A∞Ω
MATH@TD

SOLUTIONS

MCR3U Chapter 3 - Quadratic Functions Review

vertex form

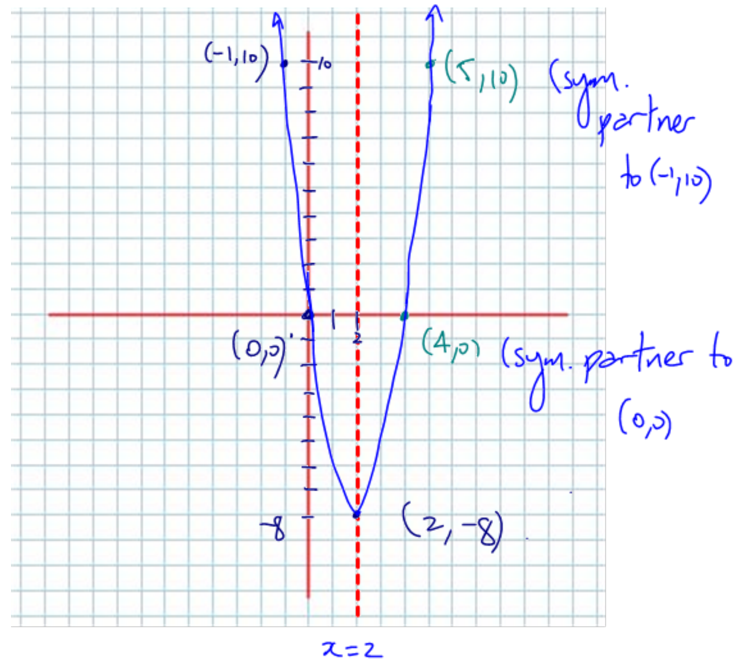
1. Graph the function $f(x) = 2(x-2)^2 - 8$. Label the vertex and axis of symmetry. Determine the y_{int} and its symmetric partner. ~~Determine the point~~ $(-1, f(-1))$ and its symmetric partner.

vertex: $(2, -8)$

AoS: $x = 2$

y_{int}: $x = 0 \Rightarrow f(0) = 2(-2)^2 - 8 = 0$
$\Rightarrow (0,0)$

$(-1, f(-1))$: $f(-1) = 2(-1-2)^2 - 8$
$= 2(-3)^2 - 8$
$= 18 - 8$
$= 10 \Rightarrow (-1, 10)$



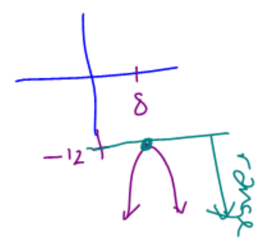
2. Does the parabola for the function $f(x) = -3(x-8)^2 - 12$ open up or down? What is the range? Explain your answer.

vertex $(8, -12)$

$f(x)$ is in vertex form.

'a' $= -3 \Rightarrow$ the parabola opens down.

Basic Sketch



Range of $f(x)$ is $\{f(x) \in \mathbb{R} \mid f(x) \leq -12\}$

3. A quadratic function has these characteristics:
$x = 1$ is the equation for the axis of symmetry. — part of vertex
$x = -1$ is an x-intercept. $\Rightarrow$ point $(-1, 0)$
$y = -4$ is the minimum value. — part of vertex
vertex is $(1, -4)$
Determine the y-intercept of this parabola.

or ZERO!

↳ need to plug $x=0$ into the fn
$\Rightarrow$ we need the fn. Use vertex form because we have the vertex

eqn: $f(x) = a(x-1)^2 - 4$ use $(\overset{x}{-1}, \overset{f(x)}{0})$ to find 'a'

$0 = a(-1-1)^2 - 4$

$0 = a(-2)^2 - 4$

$4 = 4a \Rightarrow a = 1 \quad \therefore f(x) = (x-1)^2 - 4$

y-int: $x = 0 \Rightarrow f(0) = (0-1)^2 - 4$
$= -3$

$\boxed{\therefore \text{ y-int } = -3}$

4. At a baseball game, workers toss T-shirts to spectators in the stands out of a sling-shot. The height of a T-shirt is modelled by the function $h(t) = -5t^2 + 20t + 1$ where $h(t)$ is height in metres and t is the time in seconds after the toss. What is the maximum height of the T-shirt if it is not caught? How much time does it take the T-shirt to reach maximum height?

we want the vertex!

will show 3 methods on the next page.

