Name _____

A∞Ω
MATH@TD
MCR3U

3U: U3 - Lessons 1 - 3 Review

Multiple Choice
Identify the choice that best completes the statement or answers the question.

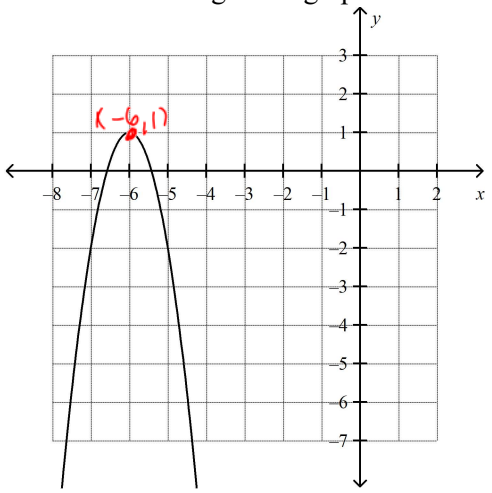
B 1. Which of the following is true about the parabola for the function $f(x) = 3(x - 4)^2 + 5$?

a. The y-intercept is 0.
c. The axis of symmetry is $x = -4$.
b. The vertex is (4, 5).
d. The parabola opens down.

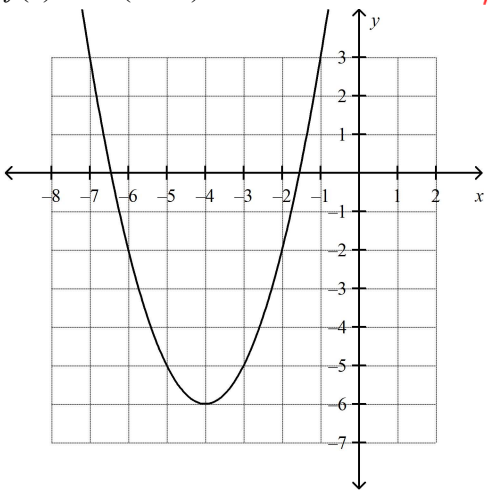
A 2. Which of the following is the graph for the function $f(x) = -3(x + 6)^2 + 1$?

opens down
vertex (−6, 1)

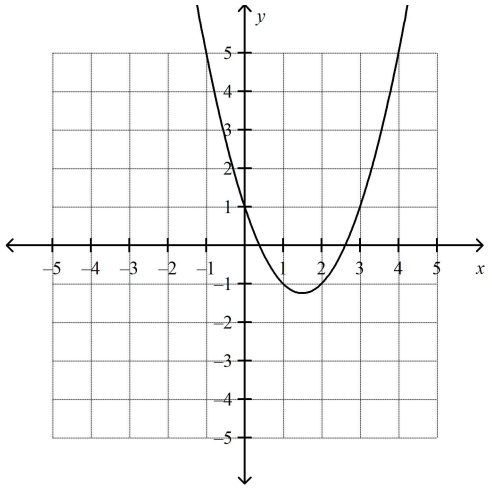
a.



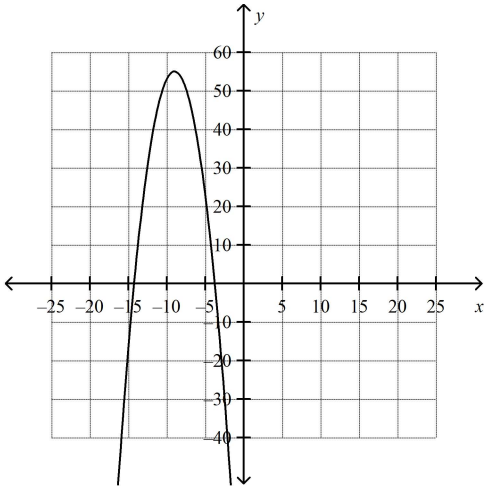
c.



b.



d.



