Math 9 – Unit 1: Real Numbers Name: Mrs. Jacobs

Lesson #6: Limits Date: Fri, Feb 7.

Learning Goal: To understand what a limit is and what it represents.

What is a Limit? Exploring the idea of limits using Geometry
https://www.geogebra.org/m/tw2ncpsf

Example of limits in real life: if you drop an ice cube in a glass of warm water and measure the temperature with time, the temperature eventually approaches the room temperature where the glass is stored. Measuring the temperature is a limit as time approaches infinity.

> **<u>Definition Limit</u> – long term behaviour of a pattern, process, or function. It is the closest result as the number of terms increase (even if we can't actually reach the final result).**

Where do we use limits?
Limits are used throughout analysis as a way of making approximations into exact quantities, as when the area inside a curved region is defined to be the limit of approximations by rectangles. It is a big piece of Calculus.

$\longrightarrow +\infty$

Problem 1: Given the set {6, 7, 8, 9…} What is the long-term behaviour or limit for the sequence?
(What happens to the numbers as we continue to write this sequence?)

The SEQUENCE approaches INFINITY

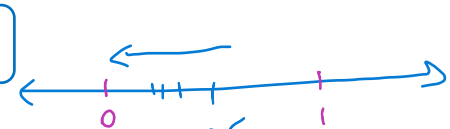
Why does this set approach infinity?

∵ (Bcz) the set looks like set of counting #s

Write a sequence of numbers that has a limit of negative infinity.

$\{-1, -2, -3, -4, \ldots\} \longrightarrow -\infty$

> **<u>Warning</u>**: ***when we say a limit =∞, technically the limit doesn't exist.***

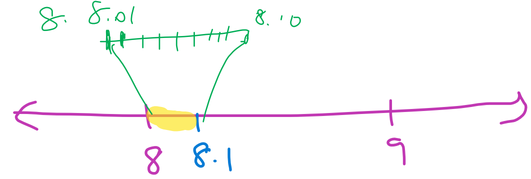


0.5 0.33 0.25 $\longrightarrow 0$

Problem 2: Given the set {$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ …} What is the limit? limit $\frac{1}{n} \longrightarrow 0$ where n could be 1, 2, 3, . . .
(What happens to the value of the fractions as we continue to write this sequence?)

Why do the fractions get smaller as we continue the sequence? Why is the limit __0__?

$\frac{1}{2}$ . . . $\frac{1}{3}$ . . . $\frac{1}{4}$ . . . $\frac{1}{5}$



Problem 3: Write the limit given the set: {8.1, 8.01, 8.001, 8.0001 …} ?
(Are the numbers getting larger or smaller? How do you know? What number are we getting closest to?)

Smaller and the numbers are getting closer to 8.

limit = 8

Write the limit given the set:
o 0.3, 0.33, 0.333, 0.3333… $\longrightarrow$ $0.\overline{3} = \frac{1}{3}$

o 1.6, 1.66, 1.666, 1.6666… $\longrightarrow$ $1.\overline{6} = \frac{5}{3}$

We learnt earlier that repeating decimals can be represented as rational numbers of the $\frac{p}{q}$ form.
What is the fractional representation for the limits above?

Let $x = 0.\overline{3} = 0.3333\ldots$

$\Rightarrow 10x = 3.3333\ldots$

Subtract the Left Side & Right side

$$10x - x = 3.333\ldots - 0.333\ldots$$

$$9x = 3$$

$$x = \frac{3}{9}$$

$$x = \frac{1}{3}$$

Let $y = 1.\overline{6} = 1.6666\ldots$

$10y = 16.6666\ldots$

$\therefore 10y - y = 16.66\ldots - 1.66\ldots$

$$9y = 15$$

$$\frac{9y}{9} = \frac{15 \div 3}{9 \div 3} = \frac{5}{3}$$

$$\boxed{y = \frac{5}{3}}$$

Problem 4: Given 3n. What happens when n gets larger and larger? n belongs to N which means n represents all counting #s {1, 2, 3, 4, ….}
(What operation is present in the expression? How does that help your understanding?)

3n

big

$n = 1 \Rightarrow 3n = 3(1) = 3$

$n = 2 \Rightarrow 3n = 3(2) = 6$

$n = 3 \Rightarrow 3n = 3(3) = 9$

$\{3, 6, 9, \ldots\ldots\} \longrightarrow \infty$

$\therefore$ Sequence has No limit.

What if the values of n were smaller and smaller fractions? How would that change the limit?

$n = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\ldots$

3n

small

$n = 1 \Rightarrow 3n = 3(1) = 3$

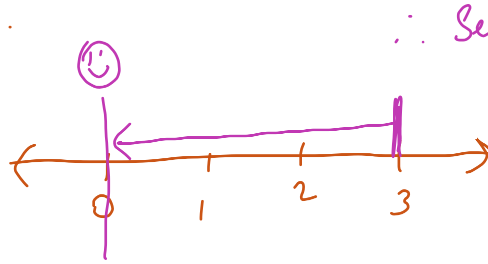
$n = 0.5 \Rightarrow 3n = 3(0.5) = 1.5$

$n = 0.3 \Rightarrow 3n = 3(0.3) = 0.9$

$n = 0.25 \Rightarrow 3n = 3(0.25) = 0.75$

$n = 0.2 \Rightarrow 3n = 3(0.2) = 0.6$

$\therefore$ Sequence has a limit = 0



$\{0.5, 1, 1.5, 2, 2.5, \ldots\}$

Problem 5: Given $\frac{1}{2}n$. What happens when n approaches infinity? $0.5n$

(What operation is present in the expression? How does that help your understanding?)