① Completing the Square

$h(t) = -5t^2 + 20t + 1$

$= -5(t^2 - 4t) + 1$

$\left(\frac{1}{2}(4)\right)^2 = (2)^2 = 4$

$= -5(\underbrace{t^2 - 4t + 4}_{\text{perfect square}} - 4) + 1$

$= -5[(t-2)^2 - 4] + 1$

$= -5(t-2)^2 + 20 + 1$

$= -5(t-2)^2 + 21$

$\therefore$ vertex is $(2, 21)$

max value

$\therefore$ The T-Shirt reaches a max height of 21 m at $t = 2$ sec.

② Partial Factoring

$h(t) = -5t^2 + 20t + 1$

$= \underbrace{-5t(t-4)}_{\text{set to zero}} + 1$

$\Rightarrow t = 0$ or $t = 4$

$\therefore$ AoS: $t = \frac{0+4}{2} = 2$

$\therefore$ The vertex is

$(2, h(2))$

$h(2) = -5(2)^2 + 20(2) + 1 = -20 + 40 + 1 = 21$

$= (2, 21)$

$\therefore$ The T-Shirt reaches a max of 21 m at $t = 2$ seconds

Note: you can also try to find the zeros, using the Quadratic Formula and use the zeros to find the AoS - then calculate the vertex as we do here

③ Using AoS $= -\frac{b}{2a}$

$h(t) = -5t^2 + 20t + 1$

$a = -5 \quad b = 20 \quad c = 1$

$\text{AoS} = -\frac{b}{2a} = \frac{-20}{2(-5)} = +2$

$\therefore$ vertex is $(2, h(2)) = (2, 21)$

$h(2) = -5(2)^2 + 20(2) + 1 = 21$

$\therefore$ 21 m is the max height

5. Determine the maximum value for the function $f(x) = -x^2 - 4x - 32$ by completing the square. (Hint: First factor the -1 out of the first two terms).

I 'could' ask for a specific technique!

$$f(x) = -(x^2 + 4x) - 32$$
$$= -(x^2 + 4x + 4 - 4) - 32$$
$$= -[(x+2)^2 - 4] - 32$$
$$= -(x+2)^2 + 4 - 32$$
$$\Rightarrow f(x) = -(x+2)^2 - 28$$

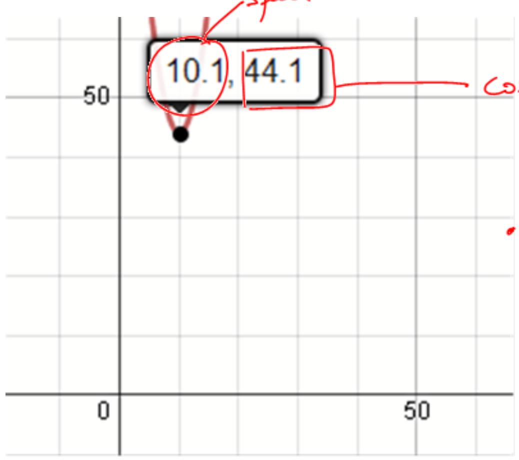
$\therefore$ The max value is -28 at $x = -2$

6. The cost, $c(x)$, in dollars per hour of running a certain fishing boat is modelled by the function $c(x) = 0.9x^2 - 18.1x + 135.1$, where x is the speed in kilometres per hour. At what approximate speed should the boat travel to achieve minimum cost? (You'll need to use graphing tech for this problem).

(speed, cost)

we want the vertex

or the Q.F. IF there are zeros (This one has no zeros)



cost = \$44.10 /hr.

$\therefore$ By Desmos, the min cost is achieved at a speed of 10.1 km/hr.

OR $AoS = -\frac{b}{2a}$

$AoS = -\frac{b}{2a} = \frac{+18.1}{2(0.9)} = 10.1$ $\therefore$ Vertex is $(10.1, c(10.1))$

speed cost

$\therefore$ a min cost is acheived at a speed of 10.1 km/hr

7. The cost function for a container company is $C(x) = 10x + 30$ and the revenue function is $R(x) = -x^2 + 24x$, where x is the number of containers sold, in thousands. Determine the profit function for the number of containers sold. Then determine the number of containers sold that maximizes profit.

need vertex

Profit = Revenue − Cost

$$P(x) = (-x^2 + 24x) - (10x + 30)$$

$$= -x^2 + 14x - 30$$

we can find the vertex in one of 4 ways: complete the square, partial factor to find the AoS, find the zeros and then the AoS, graphing tech, $AoS = -\frac{b}{2a}$

$$AoS = -\frac{b}{2a} = \frac{-14}{2(-1)} = 7.$$

$\therefore$ 7000 containers.

