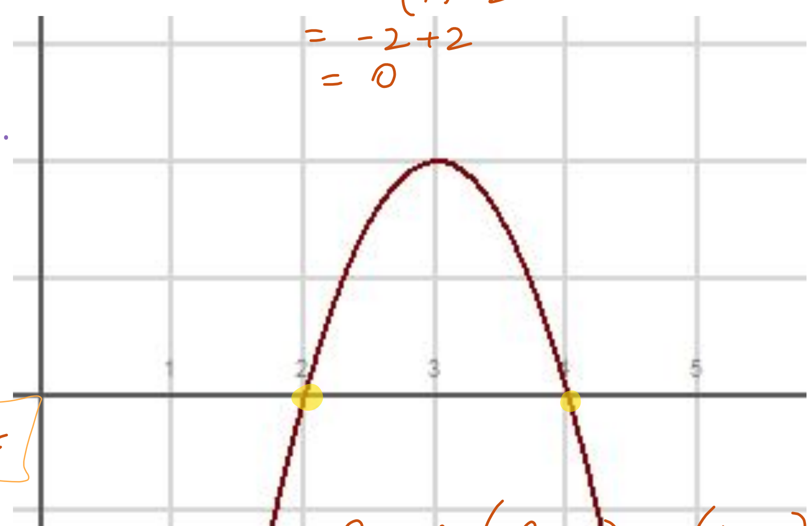
When we talk about the ZEROS of a quadratic we need to understand what we mean by that. Consider the sketch of the graph of the quadratic function $\boxed{f(x) = -2(x-3)^2 + 2}$

$x = 2$

$f(2) = -2(2-3)^2 + 2$
$= -2(1) + 2$
$= -2 + 2$
$= 0$

* $x = a$ is a zero of the quadratic $f(x)$ iff $f(a) = 0$

* Zeros form of the Quadratic:
$f(x) = a(x-r)(x-s)$
$\Rightarrow$ r and s are the zeros



