

Lesson #3: Adding and Subtracting Fractions

Date: Feb 4, 2025**Learning Goal:** We are learning to add and subtract fractions by using a **common denominator**.

If you had a third of a pizza and a quarter of another pizza (assuming same sized pizza), how much pizza would you have? This question can be done in two ways. One way is to estimate, which might be okay when eating pizza, but we are much more interested in an exact answer. To get a full answer, you would need to make sure all the pizza slices are the same size. Let's analyze these questions with fractions:

$\frac{1}{3} + \frac{1}{4} = ?$ To solve this addition problem, we cannot have different denominators, so we must create

equivalent fractions with the same (or common) denominator. Equivalent fractions are fractions that have the same value, such as $\frac{3}{6}$ and $\frac{5}{10}$ or $\frac{7}{8}$ and $\frac{14}{16}$. To create an equivalent fraction, you need to **MULTIPLY** the numerator AND denominator by the same number.

Finally, let's find out how much pizza we have:

$$\begin{aligned} & \frac{1 \times 4}{3 \times 4} + \frac{1 \times 3}{4 \times 3} \\ & \text{least common denominator} \\ & \text{L.C.D}(3, 4) = 12 \\ & = \frac{4}{12} + \frac{3}{12} = \frac{7}{12} \end{aligned}$$

Subtracting fraction is done the exact same way as adding. Find a common denominator, create equivalent fractions, then subtract. Let's practice!

$$\begin{aligned} & \frac{4 \times 4}{4 \times 5} + \frac{3 \times 5}{4 \times 5} \\ & \text{a) } \frac{16}{20} + \frac{15}{20} \quad \text{LCD}(5, 4) = 20 \end{aligned}$$

$$= \frac{16}{20} + \frac{15}{20}$$

$$= \frac{31}{20}$$

$$\begin{aligned} & \frac{11}{2} - \frac{5 \times 2}{1 \times 2} \quad \text{LCD}(2, 1) = 2 \\ & \text{c) } \end{aligned}$$

$$= \frac{11}{2} - \frac{10}{2}$$

$$= \frac{1}{2}$$

$$\begin{aligned} & \frac{7 \times -8}{7 \times 3} + \frac{6 \times 3}{7 \times 3} \quad \text{LCD}(3, 7) = 21 \\ & \text{b) } \end{aligned}$$

$$= \frac{-56}{21} + \frac{18}{21}$$

$$= \frac{-38}{21}$$

$$\begin{aligned} & \frac{2 \times 7}{2 \times 6} - \frac{9 \times 3}{4 \times 3} \quad \text{LCD}(6, 4) = 12 \\ & \text{d) } \end{aligned}$$

$$= \frac{14}{12} - \frac{27}{12}$$

$$= -\frac{13}{12}$$

$$\begin{array}{r|l} 2 & 6, 4 \\ 2 & 3, 2 \\ 3 & 3, 1 \\ \hline & 1, 1 \end{array}$$

$$\text{e) } \frac{-3}{5} - \frac{2}{7} + \frac{3}{2} \quad \text{LCD}(5, 7, 2) = 70$$

$$\text{f) } \frac{5}{6} - \frac{2}{5} - \frac{1}{15} \quad \text{LCD}(6, 5, 15) = 30$$

2	6, 5, 15
3	3, 5, 15
5	1, 5, 5
	1, 1, 1

$$= \frac{-42}{70} - \frac{20}{70} + \frac{105}{70}$$

$$= \frac{43}{70}$$

$$= \frac{25}{30} - \frac{12}{30} - \frac{2}{30}$$

$$= \frac{11}{30}$$

$$\text{g) } \frac{4}{3} + \frac{3}{9} - \frac{2}{9} \quad \text{LCD} = 9$$

$$\text{h) } \frac{5}{12} + \frac{7}{8} - \frac{7}{6} \quad \text{LCD}(12, 8, 6) = 24$$

2	12, 8, 6
2	6, 4, 3
2	3, 2, 3
3	3, 1, 3
	1, 1, 1

$$= \frac{12}{9} + \frac{27}{9} - \frac{2}{9}$$

$$= \frac{10}{24} + \frac{21}{24} - \frac{28}{24}$$

$$= \frac{37}{9}$$

$$= \frac{3}{24} = \frac{1}{8}$$

Application:

