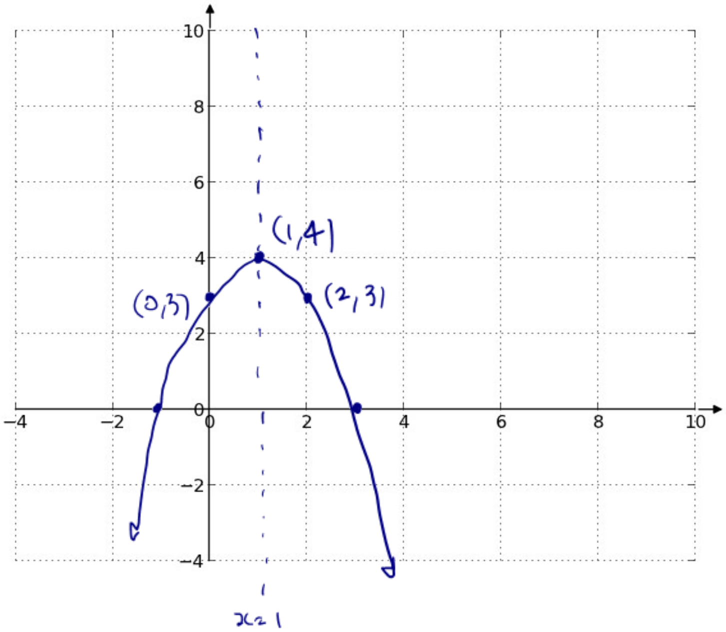
Name: Solutions

Unit 3 Practice Test – Quadratic Functions

K ___/10   T____/9   C ___/7   A ____/15

1. a) Graph $f(x) = -(x+1)(x-3)$ *(Also draw the AoS on your graph).* **K/10**

$f(1) = -(1+1)(1-3) = -(2)(-2) = +4$



b) Determine:

AoS is $x = \frac{-1+3}{2} = 1$.

Vertex is (1 , 4).

Vertex is (AoS, f(AoS))
= (1, f(1))
= (1, 4)

y_{int} = (0 , 3).

x=0! ⇒ $f(0) = -(0+1)(0-3) = -(-3) = +3$

Symmetric Partner of y_{int}:

(2 , 3). (one over from AoS)

Zeros are: (−1 , 0) and

(3 , 0).

c) This parabola opens down. The vertex contains a max (max or min?) value.

(a = −1)

d) Re-write the function in standard form: $f(x) = -x^2 + 2x + 3$

$f(x) = -(x+1)(x-3) = -(x^2 - 3x + x - 3) = -(x^2 - 2x - 3) = -x^2 + 2x + 3$

expand

e) Re-write the function in vertex form: $f(x) = -(x-1)^2 + 4$

same "a"   "h"   "k"

vertex form

$f(x) = a(x-h)^2 + k$.

$(h, k) = (1, 4)$

→ Another way to get vertex $\boxed{AoS: x = \frac{-b}{2a}} = \frac{-20}{2(-5)} = \frac{-20}{-10} = \boxed{+2}$ ← easiest way to find the AoS

2. At a Toronto Maple Leafs game workers shoot T-shirts to the fans using a sling shot. The height of a T-shirt is modelled by the function $h(t) = -5t^2 + 20t + 1$ where $h(t)$ is in metres above the ice rink, and t is in seconds after the T-shirt is shot. Determine the maximum height a T-shirt could reach, and how long it would take to reach that maximum height. **T/3**

VERTEX! → find using one of: a) zeros b) partial factoring ← find AoS

Using Partial Factoring.

$h(t) = -5t(t-4) + 1$ — set to zero ⇒ $t=0$ or $t=4$

→ AoS: $x = \frac{0+4}{2} = 2$

AoS! / y-value of AoS (the max or min value)

$\therefore$ vertex $(2, h(2)) = (2, 21)$ $\therefore$ The max height is 21 m at 2 seconds

$(h(2) = -5(2)^2 + 20(2) + 1 = 21)$

3. Write the equation for each function in any form you choose. (Show your work) **T/4**

a) Equation: $f(x) =$ $a(x+3)(x-2)$

$f(x)$ has zeros at $x = -3$ and at $x = 2$.

$f(x)$ also passes through $P(1,8)$ = $P(x, f(x))$

Need to find 'a' – use $P(1,8)$

$\Rightarrow 8 = a(1+3)(1-2)$

$\Rightarrow 8 = a(4)(-1)$

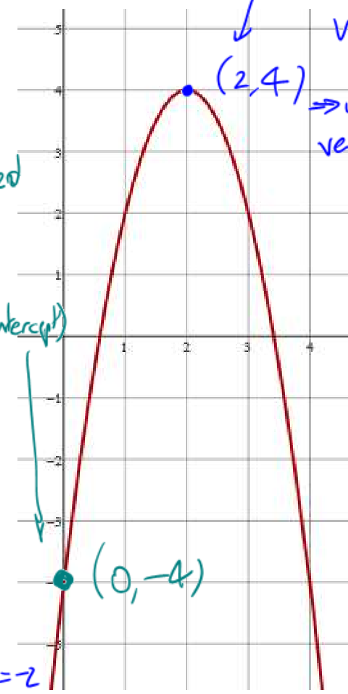
$\Rightarrow 8 = -4a$

$\Rightarrow a = \frac{8}{-4} = -2$

$\therefore f(x) = -2(x+3)(x-2)$

b) Equation: $g(x) =$ $a(x-h)^2 + k$ ← have the vertex $(2,4)$ ⇒ use vertex form

$g(x)$



$\Rightarrow g(x) = a(x-2)^2 + 4$

to find 'a' we need a point on $g(x)$ (easiest is often the y-intercept)

Using $(0,-4)$ $(x, g(x))$ find a

$-4 = a(0-2)^2 + 4$

$\Rightarrow -4 = a(4) + 4$

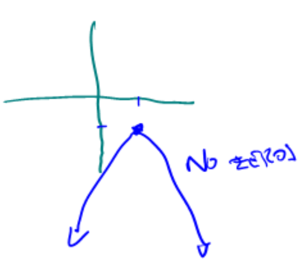
$-8 = 4a \Rightarrow a = \frac{-8}{4} = -2$

$\therefore g(x) = -2(x-2)^2 + 4$

4. Given the quadratic function $g(x) = -\frac{2}{3}(x-3)^2 - 2$ ***state*** how many zeros $g(x)$ has, with a reason. Note: you may not use the quadratic formula nor the discriminant. **T/2**

Discriminant or max/min.