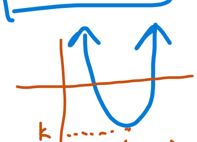
Zeros (or) x-intercepts (or) Roots

Zeros: $(2, 0)$; $(4, 0)$

$\boxed{f(x) = a(x-h)^2 + k}$

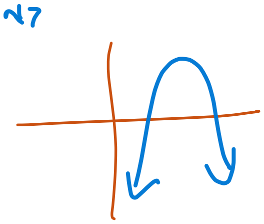
$V(h,k) \rightarrow$ TURNING POINT

CASE 1 : $a > 0$, $k < 0$

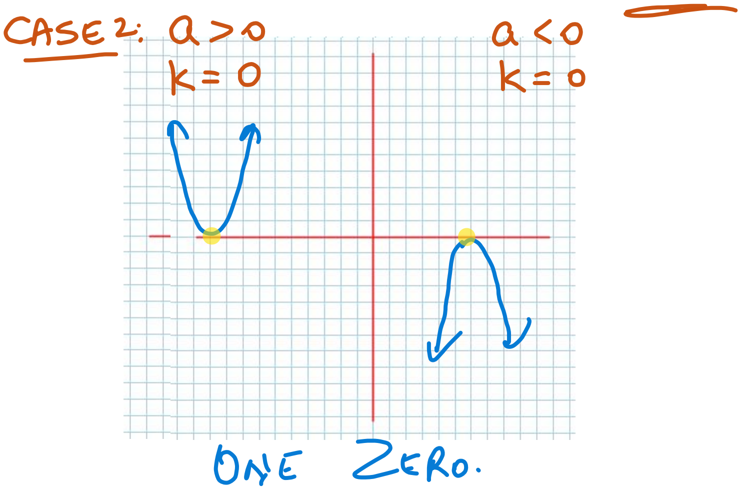


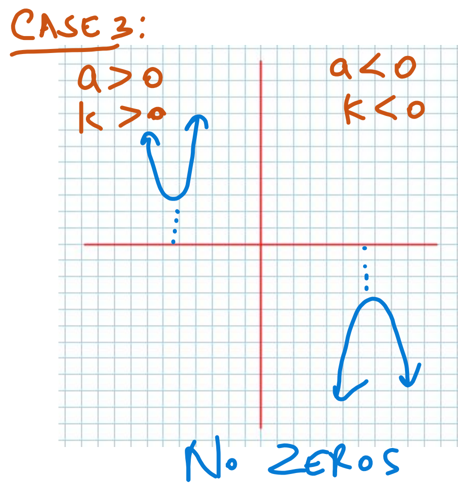
2 ZEROS

$a < 0$, $k > 0$



Q. Do all quadratics have 2 zeros? NO!!!!!!!





Finding the Zeros of a Quadratic

We can find those pesky zeros in a number of ways:

1) Writing the quadratic in zeros form (by factoring)
2) Writing the quadratic in vertex form, and doing some algebra (a bit nasty)
3) Using the quadratic formula (but the quadratic ***MUST BE IN STANDARD FORM*** - $f(x) = ax^2 + bx + c$)
4) Using graphing technology (lame, but legit)

Example 3.6.1

Determine the zeros:

a) $f(x) = x^2 - 3x - 4$

M: $-4 = (-4)(1)$
A: $-3 = (-4) + (1)$

$f(x) = 1(x-4)(x+1)$
$f(x) = a(x-r)(x-s)$

$\therefore$ zeros $r = 4$; $s = -1$
$(4, 0)$ $(-1, 0)$

b) $g(x) = 2x^2 + x - 1$

M = $-2 = (+2)(-1)$
A = $1 = (+2) + (-1)$

$\underbrace{2x^2 + 2x}_{2x} \underbrace{- x - 1}_{-1}$

$g(x) = (2x-1)(x+1)$
$0 = (2x-1)(x+1)$

$2x - 1 = 0$ $x + 1 = 0$

$x = \frac{1}{2} = 0.5$ $x = -1$

Zeros. $(0.5, 0)$; $(-1, 0)$

?

$f(x) = a(x-h)^2 + k$

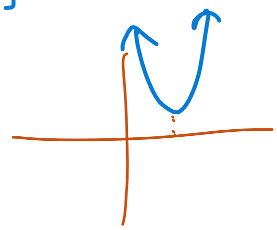
Example 3.6.2

Determine why the quadratic $\boxed{f(x) = 2(x-1)^2 + 2}$ has no zeros.

$a = 2 > 0$

$k = 2 > 0$

$\therefore$ The quadratic has No zeros.



Example 3.6.3

Determine the zeros of $\boxed{g(x) = -(x+1)^2 + 8}$ → Already VERTEX FORM.

$y = g(x) = 0$

$0 = -(x+1)^2 + 8$

$\cancel{-}8 = \cancel{-}(x+1)^2$

$\pm\sqrt{\underset{4\times 2}{8}} = x+1$

$\cancel{\sqrt{4}}\sqrt{2} = x+1$

$\pm 2\sqrt{2} = x+1$

$2\sqrt{2} = x+1$

$\Rightarrow \boxed{2\sqrt{2} - 1 = x_1}$

$-2\sqrt{2} = x+1$

$\Rightarrow \boxed{-2\sqrt{2} - 1 = x_2}$

Zeros: $(2\sqrt{2} - 1, 0)$ and $(-2\sqrt{2} - 1, 0)$

Example 3.6.4

Using the quadratic formula, determine the zeros of the quadratic:

In case you've forgotten, the quadratic formula is $\boxed{x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$.

a) $f(x) = 2x^2 + 3x - 7$ $\quad$ $f(x)=0$

$a = 2$; $b = 3$; $c = -7$

$$0 = 2x^2 + 3x - 7$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-7)}}{2(2)}$$

$$x = \frac{-3 \pm \sqrt{9 + 56}}{4}$$

$$x = \frac{-3 \pm \sqrt{65}}{4}$$

zeros: $\left(\frac{-3+\sqrt{65}}{4}, 0\right)$ and $\left(\frac{-3-\sqrt{65}}{4}, 0\right)$

b) $g(x) = 3x^2 - 2x + 4$

$a = 3$; $b = -2$; $c = 4$

$$0 = 3x^2 - 2x + 4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(4)}}{2(3)}$$

$$x = \frac{2 \pm \sqrt{4 - 48}}{6}$$

$$x = \frac{2 \pm \sqrt{-44}}{6}$$

DISCRIMINANT
$b^2 - 4ac$

No Real Zeros.

