Mathematics 10D

2.6 – Centroid and Circumcentre

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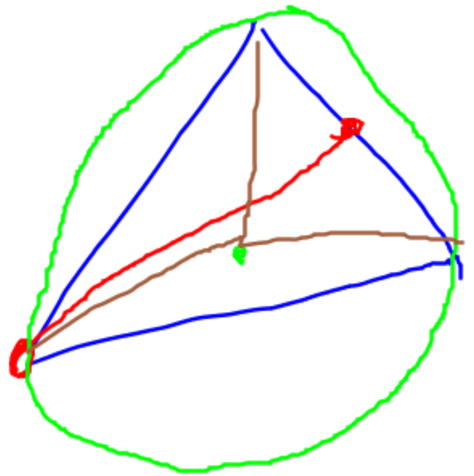
Centroid - the centre of mass of a triangle. It is the intersection of the medians.

Circumcentre - the centre of the circle created by a triangle. It is the intersection of the perpendicular bisectors.

median:



Find centroid of $A(-4, 3)$, $B(-1, -3)$, $C(2, 2)$.

(1) midpoint of AB

$$M_{AB}\left(\frac{-4-1}{2}, \frac{3-3}{2}\right) : \left(\frac{-5}{2}, 0\right)$$

(2) $y = mx + b$ from M_{AB} to C

$$m = \frac{2-0}{2+\frac{5}{2}} = \frac{2}{\frac{9}{2}} = \frac{4}{9}$$

$$0 = \frac{4}{9}\left(\frac{-5}{2}\right) + b$$

$$0 = \frac{-20}{18} + b$$

$$\frac{10}{9} = b$$

$$\therefore y = \frac{4}{9}x + \frac{10}{9}$$

(3) Midpoint of BC

$$M_{BC}\left(\frac{-1+2}{2}, \frac{-3+2}{2}\right) = \left(\frac{1}{2}, \frac{-1}{2}\right)$$

(4) $y = mx + b$ from M_{BC} to A

$$m = \frac{\frac{-1}{2} - 3}{\frac{1}{2} + 4} = \frac{-\frac{7}{2}}{\frac{9}{2}} = \frac{-7}{9}$$

$$3 = \frac{-7}{9}(-4) + b$$

$$3 = \frac{28}{9} + b$$

$$\frac{27}{9} - \frac{28}{9} = b = \frac{-1}{9}$$

$$y = \frac{-7}{9}x - \frac{1}{9}$$

(5) Find PoI of $y = \frac{4}{9}x + \frac{10}{9}$ and $y = \frac{-7}{9}x - \frac{1}{9}$

$$\left(\frac{4}{9}x + \frac{10}{9} = \frac{-7}{9}x - \frac{1}{9}\right)9$$

$$4x + 10 = -7x - 1$$

$$11x = -11$$

$$x = -1$$

$$y = \frac{-7}{9}(-1) - \frac{1}{9}$$

$$y = \frac{7}{9} - \frac{1}{9}$$

$$y = \frac{6}{9} = \frac{2}{3}$$

$\therefore$ the centroid is at $(-1, 2/3)$

Find Circumcentre of $A(-4,3)$, $B(-1,-3)$, $C(2,2)$

(1) Midpoint of AB $\quad M_{AB}\left(\frac{-5}{2}, 0\right)$

(2) Slope $\perp$ to AB

$$m = \frac{-3-3}{-1+4} = \frac{-6}{3} = -2 \quad \therefore \quad m_{\perp} = \frac{1}{2}$$

(3) $y = m_{\perp}x + b$ with M_{AB}

$$0 = \frac{1}{2}\left(\frac{-5}{2}\right) + b = \frac{5}{4}$$

$$y = \frac{1}{2}x + \frac{5}{4}$$

(4) $M_{BL}\left(\frac{1}{2}, -\frac{1}{2}\right)$

(5) Slope $\perp$ to BC

$$m = \frac{2+3}{2+1} = \frac{5}{3} \quad \therefore \quad m_{\perp} = -\frac{3}{5}$$

(6) $y = m_{\perp}x + b$ and M_{BL}

$$-\frac{1}{2} = \frac{-3}{5}\left(\frac{1}{2}\right) + b$$

$$-\frac{1}{2} = -\frac{3}{10} + b$$

$$-\frac{5}{10} + \frac{3}{10} = b$$

$$\frac{-2}{10} = b = -\frac{1}{5}$$

$$\therefore y = -\frac{3}{5}x - \frac{1}{5}$$

(7) PoI of $y = \frac{1}{2}x + \frac{5}{4}$ and $y = \frac{-3}{5}x - \frac{1}{5}$

$\left(\frac{1}{2}x + \frac{5}{4} = \frac{-3}{5}x - \frac{1}{5}\right) 20$

$10x + 25 = -12x - 4$

$22x = -29$

$x = -1.32$

$y = \frac{1}{2}(-1.32) + \frac{5}{4}$

$y = -0.66 + 1.25$

$y = 0.59$

Circumcentre is

$(-1.32, 0.59)$