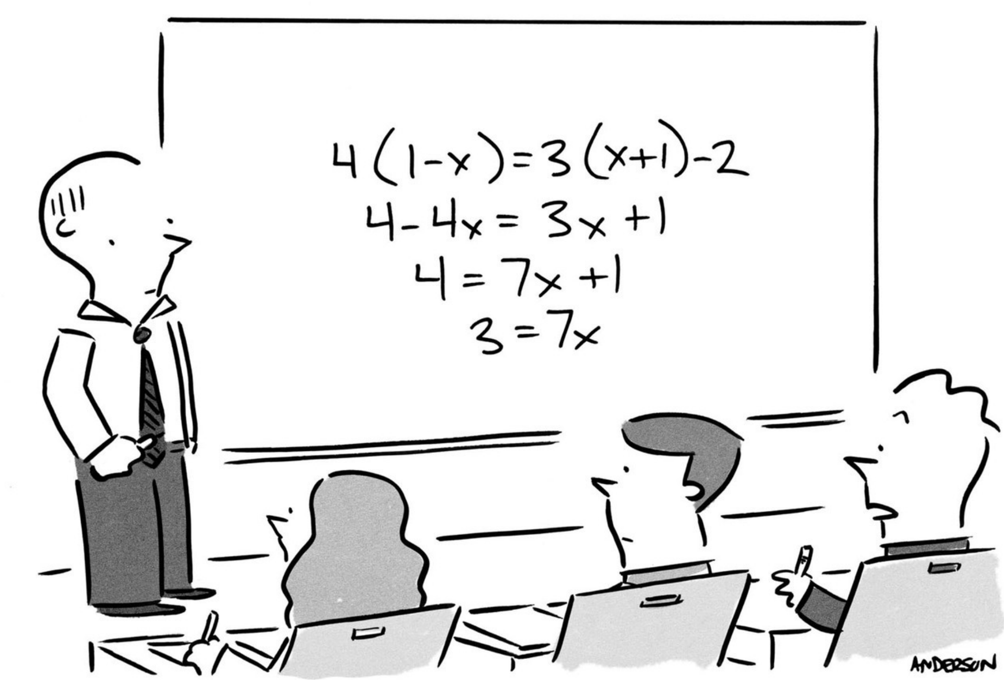
Name Mrs. Jacobs .

Math 9

(MTH1W)



"Wouldn't it be more efficient to just find who's complicating equations and ask them to stop?"

Unit 2: ALGEBRA

- An Upgrade for your Mind!!! 😊

WHAT IS ALGEBRA? Learning algebra is like learning to read. It's a skillset for working with numbers and variables that unlocks a whole amazing world of math & science!

Math 9 – Unit 2: Algebra One Name: Mrs. Jacob

Lesson 2.1: Collecting Like Terms; Adding and Subtracting Polynomials Date: Feb 14, 2025

In this unit, you will be introduced to one of the most important components to Mathematics: Algebra. Algebra comes from the Arabic word *"al-jabr"*, meaning "the coming together of broken parts", and math is about bringing together ideas to solve problems. In Algebra, we will look at how to use Mathematical symbols and the rules for manipulating them. Typically, the symbols are letters.

Learning Goal: We are learning common math terminology and using those terms to simplify algebraic expressions. Later, we also explore adding and subtracting polynomials.

To begin, let's define some terminology that is important in Algebra.

Constant: It is anything that is fixed. (remains same) In Math, we use numbers to represent fixed values. eg $\frac{1}{2}$, 700, $-0.39\overline{5}$ ...

Variable: It is anything that can change. In Math, we use letters (lower case of English Alphabet) to represent variables. eg $x, y, z, a, b, c, p, q, r, \ldots$

Algebraic Expression: Combination of constants and variables eg $3x+y-2$ (3 terms), x^2 (1 term), pqx

Terms: are the building blocks of algebraic expressions.

Coefficient (Numerical Coefficient) of a term: It is the number factor of the term.