The Discriminant

The Discriminant of the quadratic formula is called the **DISCRIMINANT** because

The Discriminant is $b^2 - 4ac$

1) If $b^2 - 4ac > 0$, then the quadratic has ____2____ zeros
   POSITIVE
2) If $b^2 - 4ac = 0$, then the quadratic has ____1____ ~~zeros~~ zero
3) If $b^2 - 4ac < 0$, then the quadratic has ____NO REAL____ zeros
   NEGATIVE

Example 3.6.5

Determine the number of zeros using the discriminant:

a) $f(x) = 2x^2 + 3x - 2$

$a = 2,\ b = 3,\ c = -2$

$D = b^2 - 4ac$
$= 3^2 - 4(2)(-2)$
$= 9 + 16$
$= 25 > 0$
$\therefore$ 2 zeros.

b) $g(x) = -x^2 + 4x - 4$

$a = -1,\ b = 4,\ c = -4$

$D = b^2 - 4ac$
$= 4^2 - 4(-1)(-4)$
$= 16 - 16$
$= 0$
$\therefore$ 1 zero

c) $h(x) = 3x^2 + 5x + 6$

$a = 3,\ b = 5,\ c = 6$

$D = b^2 - 4ac$
$= 5^2 - 4(3)(6)$
$= 25 - 72 < 0$
$\therefore$ No zeros.

HW Section 3.6

Pg 185 – 186 #1 – 3abc, 4, 6 – 9 (*these are a bit tricky…ask for help!*), 15

Success Criteria:

- I can recognize that a quadratic function may have 0, 1, or 2 zeros
- I can use the discriminant of the quadratic formula to determine the number of zeros

3.5 Solving Quadratic Equations

Learning Goal: We are learning to solve quadratic functions in different ways.

Note that last day we looked at section 3.6. We now go back to 3.5 as this is a better order for the concepts.

Before beginning we should look at the difference between a Quadratic **FUNCTION** and a Quadratic **EQUATION**. A function such as $f(x)=3x^2-5x+1$ has a graph with **infinitely** many points. On the other hand, a quadratic equation (in standard form) looks like:

$$3x^2-5x+1=0$$

(*What is the difference between the function and the equation?)*

In section 3.6 we saw how to find the **zeroes** of quadratic functions, using the techniques of factoring, the quadratic formula or using graphing technology. As it turns out, solving a quadratic equation is **Exactly the Same as finding the zeros of quadratic functions.**

Quadratic equations, therefore can have __2__, __1__, or __No__ **SOLUTIONS**.

Example 3.5.1

Solve the equations:

a) $x^2-5x-14=0$

$(x-7)(x+2)=0$

$x_1=7$ $x_2=-2$

b) $2x^2+5x=2x+4$

$2x^2+5x-2x-4=0$

$2x^2+3x-4=0$

$a=2;\ b=3;\ c=-4$ $M=-8$ $A=3$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-3\pm\sqrt{3^2-4(2)(-4)}}{2(2)}$

$x=\dfrac{-3\pm\sqrt{9+32}}{4}$

$x=\dfrac{-3\pm\sqrt{41}}{4}$

~~**Example 3.5.2**~~

Ex. 3.5.2

Solve $-2.3x^2 - 1.32x = -1.45$

$0 = 2.3x^2 + 1.32x - 1.45$

$a = 2.3 \; ; \; b = 1.32 \; ; \; c = -1.45$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1.32 \pm \sqrt{(1.32)^2 - 4(2.3)(-1.45)}}{2(2.3)}$$

$$x = \frac{-1.32 \pm \sqrt{15.0824}}{4.6} = \frac{-1.32 \pm 3.88}{4.6}$$

$x_1 = \frac{-1.32 + 3.88}{4.6}$

$x_1 = 0.55$

$x_2 = \frac{-1.32 - 3.88}{4.6}$

$x_2 = -1.13$

Example 3.5.3 (From your text: Pg. 178 #6a)