B 3. The Canada Calendar Company reduces the prices of its calendars each January. The function $f(x) = -25(x - 9)^2 + 200$ models the company's profits, where x is the price of a calendar in dollars. At what calendar price will the company receive maximum profits?

a. $5
c. $10
b. $9
d. $15

Vertex (9, 200)
price $9
profit 200 (probably in thousands)

B 4. The points (–1, –11) and (5, –11) are the same distance from the vertex of their parabola. What is the equation for the axis of symmetry of the parabola?

a. $x = -1$ c. $x = -11$

b. $x = 2$ d. $x = 5$

AoS: $\frac{-1+5}{2} = 2$

C 5. Which of the following quadratic functions will have a minimum value? opens up

a. $h(x) = -3(x+4)^2 + 8$ c. $f(x) = 2x^2 + 9x - 13$

b. $g(x) = -5x^2 + 3x + 7$ d. $y = -(x-6)^2 - 15$

C 6. What is the minimum value for the function $f(x) = 4x^2 - 8x + 16$? ← standard form

a. –12 c. 12

b. 1 d. 16

vertex. (y-value)

AoS: $x = \frac{-b}{2a} = \frac{-(-8)}{2(4)} = 1$

∴ vertex is $(1, f(1)) = (1, 12)$ ← min value

C 7. Which of the following expresses $\sqrt{96}$ in simplest form?

a. $6\sqrt{3}$ c. $4\sqrt{6}$

b. $16\sqrt{6}$ d. $8\sqrt{2}$

$\sqrt{96} = \sqrt{16 \times 6} = 4\sqrt{6}$

B 8. Which of the following expresses $\sqrt{8} \times \sqrt{32}$ in simplest form?

a. $2\sqrt{64}$ c. $8\sqrt{2}$

b. 16 d. $4\sqrt{8}$

$= \sqrt{4 \times 2} \times \sqrt{16 \times 2}$

$= 2\sqrt{2} \times 4\sqrt{2} = 8\sqrt{4} = 8(2) = 16$

B 9. Which of the following expresses $16\sqrt{10} + 2\sqrt{10}$ in simplest form?

a. 180 c. $36\sqrt{5}$

b. $18\sqrt{10}$ d. 320

D 10. Which of the following expresses $\left(4 - \sqrt{7}\right)\left(5 + \sqrt{60}\right)$ in simplest form? → using foil.

a. $20 + 40\sqrt{6} - 5\sqrt{7} - 10\sqrt{42}$ c. $-21\sqrt{97}$

b. $20 - 2\sqrt{105}$ d. $20 + 8\sqrt{15} - 5\sqrt{7} - 2\sqrt{105}$

$20 + 4\sqrt{60} - 5\sqrt{7} - \sqrt{420}$

$= 20 + 4\sqrt{4 \times 15} - 5\sqrt{7} - \sqrt{4 \times 105}$

$= 20 + 8\sqrt{15} - 5\sqrt{7} - 2\sqrt{105}$

A 11. Which of the following expresses $8\sqrt{200}$ in simplest form?

a. $80\sqrt{2}$ c. 160

b. $40\sqrt{5}$ d. $18\sqrt{2}$

$= 8\sqrt{100 \times 2} = 8(10)\sqrt{2} = 80\sqrt{2}$

D 12. Which of the following expresses $3\sqrt{7} - 2\sqrt{28}$ in simplest form?

a. –1 c. $-\sqrt{21}$

b. $-5\sqrt{7}$ d. $-\sqrt{7}$

$3\sqrt{7} - 2\sqrt{4 \times 7}$

$= 3\sqrt{7} - 2(2)\sqrt{7}$

$= 3\sqrt{7} - 4\sqrt{7}$

$= -\sqrt{7}$

Provide clear solutions to the following problems: (see next page)

13. A quadratic function has these characteristics:
$x = 1$ is the equation for the axis of symmetry.
$x = -1$ is an x-intercept.
$y = -4$ is the minimum value.
Determine the y-intercept of this parabola. Hint - you need the equation of the quadratic function. Which form (standard, vertex or zeros/factored form) should you use, and why?

