MTH1W – Unit 7: Coordinate

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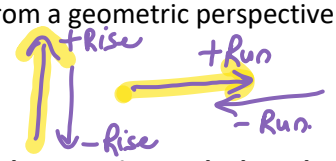
Geometry Lesson 6.3: Slope of a Line

Date: April 16, 2025.

Learning Goal: We are learning how slope impacts a linear equation. It's all downhill from here!

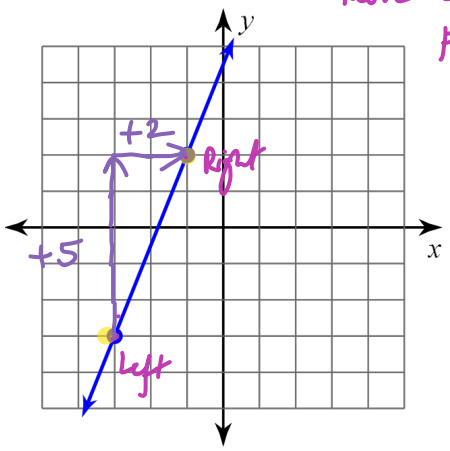
In this lesson, we will explore the most significant property of a linear relationship: the slope! The slope of a line tells us how the relationship is changing and can be thought of as how slanted/steep the line is. It has many important applications such as engineering the initial climb of a roller coaster to making safe ramps, but today we will focus on the algebra and understanding how to calculate the slope of a line.

First, let's look at the slope from a geometric perspective. The slope, defined by the letter m for no apparent reason, is: $m = \frac{Rise}{Run}$



Example 1: Given the line with two points, calculate the slope.

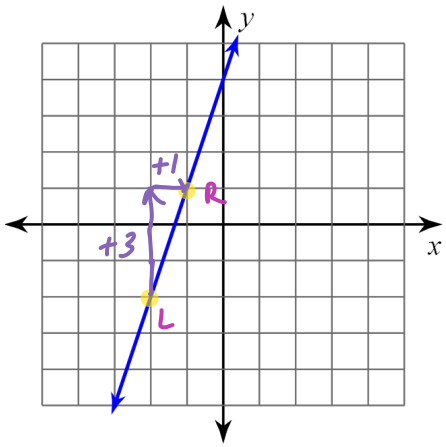
move Left point → Right point



$m = \frac{5}{2}$

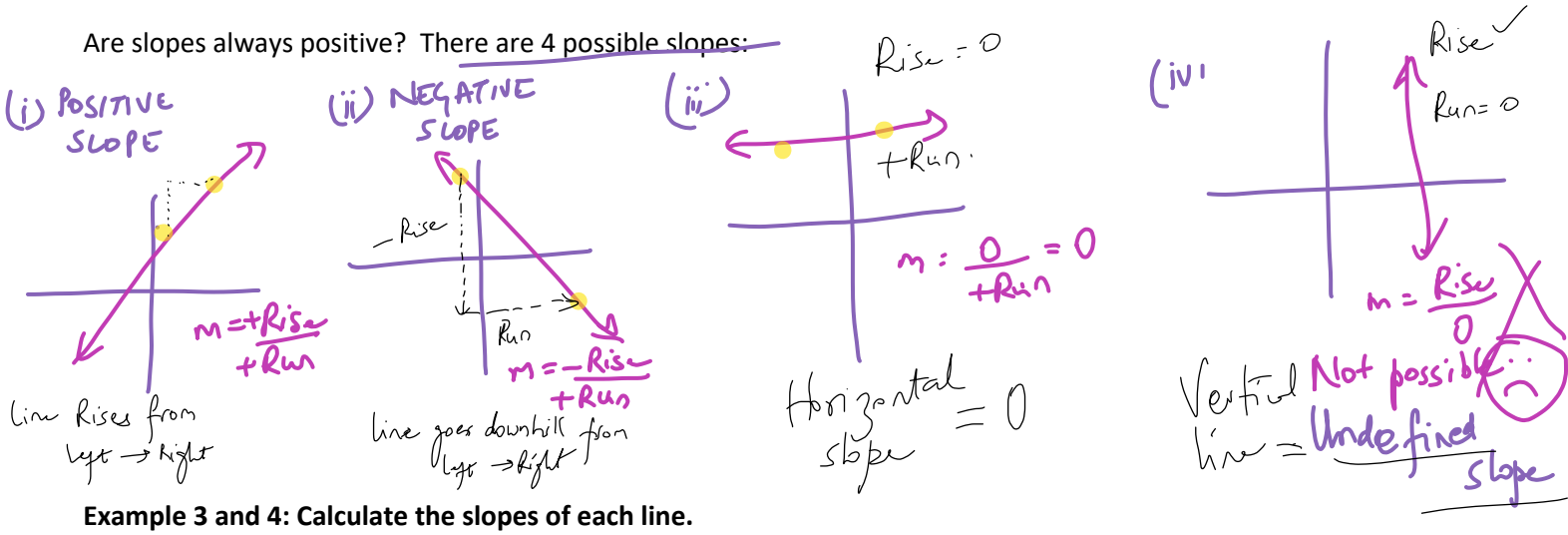
Always reduce the slope to a simplified FRACTION.