$P = 0$

6. Determine the break-even quantities for each profit function, where x is the number sold, in thousands.

$\rightarrow x = ?$

a) $P(x) = -x^2 + 12x + 28$

$0 = -x^2 + 12x + 28$

$a = -1 \; ; \; b = 12 \; ; \; c = 28$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-12 \pm \sqrt{12^2 - 4(-1)(28)}}{2(-1)}$$

$$x = \frac{-12 \pm 16}{-2}$$

$x_1 = \frac{-12 + 16}{-2}$

$x_1 = -2$

$x_2 = \frac{-12 - 16}{-2}$

$x_2 = 14$ ✓

$\therefore$ Break even quantity is 14,000.

8. The population of a region can be modelled by the function $P(t) = 0.4t^2 + 10t + 50$, where $P(t)$ is the population in thousands and t is the time in years since the year 1995.

a) What was the population in 1995?

b) What will be the population in 2010?

c) In what year will the population be at least 450 000? Explain your answer.

a) in 1995, $t = 0$

$\therefore P(t=0) = 0.4(0)^2 + 10(0) + 50 = 50$

Ans 50,000.

b) in 2010, $t = 2010 - 1995 = 15$

$\therefore P(t=15) = 0.4(15)^2 + 10(15) + 50 = 290$

Ans 290,000

c) $P = 450\,000 = 450$ thousands.

$P(t) = 450$

$P(t) = 0.4t^2 + 10t + 50$

$450 = 0.4t^2 + 10t + 50$

$0 = 0.4t^2 + 10t + 50 - 450$

$0 = 0.4t^2 + 10t - 400$

$a = 0.4$; $b = 10$; $c = -400$

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \dfrac{-10 \pm \sqrt{10^2 - 4(0.4)(-400)}}{2(0.4)}$

$t = \dfrac{-10 \pm 27.2}{0.8}$

$t_1 = 21.5$
≈ 22

$\cancel{t_2 = -37.75}$

$\therefore$ The population will be at least 450,000 in the year $= 1995 + 22 = 2017$

HW Section 3.5

Pg 177 – 178 #1bc, 2bcd, 4abef, 6cd, 7 (*Hint: what is the height of the ball when it is on the ground?*), 9, 11 (#9 and 11 are tricky – ask for help!), 14

Success Criteria:

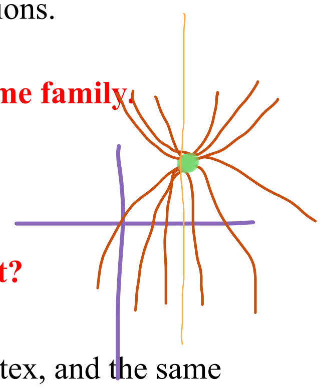
- I can solve quadratic functions by factoring, then setting each factor equal to zero
- I can solve quadratic functions by using the quadratic formula

Chapter 3 – Quadratic Functions Name___________________.

3.7 Families of Quadratic Functions

Learning Goal: We are learning the properties of families of quadratic functions.

A group of parabolas that all share a common characteristic lie in the same family.



Consider the two quadratic functions:

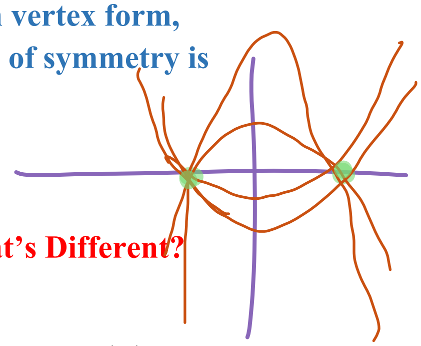
$$f(x)=2(x-3)^2+1, \text{ and } g(x)=-3(x-3)^2+1$$

What's Different?

Clearly $f(x)$ and $g(x)$ are different functions, but they do share the same vertex, and the same axis of symmetry. These quadratics are said to be in the same "family"

(some might say they are in the same vertex family)

Family 1: If the value of a is varied in a quadratic function expressed in vertex form, f(x) = a (x – h)^2 + k, a family of parabolas with the same vertex and axis of symmetry is created.