14. Does the parabola for the function $f(x) = -(x-7)^2 + 10$ open up or down? What is the range? Explain your answer by drawing a rough sketch may be helpful (plot the vertex and direction of opening - no need for the zeros, y-intercept and its symmetric partner - just a rough sketch)

15. An ice cream company varies the prices of its pint containers to maximize profit. The function $P(x) = -80(x-3)^2 + 150$ models the company's profits in thousands of dollars, where x is the price of a pint of ice cream in dollars. At what price will the company receive **maximum** profits? How much profit will the company earn? (when you see the word maximum (or minimum), bells should go off in your head and you should hear a voice which says "Find the Vertex" - you have many options, and in this case finding the vertex is ridiculously easy).

16. The cost, $c(x)$, in dollars per hour of running a certain fishing boat is modelled by the function $c(x) = 0.9x^2 - 18.1x + 135.1$, where x is the speed in kilometres per hour. At what approximate speed should the boat travel to achieve **minimum** cost? (use your favorite method for finding the vertex - personally I like finding the AoS by using $AoS = \frac{-b}{2a}$)

17. The cost function for a clock factory is $C(x) = 7x + 27$ and the revenue function is $R(x) = -4x^2 + 39x$, where x is the number of clocks sold, in thousands. Determine the profit function for the number of clocks sold. Then determine the number of clocks sold that maximizes profit.

18. Simplify.
$-4\sqrt{51} \times 6\sqrt{3}$

19. Simplify.
$3\sqrt{12} + \sqrt{24} - 2\sqrt{36}$

20. Simplify $\left(7 + \sqrt{50}\right)\left(-9 - \sqrt{32}\right)$.

21. Simplify.
$3\sqrt{2}\left(6\sqrt{6} - \sqrt{10}\right) - 12\sqrt{3}$

22. Simplify $\left(8 - 3\sqrt{6}\right)\left(2\sqrt{14} - 5\right)$.

13. A quadratic function has these characteristics:
$x = 1$ is the equation for the axis of symmetry. $\Rightarrow$ vertex $(1, -4) = (h, k)$
$x = -1$ is an x-intercept. $\Rightarrow (-1, 0)$
$y = -4$ is the minimum value.
Determine the y-intercept of this parabola. Hint - you need the equation of the quadratic function. Which form (standard, vertex or zeros/factored form) should you use, and why?

vertex form: (because we have the vertex)

$f(x) = a(x-h)^2 + k$

$\Rightarrow f(x) = a(x-1)^2 - 4$ $\Rightarrow$ need to find "a"

$\Rightarrow$ use $(-1, 0)$ (x, $f(x)$)

$0 = a(-1-1)^2 - 4$

$\Rightarrow 4 = 4a \Rightarrow a = 1$

$\therefore f(x) = (x-1)^2 - 4$

y-int $\Rightarrow x = 0$ $\quad f(0) = (0-1)^2 - 4$

$= -3$

$\therefore$ y-int is $(0, -3)$

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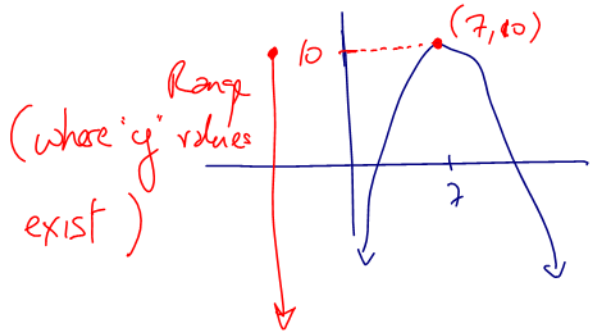
14. Does the parabola for the function $f(x) = -(x-7)^2 + 10$ open up or down? What is the range? Explain your answer by drawing a rough sketch may be helpful (plot the vertex and direction of opening - no need for the zeros, y-intercept and its symmetric partner - just a rough sketch)

$f(x)$ opens down because $a = -1 < 0$ | vertex is $(7, 10)$



Range: the y-values are "10 and below"

$\Rightarrow R: \{f(x) \in \mathbb{R} \mid f(x) \leq 10\}$

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remember - the values in points carry meaning

max! points on $P(x)$ are in this form (price, Profit)

Vertex is $(3, 150)$ $\Rightarrow$ at a price of \$3/pint, the company gets a profit of \$150 000 (profit is in thousands)

16. The cost, $c(x)$, in dollars per hour of running a certain fishing boat is modelled by the function $c(x) = 0.9x^2 - 18.1x + 135.1$, where x is the speed in kilometres per hour. At what approximate speed should the boat travel to achieve **minimum** cost? (use your favorite method for finding the vertex - personally I like finding the AoS by using $AoS = \frac{-b}{2a}$)

points here are of the form (speed, cost)

need the vertex. Recall the vertex is of the form

$(AoS, c(AoS))$ — a number we calculate once we know the AoS.

