Mathematics 10D

8.3 Cosine Law

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Goal: Explore the relationship between side lengths and angle measures in a triangle using the cosines of angles.

Cosine Law:

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

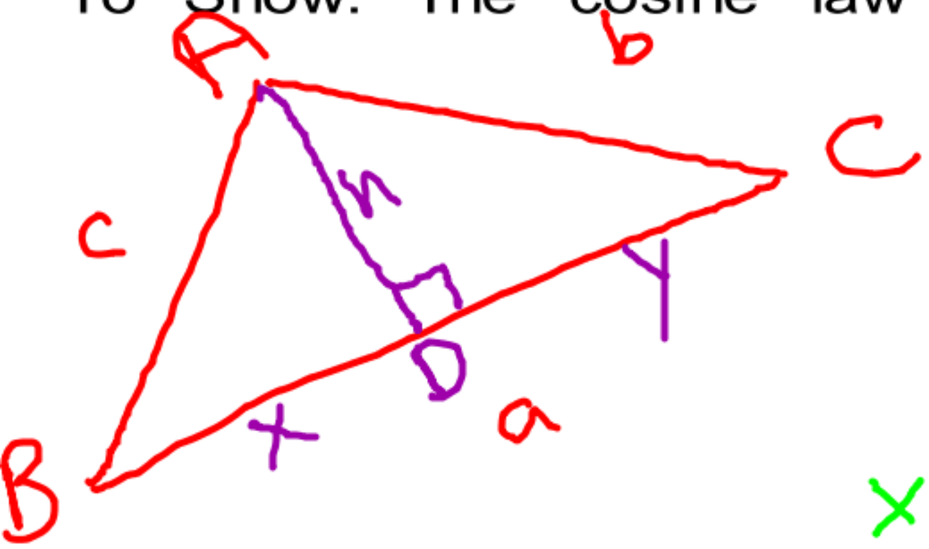
$$b^2 = a^2 + c^2 - 2ac\cos(B)$$

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos(C)}$$

Solving a side length using 2 side lengths and an angle.

Proof

To Show: The cosine law is true for all acute triangles.



$\triangle ABD$

$c^2 = x^2 + h^2$

$c^2 - x^2 = h^2$

$\triangle ACD$

$b^2 = h^2 + y^2$

$b^2 - y^2 = h^2$

$x + y = a$

$x = a - y$

$c^2 - x^2 = b^2 - y^2$

$c^2 - (a-y)^2 = b^2 - y^2$

$c^2 = b^2 - y^2 + (a-y)^2$

$c^2 = b^2 - y^2 + (a-y)(a-y)$

$c^2 = b^2 - y^2 + a^2 - 2ay + y^2$

$c^2 = b^2 + a^2 - 2ay.$

CAH

$b(\cos(C)) = \left(\frac{Y}{b}\right)b$

$b\cos(C) = y$ sub-in

$c^2 = b^2 + a^2 - 2a(b\cos(C))$

$c^2 = b^2 + a^2 - 2ab\cos(C)$

QED.