Next,

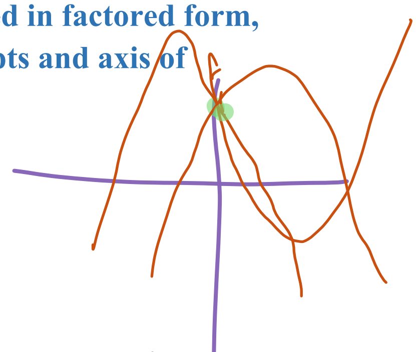
consider $h(x)=3(x+2)(x-4)$, and $f(x)=\frac{2}{3}(x+2)(x-4)$.

What's Different?

We see another kind of family here because $h(x)$ and $f(x)$ share the same zeros, and the same axis of symmetry.

(some might say these quadratics are in the same zeroes family)

Family 2: If the value of a is varied in a quadratic function expressed in factored form, f(x) = a (x – r) (x – s), a family of parabolas with the same x-intercepts and axis of symmetry is created.



Finally consider the third form of a quadratic. Consider

$$f(x) = -3x^2 - 2x + 7, \text{ and } g(x) = 2x^2 + 8x + 7$$

Family 3: If the values of a and b are varied in a quadratic function expressed in standard form, f(x) = ax ^{2} + bx + c, a family of parabolas with the same y-intercept is created.

Example 3.7.1

Determine the equation of the quadratic with zeros $x=3,$ and $x=-1$ and that passes through the point $(5,6)$.

$(x, f(x))$
$= (5,6)$

$f(x) = a(x-r)(x-s)$

$f(x) = a(x-3)(x+1)$

$6 = a(5-3)(5+1)$

$6 = a(2)(6) = 12a$

$\frac{6}{12} = a$

$0.5 = a$

$\therefore$ EQUATION:

$f(x) = 0.5(x-3)(x+1)$

Example 3.7.2

Determine the equation of the quadratic function $f(x)$ with a max value of 3 and axis of symmetry with equation $x=-5$ if $\boxed{f(2)=-18}$.

$x = h$

$k = 3$

$(x, f(x))$
$= (2,-18)$

Given:
$V(h,k) = (-5,3)$

$f(x) = a(x-h)^2 + k$

$-18 = a(2+5)^2 + 3$

$-18 = 49a + 3$

$-18 - 3 = 49a$

$\frac{-21}{49} = a = -\frac{3}{7}$

$\therefore$ EQUATION:

$f(x) = -\frac{3}{7}(x+5)^2 + 3$

HW Section 3.7

Page 192 #4 – 6, 8 – 10

Success Criteria:

- I can solve for "a" if given either the vertex or zeros

Chapter 3 – Quadratic Functions Name____________________.

3.7 Linear Quadratic Systems

Learning Goal: We are learning to solve problems involving the intersection of a linear and quadratic function.

Recall from Grade 10 that solving a **S**YSTEM **O**F **L**INEAR **E**QUATIONS could be interpreted to mean finding the point of intersection of the two lines. The solution to a **SoLE** is a point, (x, y). From an algebraic point of view, we have two techniques for solving a SoLE:

1) Substitution
2) Elimination

Example 3.8.1

Solve the SoLE

$$2x + 3y = 7 \quad (1)$$
$$\boxed{x - 2y = -7 \quad (2)}$$

Algebraically

SUBSTITUTION

$(2) \rightarrow \boxed{x = -7 + 2y}$

$(1)\ 2x + 3y = 7$

$\Rightarrow 2(-7 + 2y) + 3y = 7$

$\Rightarrow -14 + 4y + 3y = 7$

$\Rightarrow 7y = 7 + 14$

$\Rightarrow \frac{7y}{7} = \frac{21}{7}$

$\Rightarrow y = 3$

$\therefore x = -7 + 2(3)$

$x = -7 + 6$

$x = -1$

$\therefore$ PoI $(-1, 3)$

ELIMINATION.

