MTH1W – Unit 6: Coordinate Geometry

Name: _____

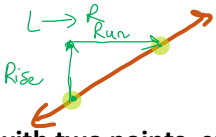
Lesson 6.3: Slope of a Line

Date: _____

Learning Goal: We are learning how slope impacts a linear equation. It's all downhill from here!

In this lesson, we will explore the most significant property of a linear relationship: the slope! The slope of a line tells us how the relationship is changing and can be thought of as how slanted/steep the line is. It has many important applications such as engineering the initial climb of a roller coaster to making safe ramps, but today we will focus on the algebra and understanding how to calculate the slope of a line.

First, let's look at the slope from a geometric perspective. The slope, defined by the letter m for no apparent reason, is: $m = \frac{Rise}{Run}$



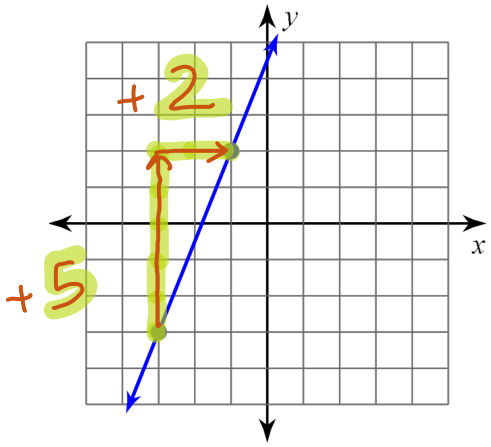
+RISE

-RISE

+RUN

-RUN

Example 1: Given the line with two points, calculate the slope.



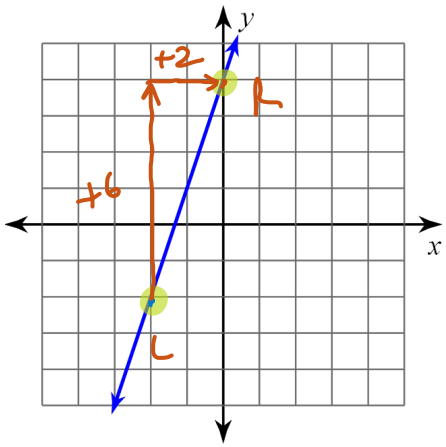
$m = \frac{RISE}{RUN} = \frac{5}{2}$

Slope is a fraction in its most reduced / simplified form

$m = \frac{-15}{3} = -5$

You must always REDUCE the slope.

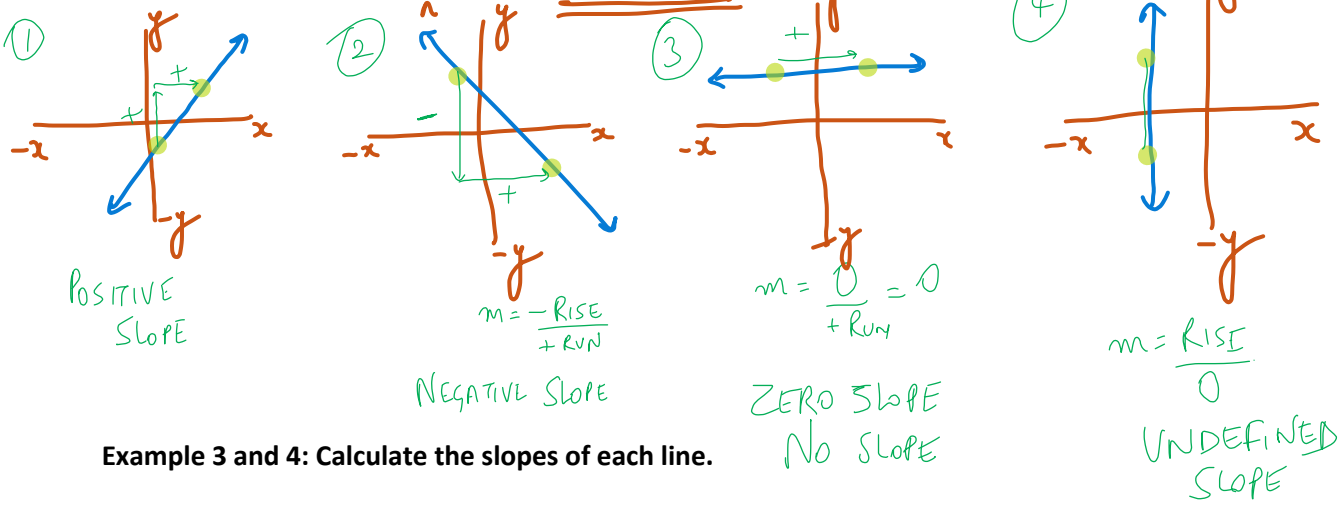
Example 2: Given the line, locate two points, then calculate the slope.



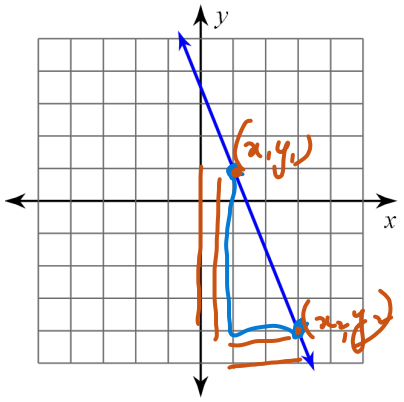
$m = \frac{Rise}{Run} = \frac{6}{2} = \frac{3}{1} = 3$

No!

