

Lesson #4: Slope as a Rate of Change

Date: April 22, 2025**Learning Goal:** We are learning to connect rate of change to the slope of a line.

To explore what “rate of change” is, we first need to refamiliarize ourselves with “rate”. A rate is a comparison of two quantities expressed as different units.

Examples: 50 pages / 2 days; 70 km / 1 hr; 500 steps / day; 5 cookies / \$2

A line on a graph is always changing (unless it is flat or $m = 0$). Rate of change, then, is the rate at which a line on a graph is changing. Thankfully, we know how to calculate this change by calculating the slope! Thus,

$$\text{Rate of change} = \text{slope} = m = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1} = \text{Rate of change}$$

$$R_oC = m$$

Example 1: Given the graph to the right:

a) Calculate the rate of change. Include the units (always include units).

$$R_oC = m = \frac{\Delta y}{\Delta x} = \frac{\$1}{4 \text{ cookies}} = \$0.25/\text{cookie}$$

b) What does the rate of change represent?

R.o.C represents the cost in \$ per cookie

c) How much would 7 cookies cost? If I spent one dollar, how many cookies would I get?

$$\therefore \text{cost of 7 cookies} = \$1.75$$

$$\therefore \text{for } \$1, \text{ I get } = 4 \text{ cookies}$$

d) The information for question c) was in the graph. The rate of change allows us to go beyond the graph. How much would 20 cookies cost?

$$\text{cost of 1 cookie} = \$0.25$$

$$\therefore \text{cost of 20 cookies} = 20(0.25) = \$5$$

INTERPOLATION

EXTRAPOLATION

Rate of Change Without a Graph

Having a graph is great as it allows us to visualize the information and actually see the steepness (or its flatness, yes, that's a word). However, we do not always have a graph:

Example 1: A climber is on a hike. After $x_1 = 2$ hours, he is at an altitude of $y_1 = 400$ feet. After $x_2 = 6$ hours, he is at an altitude of $y_2 = 700$ feet. What is the average rate of change?

Wait—why are we asking for the average rate of change?

Average RoC because the change in altitude with respect to time may not be constant. It could vary if the climber slows down or gets tired.

Since rate of change = slope, the rate of change is also $m = \frac{y_2 - y_1}{x_2 - x_1}$. If we could create two points, we could

then calculate the slope/RoC!

Solve Example 1:

$$\begin{aligned} \text{RoC} = m &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{(700 - 400) \text{ feet}}{(6 - 2) \text{ hours}} \\ &= \frac{300}{4} \text{ ft/h.} \\ &= 75 \text{ ft/h.} \end{aligned}$$

$\therefore$ The average rate of change is 75 ft/h.

Example 2: A scuba diver is $y_1 = -30$ feet below the surface of the water $x_1 = 10$ seconds after he entered the water and 100 feet below the surface after 40 seconds. What is the scuba diver's rate of change?

$$y_2 = -100 \text{ ft}$$

$$x_2 = 40 \text{ s.}$$

$$\text{RoC} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-100) - (-30)}{(40) - (10)} = \frac{-100 + 30}{40 - 10} = \frac{-70}{30} = -\frac{7}{3} \text{ ft/s.}$$

$\therefore$ The scuba diver's rate of change is $-\frac{7}{3}$ ft/s.

Success Criteria

- I can recognize that slope and rate of change are the same thing
- I can find rate of change on a graph, by finding its slope
- I can create two ordered pairs from a given scenario or equation and find the average rate of change between them
- I can use the slope formula to calculate average rate of change