Math 9 – Unit 2: Algebra One Name: Mrs. Jacob

Lesson 2.3: Powers of Monomials and Dividing by Monomials Date: Feb 20, 2025

Learning Goal: We are learning to expand and simplify more complicated expressions. Also, learning how to divide by monomials.

Let's start off by discussing monomials. How do we simplify $\boxed{(3x^2y^5)^3}$? This is called a monomial raised to a power. How does the outside exponent affect the question? First, how does it work with just a number?

Simplify $(4^3)^2 = (4^3)(4^3) = 4^{3+3} = 4^6 = 4^{3\times 2} = 4096$

The initial exponents were *3* and *2*, with the final exponent a 6. So, 3 x 2 = 6! This leads to our second exponent law. **When raising a power to a power,** MULTIPLY **the exponents.** Try it out!

a) $(x^4)^5$ $= x^{4\times 5} = x^{20}$

b) $(y^2)^8$ $= y^{16}$

c) $(m^3n^6)^4$ $= (m^3)^4(n^6)^4 = m^{12}n^{24}$

That's all well and good (hopefully), but how do you handle a question with a coefficient?

Consider the expression from before, $(3x^2y^5)^3$. Expand it without using the laws.

$3^3(x^2)^3(y^5)^3 = 27x^6y^{15}$

The coefficient was just raised to the power of 3! Awesome. Try out some more, this time following the laws.

a) $(2x^4y^2)^5$ $= 2^5(x^4)^5(y^2)^5 = 32x^{20}y^{10}$

b) $(-3m^7n)^2$ $= (-3)^2(m^7)^2n^2 = 9m^{14}n^2$

c) $(5a^2b^3c^4d^5)^6$ 15625 $a^{12}b^{18}c^{24}d^{30}$

d) $(3x^2y^5)^2(2xy^3)$ $= (9x^4y^{10})(2xy^3) = 18x^5y^{13}$

e) $(-4m^3n^2)^3(3m^4n^3)^2$ $= (-64m^9n^6)(9m^8n^6) = -576m^{17}n^{12}$

We've added, subtracted, multiplied, and even raised monomials to powers. All that is left is dividing by monomials. First, let's develop a rule with numbers.

Simplify $\frac{4^5}{4^3} = \frac{4\times 4\times \cancel{4}\times \cancel{4}\times \cancel{4}}{\cancel{4}\times \cancel{4}\times \cancel{4}} = 4^2$

This leads to our 4th exponent law. **When dividing,** SUBTRACT **the exponents**. Time to put it into practice!

a) $\frac{x^8}{x^5} = x^3$ b) $\frac{y^{72}}{y^{46}} = y^{72-46} = y^{26}$ c) $\frac{m^5n^3}{m^2n} = \left(\frac{m^5}{m^2}\right)\left(\frac{n^3}{n}\right) = m^3n^2$ d) $\frac{18p^7q^9}{3p^2q^2} = 6p^5q^7$

The final step is to divide a monomial into a polynomial, such as $\frac{(4x^5-2x^3+6x^2)}{2x^2}$. However, first let's look back at adding fractions so we can see an integral step that we will need to use:

$$\frac{1}{2}+\frac{3}{4}+\frac{5}{8} = \frac{4}{8}+\frac{6}{8}+\frac{5}{8} = \frac{4+6+5}{8}$$

Now time for Algebra! Remember that the denominator gets applied to all the terms in the numerator.

a) $\frac{4x^5-2x^3+6x^2}{2x^2}$

$= \frac{4x^5}{2x^2} - \frac{2x^3}{2x^2} + \frac{6x^2}{2x^2}$

$= 2x^3 - x + 3$

b) $\frac{16x^3y^5+8x^2y^4}{4x^2y}$

$= \frac{16x^3y^5}{4x^2y} + \frac{8x^2y^4}{4x^2y}$

$= 4xy^4 + 2y^3$

c) $\frac{40a^3b^6-50a^2b^3+10ab}{10ab}$

$= \frac{40a^3b^6}{10ab} - \frac{50a^2b^3}{10ab} + \frac{10ab}{10ab}$

$= 4a^2b^5 - 5ab^2 + 1$

d) $\frac{9x^7+27x^5-15x^4}{(-3x^3)}$

$= \frac{9x^7}{-3x^3} + \frac{27x^5}{-3x^3} \frac{-15x^4}{-3x^3}$

$= -3x^4 - 9x^2 + 5x$

e) $\frac{192r^{78}s^{34}-144r^{65}s^{53}-256r^{98}s^{23}+80r^{88}s^{45}}{16r^{33}s^{21}}$

$= \frac{192r^{78}s^{34}}{16r^{33}s^{21}} - \frac{144r^{65}s^{53}}{16r^{33}s^{21}} - \frac{256r^{98}s^{23}}{16r^{33}s^{21}} + \frac{80r^{88}s^{45}}{16r^{33}s^{21}}$

$= 12r^{45}s^{13} - 9r^{32}s^{32} - 16r^{65}s^2 + 5r^{55}s^{24}$

Success Criteria:

- I can simplify a monomial raised to a power by multiplying the exponents of each variable
- I recognize that when a coefficient is raised to a power, it is NOT NOT NOT multiplied
- I can divide like variables by subtracting their exponents and understand the difference between dividing coefficients and dividing variables
- I can divide the monomial into each term of a polynomial separately and recognize that when you divide two identical monomials, the result is one.