$\Rightarrow$ to find the vertex we need the AoS

standard form

$c(x) = ax^2 + bx + c$

$$AoS: \ x = \frac{-b}{2a} = \frac{-(-18.1)}{2(0.9)} \doteq 10.1$$

$\therefore$ the vertex is $(10.1, c(10.1))$ $\quad c(10.1) = 0.9(10.1)^2 - 18.1(10.1) + 135.1 \doteq 44.1$

$= (10.1, 44.1)$ $\quad \therefore$ at 10.1 km/hr the boat will have a min cost of \$44.10.

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17. The cost function for a clock factory is $C(x) = 7x + 27$ and the revenue function is $R(x) = -4x^2 + 39x$, where x is the number of clocks sold, in thousands. Determine the profit function for the number of clocks sold. Then determine the number of clocks sold that maximizes profit.

vertex!

Points are of the form (# clocks sold, Profit)

Profit equals Revenue minus Cost

domain value

$\Rightarrow P(x) = R(x) - C(x)$

$= [-4x^2 + 39x] - [7x + 27]$

$\Rightarrow P(x) = -4x^2 + 32x - 27.$

to find the AoS I will use partial factoring:

$P(x) = -4x(x - 8) - 27$

set to zero

$\Rightarrow x = 0, \ x = 8$

$$AoS: \ x = \frac{0 + 8}{2} = 4$$

$\therefore$ vertex is $(4, P(4))$

want this value (! it is in thousands)

$\therefore$ The profit is maximized if 4000 clocks are sold

18. Simplify.
$-4\sqrt{51} \times 6\sqrt{3}$

$= -24\sqrt{153}$

$= -24\sqrt{9 \times 17}$

$= -24(\sqrt{9})(\sqrt{17})$

$= -24(3)\sqrt{17}$

$= -72\sqrt{17}$

19. Simplify.
$3\sqrt{12} + \sqrt{24} - 2\sqrt{36}$

$= 3\sqrt{4 \times 3} + \sqrt{4 \times 6} - 2(6)$

$= 6\sqrt{3} + 2\sqrt{6} - 12$ //

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20. Simplify $\left(7 + \sqrt{50}\right)\left(-9 - \sqrt{32}\right)$.

(I choose to simplify the radicals before using 'FOIL')

$= \left(7 + \sqrt{25 \times 2}\right)\left(-9 - \sqrt{16 \times 2}\right)$

$= \left(7 + 5\sqrt{2}\right)\left(-9 - 4\sqrt{2}\right)$

$\sqrt{2} \times \sqrt{2} = (\sqrt{2})^2 = 2$

$= -63 - 28\sqrt{2} - 45\sqrt{2} - 20(2)$

$= -63 - 73\sqrt{2} - 40$

$= -103 - 73\sqrt{2}$

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21. Simplify.
$3\sqrt{2}\left(6\sqrt{6} - \sqrt{10}\right) - 12\sqrt{3}$

$= 18\sqrt{12} - 3\sqrt{20} - 12\sqrt{3}$

$= 18\sqrt{4 \times 3} - 3\sqrt{4 \times 5} - 12\sqrt{3}$

"like"

$= 36\sqrt{3} - 6\sqrt{5} - 12\sqrt{3}$

$= 24\sqrt{3} - 6\sqrt{5}$

22. Simplify $\left(8 - 3\sqrt{6}\right)\left(2\sqrt{14} - 5\right)$.

$= 16\sqrt{14} - 40 - 6\sqrt{84} + 15\sqrt{6}$

$= 16\sqrt{14} - 40 - 6\sqrt{4 \times 21} + 15\sqrt{6}$

$= 16\sqrt{14} - 40 - 12\sqrt{21} + 15\sqrt{6}$ //