$n \longrightarrow \infty = \frac{1}{2}n \longrightarrow \infty$ $\therefore$ does not have a limit

What if the values of n were smaller and smaller fractions? How would that change the limit?

$0.5n$

$n = 1 \Rightarrow 0.5(1) = 1$

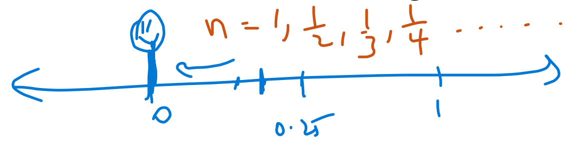
$n = 0.5 \Rightarrow 0.5(0.5) = 0.25$

$n = 0.3 \Rightarrow 0.5(0.3) = 0.150$

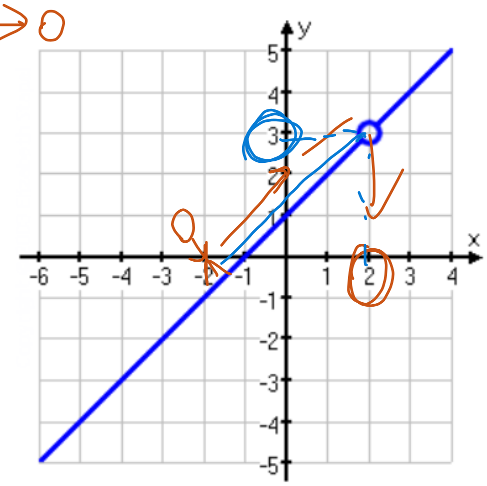
$n = 0.25 \Rightarrow 0.5(0.25) = 0.125$

$\longrightarrow 0$

$n = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots\ldots$



Problem 6: Now, let's say we want to know the y-value when x = 2 for this graph.



But this graph has a hole at x = 2. So, if you try to go right to x = 2, you'll fall through!!!

What you need to do is "take the limit as x approaches 2" (aka. Get really close to x = 2, but don't fall through the hole) $x \longrightarrow 2$

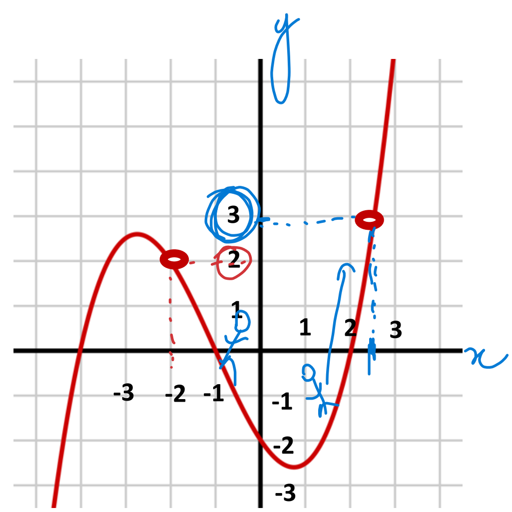
And that limit = 3

what is corresponding y-value

Problem 7: What is the limit as x approaches 2.5 on this graph?

$x \longrightarrow 2.5 \Rightarrow y \longrightarrow 3$

$\therefore$ limit $= 3$



What is the limit as x approaches –2 on this graph?

$x \longrightarrow -2 \Rightarrow$ limit $= 2$

Limit Notation:

Here's how we write "***take the limit as x approaches 2***" in mathy notation:

$$\lim_{x \to 2}$$

Another place we can't actually get to is **infinity**.
So, we might "*take the limit as x approaches infinity*" to see what happens in the long-term behaviour of a process or function.

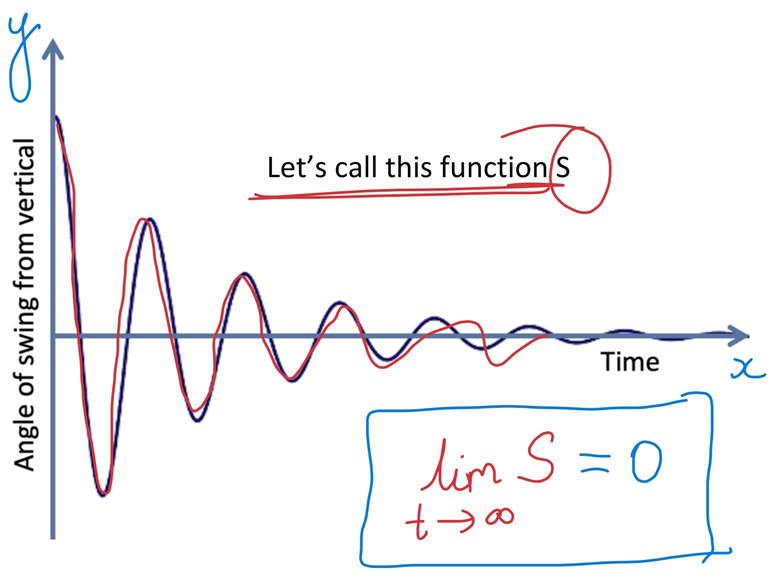
Here's how we write that:

$$\lim_{x \to \infty}$$

Application: Easter Seals Camp Woodeden Giant Swing

- Campers are pulled up to about 60ft in the air by a pulley
- The camper pulls a level that releases them, and they start to swing!

If you measure the angle between the rope and a vertical line, the angle will change with time like this:



$$\lim_{t \to \infty} S = 0$$

Calculate the limit of S (note that S depends on time t). Remember to use the mathy notation to represent it.

Success criteria:

- I understand what a limit represents and why we use it.
- I know the mathy notation for a limit.
- I can find the limit of a given pattern, sequence, or function.
- I can apply the concept of limits to the real world.