Example- eg $13x^2+2x$ → 2 is coefficient of $2x$; 13 is coefficient of $13x^2$

Like terms: Terms with same variable combinations.

Example- $3x^2$ and $5x^2$, $-15xy$ and $3xy$, $5ab$ and $7ba$

Unlike terms: Terms that are not like.

Example- $3x^2$ and $3y^2$, $13xy$ and $2x$

Polynomials: Special algebraic expressions with exponents of variables are whole numbers.

Degree of a polynomial: It is the highest exponent carried by the variable of our polynomial.

Classification of polynomials:

eg degree $(3x^2+2x^1+5) = 2$

degree $(x^4-1) = 4$

degree $(y^5-y^3+2y-7) = 5$

Polynomial

(# of terms)
1. 1-term (eg $3x$) MONOMIAL
2. 2-terms (eg $5x+y$, $2x-1$) BINOMIAL
3. 3-terms (eg $x+y+3$) TRINOMIAL
4. 4-terms – QUADRINOMIAL
5. 5-terms – PENTANOMIAL

degree
1. Linear eg $x+3$; $5x$
2. QUADRATIC eg x^2+3x+5; x^2-5; x^2
3. CUBIC eg x^3; x^3-1, x^3-2x^2+3x+5
4. QUARTIC
5. QUINTIC
⋮

Example: ***Complete the chart.***

Degree	***Name using Degree***	***Polynomial Example***	***Number of Terms***	***Name using Number of terms***
0	CONSTANT POLYNOMIAL	***5*** $= 5x^0$	1	MONOMIAL
1	LINEAR POLYNOMIAL	***x+4***	2	BINOMIAL
2	QUADRATIC POLYNOMIAL	$4x^2$	1	MONOMIAL
3	CUBIC "	$4x^3-2x^2+x$	3	TRINOMIAL
4	QUARTIC "	$2x^4+5x^2$	2	BINOMIAL
5	QUINTIC "	$-x^5+4x^2+2x+1$	4	QUADRINOMIAL

Example: Given the following expressions, state the number of terms, the coefficients of the different terms, and the constant term.

a) $3x^2 - 5x + 7$ b) $-5y + 10x + 8 - 12y$

Polynomial	Number of Terms	Coefficients	Constants
a) $3x^2 - 5x + 7$ QUADRATIC TRINOMIAL	3	$3x^2 \rightarrow$ coef. = 3 $-5x \rightarrow$ coef. = -5	7
b) $-5y + 10x + 8 - 12y$ $= -17y + 10x + 8$ LINEAR TRINOMIAL	3	$-17y \rightarrow$ coeff. = (-17) $10x \rightarrow$ coeff. = 10	8

In the above example, the second expression has 4 terms, but two of them had the same variable. This means that we can combine them together. All you need to do is add, or subtract, their coefficients. This process is called collecting like terms.

Collect the like terms in the above example: $-5y + 10x + 8 - 12y$

$-17y + 10x + 8$

More examples:

a) $-6 - 3r^2 - 4r + 2 + 6r$

$= -3r^2 + 2r - 4$

b) $-4k^3 - 8k^2 + 4 + 7k^4 - k^3 - 8k^2 - 1$

$= 7k^4 - 5k^3 - 16k^2 + 3$

c) $7a^2b^2 + 2a^4 - 8a^3b^3 - 4a^2b - 2a^4 - 2a^3b^3 + 8a^2b^2$

$= 15a^2b^2 - 10a^3b^3 - 4a^2b$

Now for a super duper big example:

d) $-8x - x^2y^2 - 8x^3y^5 + 3x^3y + 2x^3y + 6x + 2x^2y^2 + 2xy - 2x^2y^2 + 5x^3y^4 + 3xy + 5x$

$= 3x - x^2y^2 - 8x^3y^5 + 5x^3y + 5x^3y^4 + 5xy$

There's more! Did you ask, "what term should I write first?" If you did, good thinking! There is a definite order to writing out expressions. It is called descending order.