$2x + 3y = 7$

$-2(x - 2y = -7)$

$\cancel{2x} + 3y = 7$

$+\ \cancel{-2x} + 4y = 14$

$7y = 21$

$y = \frac{21}{7} = 3$

$x - 2y = -7$

$x - 2(3) = -7$

$x - 6 = -7$

$x = -7 + 6$

$x = -1$

PoI $(-1, 3)$

Graphically

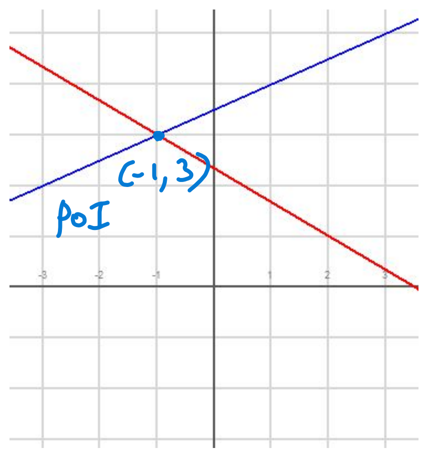


Figure 3.8.1

Solving a Linear-Quadratic System is more difficult, but we have the tools to succeed!
We will need to make use of (at least) one Property (or Rule) of Algebra:

THE TRANSITIVE PROPERTY OF EQUALITY

Rule: Given three numbers (or more generally, three mathematical objects) a, b, and c, and **if** $c = a$ and $c = b$, **then** $a = b$.

Example: If $f(x) = -2x - 4$, and $g(x) = x^2 - 3x - 10$, and if $f(x) = g(x)$, then $x^2 - 3x - 10 = -2x - 4$

Note: **Solving a Linear-Quadratic System** is equivalent to **finding the solution(s) to a quadratic equation**. For L-QS`s we can therefore have 0, 1, or 2 solutions.

We will apply the techniques for solving quadratic equations!

Example 3.8.2

Solve the Linear-Quadratic System given directly above.

$f(x) = g(x)$

$-2x - 4 = x^2 - 3x - 10$

$0 = x^2 - 3x - 10 + 2x + 4$

$0 = x^2 - x - 6$ $\quad {}^{-3}_{2}$

$0 = (x-3)(x+2)$

$x - 3 = 0$ $\qquad$ $x + 2 = 0$

$x_1 = 3$ $\qquad$ $x_2 = -2$

$\therefore y_1 = f(x) = -2x_1 - 4$
$= -2(3) - 4$
$= -6 - 4$
$= -10$

$y_2 = f(x) = -2x_2 - 4$
$= -2(-2) - 4$
$= 4 - 4$
$= 0$

$\therefore$ POI$_1$ = $(3, -10)$; POI$_2$ = $(-2, 0)$

Example 3.8.3 (#2c, on Page 198 from your text)

Determine the point(s) of intersection of the two functions algebraically:

$f(x)=3x^2-2x-1, \;\; g(x)=-x-6$

$f(x)=g(x)$

$3x^2-2x-1=-x-6$

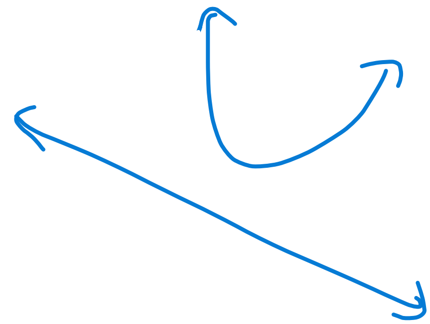
$3x^2-2x-1+x+6=0$

$3x^2-x+5=0$ $\quad a=3;\ b=-1;\ c=5$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{+1\pm\sqrt{(-1)^2-4(3)(5)}}{2(3)}$

$x=\dfrac{1\pm\sqrt{1-60}}{6}=\dfrac{1\pm\sqrt{-59}}{6}$



$\therefore$ No points of intersection.

Example 3.8.4

Determine the number of points of intersection without solving the System:

$f(x)=x^2+2x+14, \;\; g(x)=8x+5$ (Hint: To solve this problem you must be very discriminating)

$f(x)=g(x)$

$x^2+2x+14=8x+5$

$x^2+2x+14-8x-5=0$

$x^2-6x+9=0$

$a=1, \;\; b=-6, \;\; c=9$

$D=b^2-4ac=(-6)^2-4(1)(9)$

$D=36-36$

$D=0$

$\therefore$ It has only one POI

Example 3.8.5 (#9 on Page 199 in your text)

9. Determine the value(s) of k such that the linear function $g(x) = 4x + k$ does not intersect the parabola $f(x) = -3x^2 - x + 4$.