Sketch



$g(x)$ opens down ⇒ vertex contains a max!

vertex is $(3, -2)$

$\therefore$ max value is -2 ⇒ A negative max means No zeros.

5. Simplify: **A/4**

$$3\sqrt{2}\left(4\sqrt{6}-\sqrt{10}\right)-\left(5\sqrt{3}+3\sqrt{20}\right)$$

Simplify 1ST

$3\sqrt{20} = 3\sqrt{4\times5}$
$= 3\sqrt{4}\times\sqrt{5}$
$= 3(2)\times 5$
$= 6\sqrt{5}$

$3\sqrt{2}\left(4\sqrt{6}-\sqrt{10}\right)-\left(5\sqrt{3}+6\sqrt{5}\right)$

this negative distributes!

$= 12\sqrt{12} - 3\sqrt{20} - 5\sqrt{3} - 6\sqrt{5}$

$= 12\sqrt{4\times3} - 3\sqrt{4\times5} - 5\sqrt{3} - 6\sqrt{5}$

$= 24\sqrt{3} - 6\sqrt{5} - 5\sqrt{3} - 6\sqrt{5}$

$= 19\sqrt{3} - 12\sqrt{5}$

6. The height, $h(t)$, of a baseball, in metres, at time t seconds after it is tossed out of a window is modelled by the function $h(t)=-5t^2+20t+15$.

A nefarious young girl shoots at the baseball with a paintball gun. The trajectory of the paintball is given by the function $g(t)=3t+3$.

Will the paintball hit the baseball? If so, when? At what height will the baseball be? **A/5**

This is a quadratic - linear system

$\Longrightarrow -5t^2+20t+15 = 3t+3$

$\Longrightarrow -5t^2+17t+12 = 0$

find the zeros

"stuff = 0!"

factor or quadratic formula

×	+
−60	+17

20, −3

$\Rightarrow (-5t-3)(t-4)=0$

$t = -\frac{3}{5}$ OR $t = 4$

$= -0.6$ inadmissible (can't have negative time)

The baseball gets hit at $t=4$ seconds

The height is $g(4) = 3(4)+3$
$= 15\,m.$

used the "linear" eqn, but you could use the quadratic (both give height!)

using the quadratic height is $h(4) = -5(4)^2 + 20(4) + 15$

$= (-5)(16) + 80 + 15 = -80 + 80 + 15 = 15\,m.$

7. Using algebraic techniques determine the zeros of the functions (if necessary leave answers in radical form. But, if you must use decimals, round to two decimal places):

a) $f(x) = 2x^2 + x + 5$ b) $g(x) = 2(x-3)^2 - 20$ **A/3,3**

QF

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(5)}}{2(2)}$$

$$= \frac{-1 \pm \sqrt{-39}}{4}$$ no sol'ns

To get zeros $\Rightarrow g(x) = 0$

$\Rightarrow 2(x-3)^2 - 20 = 0$ (+20, +20)

$\Rightarrow 2(x-3)^2 = 20$ (÷2, ÷2)

$\Rightarrow (x-3)^2 = 10$ square root

square root

$\Rightarrow x - 3 = \pm\sqrt{10}$

don't forget "±"!

$\therefore x = +\sqrt{10} + 3 \doteq 6.16$

OR

$x = -\sqrt{10} + 3 \doteq -0.16$

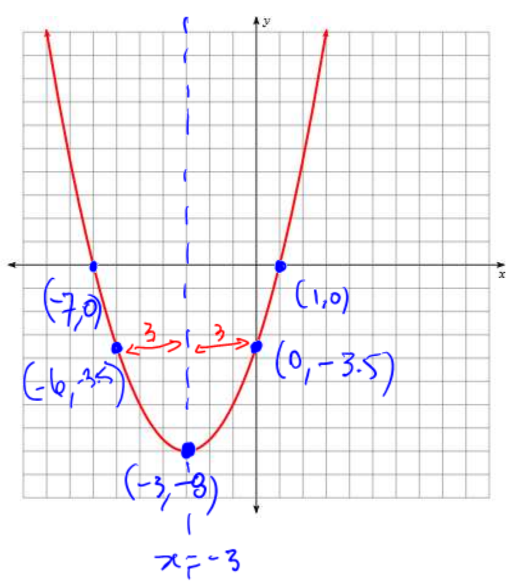
9. Explain why a vertex is NOT considered a maximum or a minimum. **C/2**

A vertex CONTAINS the max or min.

A vertex is of the form (h, k) "k" is the max/min.

"h" is the domain value where the max/min occurs

10. Tell me **everything** you can about the given parabola. You may use words and/or numbers in your description. In fact, label ***anything of importance***. **C/5**



This parabola opens up

There are two zeros: $x = 1$, $x = -7$

The vertex is $(-3, -8)$

The min value is -8 at $x = -3$.

The AoS is $x = -3$

The y-int: $(0, -3.5)$

symmetric partner of y-int $(-6, -3.5)$