Example 2: Given the line, locate two points, then calculate the slope.

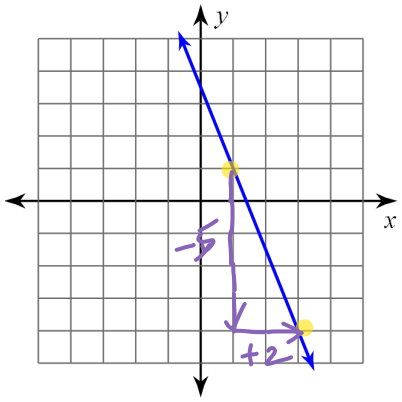


$m = \frac{3}{1} = 3$

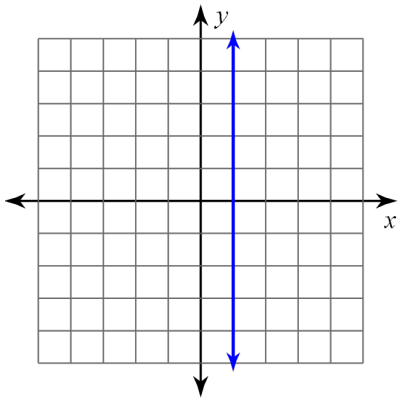
Are slopes always positive? There are 4 possible slopes:



Example 3 and 4: Calculate the slopes of each line.



$m = \frac{-5}{2}$



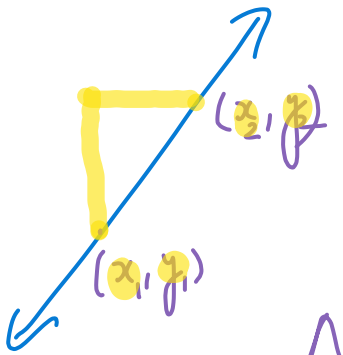
m = undefined

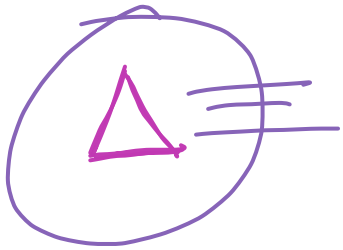
Now that we know about slope, we can derive a formula so that we do not need a graph.

Ex1 Today's Temp = 5°C
Yesterday's Temp = 2°C
Change Temp = 5 − 2 = 3°C

Ex2 Today's Temp = 2°C
Yesterday's Temp = 5°C.
Change in temp = 2 − 5 = −3°C

Ex3 Today's Temp = 2°C
Yesterday's Temp = −5°C
Change in Temp = 2 − (−5) = 2 + 5 = 7°C





DELTA (Symbol to represent Change)

$\Delta T = T_2 - T_1$

$$m = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Examples 5-8: Given the points, calculate the slope.

