Main Things to Remember for Exponential Functions

(1) The exponent rules (especially related to "sameness") — same base or same exponent

(1b) A **power** is a "base to an exponent" eg 3^x — power. base 3, exponent x

(2) ALL Exponential Functions have a Horizontal Asymptote

eg $f(x) = 3\left(2^{(x-5)}\right) + 1$ — horizontal shift; vertical stretch; vertical shift is the Horizontal asymptote : $y = 1$

(2b) Sketching exponentials can be tricky - at most one simple sketch

eg Sketch $f(x) = 3 \cdot 2^{x-1} + 1$ (3: V, −1: H, +1: V)

parent $g(x) = 2^x$

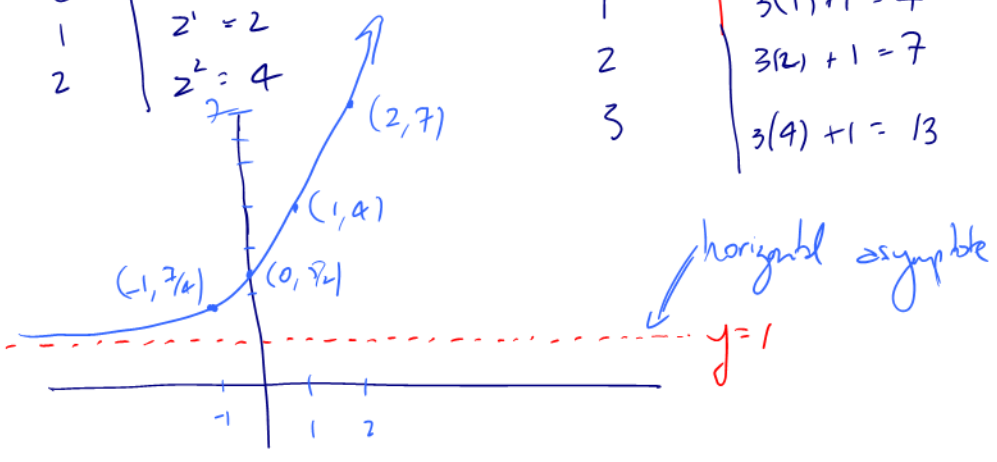
ToV

parent

x_p	$g = 2^x$
−2	$2^{-2} = \frac{1}{4}$
−1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$

transformed

$x_t = x_p + 1$	$f = 3 \cdot g + 1$
−1	$3(\frac{1}{4}) + 1 = \frac{7}{4} = 1.75$
0	$3(\frac{1}{2}) + 1 = \frac{5}{2}$
1	$3(1) + 1 = 4$
2	$3(2) + 1 = 7$
3	$3(4) + 1 = 13$



③ Exponential Functions can describe growth or decay.

Growth: "base" is bigger than 1 eg $f(x) = 2^x$, $g(x) = 1.5^x$

Decay: "base" is between 0 and 1 eg $h(t) = 0.5^t$, $f(x) = \left(\frac{1}{3}\right)^x$

General Functions/Equations

initial amount/population

growth rate

Growth: $P(t) = P_0(1+r)^t$

decay

Decay $A(t) = A_0(1-r)^t$

initial amount

decay rate

④ We can use logarithms to solve for (or find) an unknown exponent.

a base with an exponent

Remember: ① Get the "power" done! (using SAMDEB)

② Take the "log" of both sides of your equation

③ Bring down the exponent

④ Solve for the unknown on your calculator

eg Solve for x

$$3 \cdot 2^{x-1} + 7^{-7} = 8^{-7}$$

subtract 7^{-7} from both sides

$$\Rightarrow 3 \cdot 2^{x-1} = 1$$

$\div 3$ both sides

$$\frac{3 \cdot 2^{x-1}}{3} = \frac{1}{3}$$

power is done!

$$\Rightarrow 2^{x-1} = \frac{1}{3}$$

$$\Rightarrow \log\left(2^{(x-1)}\right) = \log\left(\frac{1}{3}\right)$$

bring the "exponent down"

$$\Rightarrow (x-1)\cdot \log(2) = \log\left(\frac{1}{3}\right)$$

divide both sides by "$\log(2)$"
⇒ we want x - alone.

usually just "x"

$$\Rightarrow x-1 = \frac{\log\left(\frac{1}{3}\right)}{\log(2)}$$ - calculator.

but be prepared for a more complicated exponent

$$\Rightarrow x-1 = -1.58$$

$$\therefore x = -1.58 + 1$$

$$= -0.58$$

(see next page for an example "using power rules"

#4 from "Extra Review Problems"

Simplify: only positive exponents please.

$$\left(\frac{(4x^6)^3(4y^{-9})}{(2x)^4(12y^3)^2}\right)^{\frac{1}{2}}$$

$$= \left(\frac{(4^3x^{18})(4y^{-9})}{(2^4x^4)(12^2y^6)}\right)^{\frac{1}{2}}$$

$$= \left(\frac{4^4x^{18-4}y^{-9-6}}{(2^4)(12)^2}\right)^{\frac{1}{2}}$$

$$= \left(\frac{4^4x^{14}y^{-15}}{(2^4)(12^2)}\right)^{\frac{1}{2}}$$

$$= \frac{(4)^2x^7y^{-7}}{(2^2)(12)}$$

$$= \frac{16x^7}{48y^7}$$

$$= \frac{x^7}{3y^7}.$$