Partial Factoring

$$P(x) = \underbrace{-x(x-14)}_{\text{set to zero}} - 30$$

$\therefore x = 0$ or $x = 14$

$$AoS: \; x = \frac{0+14}{2} = 7$$

$\therefore$ The vertex is $(7, P(7))$

BUT we don't need the functional value (the max value) since we just want the number of containers

$\therefore$ 7000 containers will maximize profit.

vertex!

8. Determine the minimum value for the function $f(x) = 2x^2 + 8x + 6$ by factoring and finding the AoS.

① Always look for common factors 1ST (!!!)$^\infty$

$$f(x) = 2(x^2 + 4x + 3)$$
$$= 2(x+3)(x+1) \Rightarrow \text{zeros } x = -3, -1$$

$AoS = -\frac{b}{2a}$
$= \frac{-8}{2(2)}$
$= -2$!

Same after.

AoS: $x = \frac{-3 + (-1)}{2} = -2$

$\therefore$ vertex is $(-2, f(-2))$

$= (-2, -2)$

$f(-2) = 2(-2)^2 + (-2) + 6$
$= 8 - 16 + 6$
$= -2$

$\therefore$ min value is -2 (at $x = -2$)

9. Simplify. Explain how you found your answer.
$5\sqrt{192}$

$= 5\sqrt{64 \times 3}$
$= 5(8)\sqrt{3}$
$= 40\sqrt{3}$

look for a perfect square factor which divides into 192 evenly.

$192 \div 4 = 48$ still has a square factor of 16!

$48 \div 16 = 3$

$16 \times 4 = 64$

10. **Simplify.**

$-4\sqrt{51} \times 6\sqrt{3}$

Coefficients w/ Coefficients, Radicands w/ Radicands

$= -24\sqrt{153}$ — simplify $\quad 153 = (9)(17)$

$= -24\sqrt{9 \times 17}$ $\quad = (-24)(3)\sqrt{17}$ $\quad \sqrt{9}$!

$= -72\sqrt{17}$

11. **Simplify.**

$3\sqrt{12} + \sqrt{24} - 2\sqrt{36}$

$= 3\sqrt{4 \times 3} + \sqrt{4 \times 6} - 2(6)$

$= 6\sqrt{3} + 2\sqrt{6} - 12$ (no more simplifying can be done!)

12. **Simplify** $\left(7+\sqrt{50}\right)\left(-9-\sqrt{32}\right)$.

FOIL (But - simplify the radicals 1st)

$= \left(7 + \sqrt{25 \times 2}\right)\left(-9 - \sqrt{16 \times 2}\right)$

$= \left(7 + 5\sqrt{2}\right)\left(-9 - 4\sqrt{2}\right)$

$= -63 - 28\sqrt{2} - 45\sqrt{2} - 20\sqrt{4}$

$= -63 - 73\sqrt{2} - 40$

$= -103 - 73\sqrt{2}$

13. Simplify.

$$3\sqrt{2}\left(6\sqrt{6}-\sqrt{10}\right)-12\sqrt{3}$$

$= 18\sqrt{12} - 3\sqrt{20} - 12\sqrt{3}$

$= 18\sqrt{4\times 3} - 3\sqrt{4\times 5} - 12\sqrt{3}$

$= 36\sqrt{3} - 6\sqrt{5} - 12\sqrt{3}$

$= 24\sqrt{3} - 6\sqrt{5}$

14. Determine the roots of the equation $-2x^2+4x+96=0$ by factoring.

$\div -2$ on BOTH sides

$\Rightarrow x^2 - 2x - 48 = 0$

$\times$	$+$
-48	-2

$\Rightarrow -8, +6$

$\Rightarrow (x-8)(x+6)=0$

$\therefore x = 8$ or $x = -6$

15. Use the quadratic formula to determine each of the roots of $12x^2 - 11x + 2 = 0$ to two decimal places.

$a = 12$ $b = -11$ $c = 2$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{11 \pm \sqrt{(-11)^2 - 4(12)(2)}}{2(12)}$

$= \dfrac{11 \pm \sqrt{25}}{24}$

$= \dfrac{11 \pm 5}{24}$

$\therefore x = \dfrac{11+5}{24}$ or $x = \dfrac{11-5}{24}$

$= \dfrac{16}{24}$ $= \dfrac{6}{24}$

$= 0.67$ $= 0.25$

16. Determine the number of zeros for the function $f(x) = x^2 - 3x - 5$. (be discriminating!)

$a = 1 \quad b = -3 \quad c = -5$

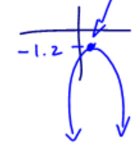
$$b^2 - 4ac = (-3)^2 - 4(1)(-5)$$
$$= 9 + 20$$
$$= 29 > 0 \quad \therefore f(x) \text{ has two zeros.}$$

— opens down.