5. $(7,-10),(9,-7)$

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-7 - -10}{9 - 7} = \frac{-7+10}{9-7}$

$\boxed{m = \frac{3}{2}}$

6. $(-6,-17),(-20,11)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - -17}{-20 - -6} = \frac{11+17}{-20+6} = \frac{28}{-14} = \frac{-2}{1}$

$\boxed{m = -2}$

7. $(6,-12),(6,1)$

$m = \frac{1 - -12}{6-6} = \frac{1+12}{0}$ ☹

$\boxed{m = \text{Undefined}}$

8. $(-3,9),(3,9)$

$m = \frac{9-9}{3--3} = \frac{0}{3+3} = 0$

$\boxed{m = 0}$

Example 9: A ramp needs to be constructed to go from the ground to a doorway. The doorway is 90 cm from the ground and the ramp needs a slope of $\frac{2}{9}$.

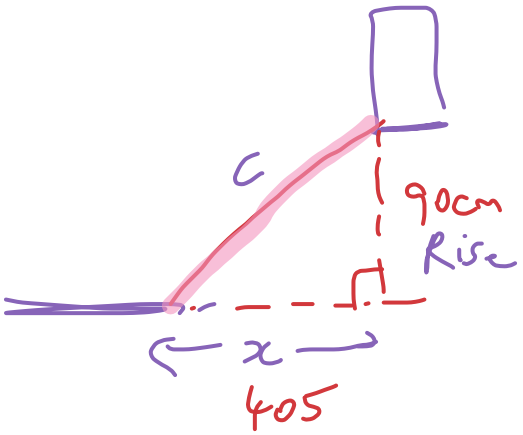
a) Calculate how far the ramp will start from the edge of the house.

$m = \frac{2 \leftarrow \text{Rise}}{9 \leftarrow \text{Run}}$ $\quad m = \frac{\text{Rise}}{\text{Run}}$

$\frac{2}{9} = \frac{90}{x}$

$\Rightarrow 2x = 810$

$\Rightarrow x = \frac{810}{2} = 405\text{cm}.$



b) Calculate the length of the ramp.

$c = \sqrt{a^2 + b^2}$

$= \sqrt{(90)^2 + (405)^2}$

$= \sqrt{8100 + 164025}$

$= \sqrt{172125} \approx 414.88\text{cm}$

$\therefore$ length of ramp $= 414.88\text{cm}.$

Example 10 and 11: Calculate the missing coordinate.

10. $(2, y)$ and $(-3, -2)$; slope $\frac{3}{5}$

$x_1\ y_1$ $x_2\ y_2$ $= m$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{3}{5} = \frac{-2-y}{-3-2}$$

$$\Rightarrow \frac{3}{5} = \frac{(-2-y)}{-5}$$

$$\Rightarrow -5(3) = 5(-2-y)$$

$$\Rightarrow -15 = -10 - 5y$$

$$\Rightarrow -15 + 10 = -5y$$

$$\frac{-5}{-5} = \frac{-5y}{-5}$$

$$1 = y$$

11. $(x, 4)$ and $(-5, 10)$; slope: $\frac{3}{2}$

$x_1\ y_1$ $x_2\ y_2$ $= m$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{3}{2} = \frac{10-4}{(-5-x)}$$

$$\frac{3}{2} = \frac{6}{(-5-x)}$$

$$3(-5-x) = 6(2)$$

$$-15 - 3x = 12$$

$$-3x = 12 + 15$$

$$x = \frac{27}{-3}$$

$$x = -9$$

Success Criteria

- I can identify the four types of slope: positive, negative, zero, undefined
- I can find the slope of a line graphically by studying its $\frac{rise}{run}$
- I can calculate the slope of a line algebraically by using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$
- I can find a missing coordinate, if given the slope