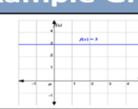
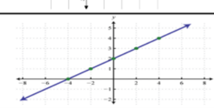
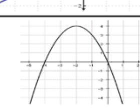
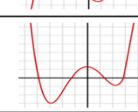
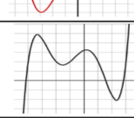
Descending order is: greatest to lowest

Now go back to the above examples and put them in descending order.

REVIEW:

When considering the number of terms, $4x^2$ is called a MONOMIAL, while $3x^5 - 2xy$ is called a BINOMIAL, and $7y^2 + 5y - 1$ is called a TRINOMIAL

Whether it be a monomial, binomial, trinomial, quadrinomial, quintrinomial (or pentanomial), they all fall under the big category called POLYNOMIAL.

Degree	Classification	Example Expression	Example Graph
0	CONSTANT	−14	
1	LINEAR BINOMIAL	3x − 2	
2	QUADRATIC TRINOMIAL	−4x² + 3x − 5	
3	CUBIC QUADRINOMIAL	x³ + 2x² − 3x + 4	
4	QUARTIC QUADRINOMIAL	2x⁴ − 5x³ + x − 7	
5	QUINTIC HEXANOMIAL	−3x⁵ + x⁴ − 5x³ + 9x² − 2x − 7	

Examples: For each expression, collect the like terms and state the type of polynomial.

a) $-2v - \cancel{2v^5} - 8 + \cancel{2v^5} + 7v$

$= 5v - 8$

b) $3xy - 4x^2y + \boxed{8x^4y} + 6xy - 7x^2y - \boxed{7x^4y}$

$= 9xy - 11x^2y + x^4y$

c) $1.75x^5 - 0.6x^4 - 1.6x^4 + 0.85x^5$

$= 2.60x^5 - 2.2x^4$

Now that we feel comfortable with what terms, coefficients, variables, and constants are, and on how to collect like terms, we can start to work on the arithmetic of algebra. Today we will add and subtract polynomials. Essentially, adding and subtracting polynomials are just collecting like terms. Let's dive in!

Examples: Add the polynomials, putting the answer in descending order.

a) $(x^3 + \boxed{7x} - 6) + (2x^3 \boxed{-5x} - 2)$

$$\begin{array}{r} 1x^3 + 7x - 6 \\ +\ 2x^3 - 5x - 2 \\ \hline 3x^3 + 2x - 8 \end{array}$$

b) $(3p^2 + 4 + 3p) + (3p^2 - 8p + 7) + (2p^2 - 4)$

$$\begin{array}{r} 3p^2 + 3p + 4 \\ 3p^2 - 8p + 7 \\ 2p^2 \qquad\quad - 4 \\ \hline 8p^2 - 5p + 7 \end{array}$$

Examples: Subtract the polynomials, putting the answer in descending order.

a) $(m^4 + 7m^2 - 5) - (6m^4 - 4m^2 + 3)$

$-6m^4 + 4m^2 - 3$

$= -5m^4 + 11m^2 - 8$

b) $(x^2 - 2x^3) - (6x^2 - 4x^4 + 5x^3)$

$-6x^2 + 4x^4 - 5x^3$

$= 4x^4 - 7x^3 - 5x^2$

c) $(7x + 7) - (7x^2 + 8x^3 + 4x^4) - (3x^4 + 8)$

$-7 \quad -8 \quad -4 \qquad -3 \quad -8$

$$\begin{array}{r} 7x + 7 \\ -4x^4 - 8x^3 - 7x^2 \qquad\quad \\ -3x^4 \qquad\qquad\qquad\qquad - 8 \\ \hline -7x^4 - 8x^3 - 7x^2 + 7x - 1 \end{array}$$

d) $(8u^3v^4 + 2v^4) - (6v^4 + 7u^2v^4 - 3u^3v^4) + (8u^2v^4 - 8u^3v^4)$

$-6v^4 - 7u^2v^4 + 3u^3v^4$

$= 3u^3v^4 - 4v^4 + u^2v^4$

Note: If you were to be asked to subtract 6 from 9, how would you answer? Essentially, you are doing $9-6$. The same holds true with polynomials:

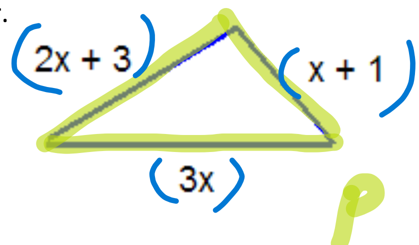
Example: Subtract $(4x^2+5x-3)$ from $(9x^2+3x-7)$ -- this is not an expression. Turn it into an expression by writing $(2^{nd}\ polynomial)-(1^{st}\ polynomial)$

$= (9x^2+3x-7)-(4x^2+5x-3)$

$-4 \quad -5 \quad +3$

$= 5x^2-2x-4$

Example: a) Given the following triangle, determine an expression for the perimeter.



$P = (2x+3)+(x+1)+(3x)$

$\boxed{P = 6x+4}$

b) Let $x = 6cm$, determine the perimeter of the triangle.

$P = 6(6)+4$

$\boxed{P = 40\text{ cm}}$

Success Criteria:

- I can correctly define the following terms: expression, variable, coefficient, constant, like term, unlike term, monomial, binomial, trinomial, polynomial, and degree
- I can group like terms within algebraic expressions
- I can identify the degree and type of various polynomials
- I can add/subtract polynomials by grouping like terms
- I can distribute the negative into a polynomial
- I can arrange polynomials in descending order

Build your Skills: :)

1. Create an algebraic expression to represent each of the following.

   a) A number x is tripled and then 19 is subtracted from the result.

   $3x - 19$

   b) The variable y is squared and then the result is increased by 10.

   $y^2 + 10$

   c) The variable n is decreased by 6 and then the result is multiplied by −8.

   $-8(n-6)$

   d) A number p is increased by 70 and then the result is divided by 6.

   $\frac{p+70}{6}$ or $(p+70) \div 6$

2. The sum of the interior angles of a polygon can be found by subtracting 2 from the number of sides and multiplying the result by 180°.

   a) Determine an algebraic expression to represent the sum of the interior angles for a polygon with n sides.

   $\boxed{S_n = (n-2)180^\circ}$

   b) Use your expression from part (a) to determine the sum of the interior angles for an octagon.

   $n = 8$

   $\therefore S_8 = (8-2)180 = 6(180) = 1080^\circ$

   c) A regular polygon has equal side lengths and equal interior angles. Determine the value of each interior angle in a regular hexagon.

   $n = 6$

   $\therefore S_6 = (6-2)180 = 4(180) = 720^\circ$

   Now, Since all interior angles equal

   $\therefore$ Each angle $= \frac{720}{6} = 120^\circ$

   d) The sum of the interior angles for a particular polygon is 1440°. How many sides does this polygon have?

   S_n $\quad n = ?$

   $S_n = (n-2)180$

   $\Rightarrow \frac{\cancel{1440}^{8}}{\cancel{180}} = \frac{(n-2)\cancel{180}}{\cancel{180}} \Rightarrow 8 = n-2 \Rightarrow 8+2 = n \Rightarrow 10 = n$

   $\therefore$ Polygon has 10 sides.

   e) For a given regular polygon, each interior angle is 156°. How many sides does the polygon have?

   all sides equal.
   all angles equal.

   $n = ?$

   $\therefore$ Sum of all angles $= 156n = S_n$

   $S_n = (n-2)180$

   $156n = 180n - 360$

   $\Rightarrow 156n - 180n = -360$

   $\Rightarrow -24n = -360$

   $\Rightarrow n = \frac{\cancel{-360}^{15}}{\cancel{-24}}$

   $\Rightarrow n = 15$

   $\therefore$ Polygon has 15 sides.