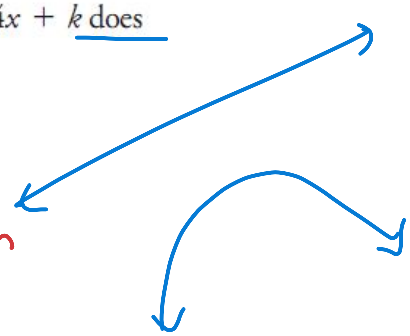
$f(x) = g(x)$

$-3x^2 - x + 4 = 4x + k$

$-3x^2 - x + 4 - 4x - k = 0$

$-3x^2 - 5x + (4 - k) = 0$

$a = -3 \; ; \; b = -5 \; ; \; c = 4 - k$



No. Solution

$\Rightarrow D < 0$

$b^2 - 4ac < 0$

$(-5)^2 - 4(-3)(4-k) < 0$

$25 + 12(4 - k) < 0$

$25 + 48 - 12k < 0$

$73 - 12k < 0$

$\Rightarrow \frac{73}{12} < \frac{12k}{12}$

$\Rightarrow \frac{73}{12} < k$

$\therefore$ k will take all real values more than $\frac{73}{12}$

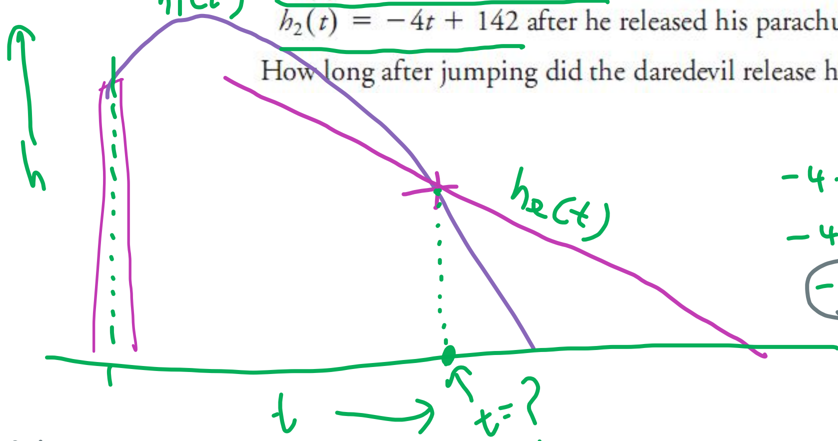
Example 3.8.6 (#10 in your text)

10. A daredevil jumps off the CN Tower and falls freely for several seconds before releasing his parachute. His height, $h(t)$, in metres, t seconds after jumping can be modelled by

$h_1(t) = -4.9t^2 + t + 360$ before he released his parachute; and

$h_2(t) = -4t + 142$ after he released his parachute.

How long after jumping did the daredevil release his parachute?



$h_1(t) = h_2(t)$

$-4.9t^2 + t + 360 = -4t + 142$

$-4.9t^2 + t + 360 + 4t - 142 = 0$

$-4.9t^2 + 5t + 218 = 0$

$t = \frac{-5 \pm \sqrt{25 - 4(-4.9)(218)}}{2(-4.9)}$

$t = \frac{-5 \pm 65.56}{-9.8}$

$t_1 = \frac{-5 + 65.56}{-9.8} = \cancel{-6.18}$

$t_2 = \frac{-5 - 65.56}{-9.8} = 7.20$

$\therefore$ After 7.2 sec, of jumping, the daredevil released his parachute.

HW Section 3.8

Pg198 – 199 #1ab, 2ab, 3, 4bcd, 6, 8, 11 (*tangent means touching at one point!*), 12

Success Criteria:

- I can solve for the points of intersection by
  1. Making the functions equal to each other
  2. Solving for the zeros (x-coordinates) of the resulting quadratic function
  3. Substituting the zeros into the linear equation to determine the corresponding y-values
- I can identify when solutions are inadmissible