17. A certain quadratic function has a *maximum* value of -1.2. How many zeros does the function have?

THIS IS A BEAUTIFUL QUESTION – AMEN

Sketch: opens down



a negative max means there are NO zeros

18. Calculate the discriminant for the function $f(x) = x^2 - x + 8$. How many times will the graph of the function intersect the x-axis? Explain your answer.

$a = 1 \quad b = -1 \quad c = 8$

$$b^2 - 4ac = (-1)^2 - 4(1)(8)$$
$$= -31 < 0 \quad \therefore \text{No zeros} \Rightarrow \text{NEVER intersects the } x\text{-axis.}$$

19. For what value(s) of k will the function $h(x) = 4x^2 - kx + 25$ have only one zero? Explain your answer.

$a = 4 \quad b = -k \quad c = 25$ → discriminant is equal to zero.

$$b^2 - 4ac = (-k)^2 - 4(4)(25) = 0$$
$$\Rightarrow k^2 - 400 = 0$$
$$\Rightarrow k^2 = 400 \Rightarrow k = \pm 20.$$

careful!

20. Neal dropped a small stone off a bridge that is 21 m above the water. The height of the stone is given by the function $h(t) = -4.9t^2 + t + 21$, where $h(t)$ is the height in metres and t is the time in seconds. How long will it take for the stone to hit the water?

→ hits water means $h(t) = 0$ ⇒ we need to find the zeros of $h(t)$

$$h(t) = -4.9t^2 + t + 21$$

does not factor

$a = -4.9 \quad b = 1 \quad c = 21$

⇒ Quadratic formula.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{(1)^2 - 4(-4.9)(21)}}{2(-4.9)}$$

$$= \frac{-1 \pm \sqrt{412.6}}{-9.8}$$

$$= \frac{-1 \pm 20.31}{-9.8}$$

$$\therefore t = \frac{19.31}{-9.8} \quad \text{or} \quad t = \frac{-21.31}{-9.8}$$

$$= -1.97 \text{ sec} \quad \text{or} \quad = +2.17 \text{ sec}$$

inadmissible (negative time)

∴ The stone hits the water after 2.17 seconds.

21. Use the quadratic formula to determine each of the roots of $-7x^2 - 23x = -10$ to two decimal places.

not standard form!

$$\Rightarrow -7x^2 - 23x + 10 = 0$$

$a=-7 \quad b=-23 \quad c=10$

$$\therefore x = \frac{23 \pm \sqrt{(-23)^2 - 4(-7)(10)}}{2(-7)} = \frac{23 \pm \sqrt{809}}{-14}$$

$$\therefore x = \frac{23+\sqrt{809}}{-14} \quad \text{OR} \quad x = \frac{23-\sqrt{809}}{-14}$$

$$= -3.67 \qquad\qquad = +0.39$$

22. Tasha is trying to toss a ball over a wall that is 9.4 m high. The height of the ball is modelled by the function $h(t) = -4.9t^2 + 10t + 1.7$ where $h(t)$ represents the height in metres and t models the time in seconds. Will Tasha be able to toss the ball over the wall? Use the discriminant to explain your answer.

Picture

will $h(t)$ be able to reach 9.4 m?

ie) $9.4 = -4.9t^2 + 10t + 1.7$? (standard form!)

$$\Rightarrow 4.9t^2 - 10t + 7.7 = 0$$

$a=4.9 \quad b=-10 \quad c=7.7$

$$b^2 - 4ac = (-10)^2 - 4(4.9)(7.7)$$

$$= -50.92$$

$\therefore 4.9t^2 - 10t + 7.7 = 0$ has no zeros

$\Rightarrow$ <u>NO</u> Tasha can't throw the ball over the wall.

point $(0,1)$ at $t=0$ she initially throws the ball.

vertex!

23. Jacqueline threw a ball from 1 m above the ground. The ball reached a maximum height of 46 m at 3 seconds. Determine the equation that will model the parabola for this situation.

vertex is $(3, 46)$ use vertex form

$$h(t) = a(t-3)^2 + 46$$

use $(0, 1)$ to find "a" ($t = 0$, $h = 1$)

$$1 = a(0-3)^2 + 46$$

$$-45 = 9a \Rightarrow a = -5$$

$$\therefore h(t) = -5(t-3)^2 + 46$$

you should be able to get the eqn of a parabola given a sketch

Look at the given info!

If you are given zeros ⟹ use zeros form

" " " " vertex ⟹ " vertex "

Use the "other point" to find "a"!