About $\frac{2}{5}$ of Canada's gold production come from Ontario. About $\frac{3}{10}$ comes from Quebec and $\frac{1}{10}$ from British Columbia. What fraction of Canada's gold production comes from the rest of the country? Write your answer in lowest terms.

$$1 - \left(\frac{2}{5} + \frac{3}{10} + \frac{1}{10} \right)$$

$\therefore \frac{1}{5}$ of the total gold production comes from the rest of the country.

$$\frac{10}{10} - \left(\frac{4}{10} + \frac{3}{10} + \frac{1}{10} \right) = \frac{10}{10} - \frac{8}{10} = \frac{2}{10} = \frac{1}{5}$$

Success Criteria:

- I can find the common denominator between 2 fractions
- I can create equivalent fractions using common denominators
- I can recognize that when adding/subtracting, only the numerator is added/subtracted

Build your Skills: :)

While adding unit fractions, Jamal believed he had discovered a shortcut. He hypothesized that when adding two unit fractions, he could determine the resulting fraction's numerator by adding the original two denominators, and he could find the resulting fraction's denominator by multiplying the original two denominators.

- Prove that Jamal's hypothesis is correct.
- What are some drawbacks of Jamal's shortcut?

Let $\frac{1}{x}$ and $\frac{1}{y}$ be any two unit fractions
(Note: $x \neq 0, y \neq 0$)

$$\begin{aligned}\text{Then, } \frac{1}{x} + \frac{1}{y} &= \frac{y}{xy} + \frac{x}{xy} \\ &= \frac{y+x}{xy}\end{aligned}$$

$$= \frac{\text{Sum of the denominators}}{\text{Product of the denominators}}$$



I need to add $\frac{1}{3} + \frac{1}{5}$.

$$3+5=8$$

$$3 \times 5 = 15$$

$$\text{So, } \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$$

$\therefore$ Jamal's hypothesis is correct! 😊

Drawbacks: Jamal's shortcut method works only for unit fractions. It does not work for all fractions. 😞

Evaluate.

$$\begin{aligned}\text{a) } 2\frac{7}{9} + 1\frac{2}{3} - \frac{4}{5} &= (2+1) + \left(\frac{7}{9} + \frac{2}{3} - \frac{4}{5}\right) \\ &= 3 + \left(\frac{35+30-36}{45}\right) \\ &= 3 + \frac{29}{45} \\ &= 3\frac{29}{45}\end{aligned}$$

$$\begin{aligned}\text{b) } \frac{7}{16} + \left(\frac{5}{8} - \frac{3}{4}\right) &= \frac{7+10-12}{16} \\ &= \frac{5}{16}\end{aligned}$$

$$\begin{aligned}\text{c) } 10\frac{3}{5} - \left(5\frac{1}{4} + 2\frac{2}{3}\right) &= 10\frac{3}{5} - \left(5+2 + \frac{1}{4} + \frac{2}{3}\right) \\ &= 10\frac{3}{5} - \left(7 + \frac{3+8}{12}\right) \\ &= 10\frac{3}{5} - 7\frac{11}{12} \\ &= (10-7) + \left(\frac{3}{5} - \frac{11}{12}\right) \\ &= 3 + \frac{36-55}{60} \\ &= 3 + \frac{-19}{60} \\ &= 2 + 1 - \frac{19}{60} \\ &= 2\frac{41}{60}\end{aligned}$$

$$\begin{aligned}\text{d) } -9\frac{7}{10} - \left(2\frac{5}{6} - 4\frac{2}{9}\right) &= -\frac{97}{10} - \left(\frac{17}{6} - \frac{38}{9}\right) \\ &= -\frac{97}{10} - \left(\frac{51-76}{18}\right) \\ &= -\frac{97}{10} - \left(-\frac{25}{18}\right) \\ &= -\frac{97}{10} + \frac{25}{18} \\ &= \frac{-873+125}{90} \\ &= \frac{-748}{90} \div 2 = \frac{-374}{45}\end{aligned}$$