Are slopes always positive? There are 4 possible slopes:

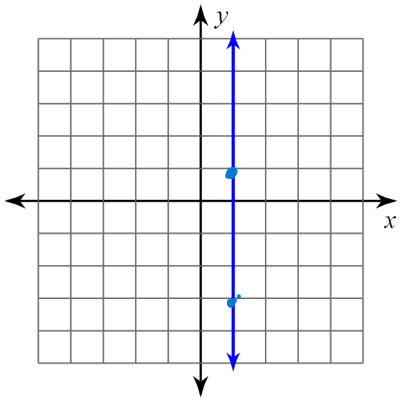


Example 3 and 4: Calculate the slopes of each line.



$m = \frac{\text{rise}}{\text{run}}$

$m = \frac{-5}{2}$



$m = \frac{\text{rise}}{\text{run}}$

$m = \frac{4}{0}$

m = undefined

Now that we know about slope, we can derive a formula so that we do not need a graph.

CASE 1:

T_2 Temp today = 5°C

T_1 Temp yesterday = 2°C

Change in temp = $T_2 - T_1$

$5 - 2$

$= +3°C$

CASE 2:

T_2 Temp today = 2°C

T_1 Temp yesterday = 5°C

Change in temp = $T_2 - T_1$

$= 2 - 5$

$= -3°C$

CASE 3:

$T_2 = 5°C$ (today)

$T_1 = -2°C$ (yesterday)

CHANGE = $\Delta T = T_2 - T_1$

↑ DELTA

$5 - -2$

$5 + 2$

$= 7°C$

$$\text{Slope} = m = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Examples 5-8: Given the points, calculate the slope using the slope formula.

$P(x, y)$

5. $(7,-10),(9,-7)$

$x_1\ y_1\ \ x_2\ y_2$

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-7 - -10}{9 - 7} = \frac{-7+10}{9-7}$

$m = \frac{3}{2}$

6. $(-6,-17),(-20,11)$

$x_1\ y_1\ \ x_2\ y_2$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$

$m = \frac{11 - -17}{-20 - -6} = \frac{28}{-14} = -\frac{2}{1} = -2$

7. $(6,-12),(6,1)$

$x_1\ y_1\ \ x_2\ y_2$

$m = \frac{\Delta y}{\Delta x} = \frac{1 - -12}{6 - 6} = \frac{13}{0}$

m = undefined

8. $(-3,9),(3,9)$

$x_1\ y_1\ \ x_2\ y_2$

$m = \frac{\Delta y}{\Delta x} = \frac{9 - 9}{3 - -3} = \frac{0}{6} = 0$

$m = 0$

Examples 9 and 10, use the idea of "change" to calculate the slope:

9. $(5,8),(10,2)$

$\Delta y = -6$, $\Delta x = 5$

$m = \frac{\Delta y}{\Delta x} = \frac{-6}{5}$

10. $(-7,9),(-15,-11)$

$\Delta y = -20$, $\Delta x = -8$

$m = \frac{\Delta y}{\Delta x} = \frac{+20}{+8} = \frac{5}{2}$

Example 9: A ramp needs to be constructed to go from the ground to a doorway. The doorway is 90 cm from the ground and the ramp needs a slope of $\frac{2}{9}$.

$m = \frac{2}{9} = \frac{\text{RISE}}{\text{RUN}}$

a) Calculate how far the ramp will start from the edge of the house.

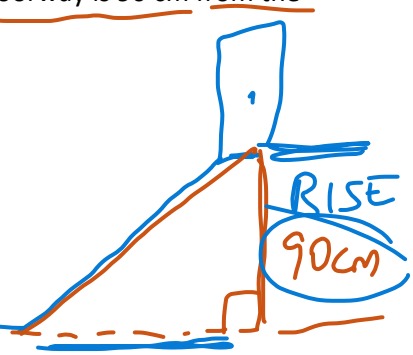
Let x

$\frac{\text{RISE}}{\text{RUN}} = \frac{2}{9} = \frac{90}{x}$

$\Rightarrow 2x = 810$

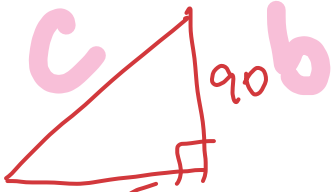
$x = \frac{810}{2} = 405\text{cm}$

∴ The ramp will start 405cm from the edge of the house



a) ? 405 RUN

b) Calculate the length of the ramp.



$c^2 = a^2 + b^2$

$c^2 = 405^2 + 90^2$

$c^2 = 164025 + 8100$

$c^2 = 172125$

$c = \sqrt{172125}$

$c \approx 414.9\text{cm}$

∴ length of ramp is approx 414.9cm.

Success Criteria

- I can identify the four types of slope: positive, negative, zero, undefined
- I can find the slope of a line graphically by studying its $\frac{rise}{run}$
- I can calculate the slope of a line algebraically by using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$
- I can find a missing coordinate, if given the slope